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ELEMENTS OF ELECTRICITY

BY

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Cambridge, Mass.

THIRD EDITION

Eighteenth Printing

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PREFACE TO THE THIRD EDITION

In putting out a third edition of this text the author has endeavored to bring up to date the electrical and magnetic data and theories and their applications to modern electrical machinery and devices. The major changes are in the discussion of electronics and vacuum tubes. It is in this part of the electrical field that the greatest advances in industrial applications have been made.

Throughout, the author has had the able assistance of Dr. Samuel H. Caldwell, Assistant Professor of Electrical Engineering at Massachusetts Institute of Technology, and to him the author expresses his deep appreciation and thanks for the valuable assistance rendered.

The plan followed in the former editions of presenting facts and theories by simple direct statements has been retained. New theories are approached through familiar illustrations and applied to the solution of the common practical problems of modern industry.

The author believes that the experience of teachers using the former editions has demonstrated that this text has distinctly the four following desirable qualities:

1. It contains an adequate amount of information concerning electrical laws and practice in an immediately usable form.

2. It applies the information contained to real things and not to abstract theory and conditions.

3. It provides sufficient drill in concrete practical problems, which not only afford in themselves an additional amount of profitable information, but also develop in the student a capacity for applying what he has learned; the large number of diagrams aiding materially in providing precise and clean-cut conceptions of the conditions of the problems.

4. It presents, out of the great mass of electrical phenomena only those facts and principles which a technical student needs to know, and to know well. History and general theory, interesting as they are, have been omitted.

The text is confined to what is believed to be an adequate treatment of a *few* fundamental ideas rather than to a discursive treat-

ment of *many*. Therefore it includes only the elementary principles of Direct- and Alternating-Current generation, distribution and utilization in light, power and communication. The object is to afford a sure foundation on which to build more advanced courses, and yet to present a practical and well-rounded course for the man who is to receive no further electrical education.

W. H. TIMBIE

CAMBRIDGE, MASS., *November*, 1936.

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ELEMENTS OF ELECTRICITY

CHAPTER I

OHM'S LAW

Electrical science deals primarily with the production, transmission, control, and utilization of electric power. While in certain cases, the electricity we deal with is practically at rest and is called static electricity, we are usually concerned with electricity in motion; that is, with electric currents.

1. The Flow of Electricity. We are all more or less familiar with the flow of water through pipes and the simple laws governing this flow. Accordingly, we can help to make real our knowledge of the flow of electric currents along conductors by calling to mind the many points of resemblance to the flow of water. Whether we are considering the flow of water or of electricity we always have to deal with three things:

(*a*) **Current** (flow of electricity, usually along a conductor).
(*b*) **Pressure** (that which causes the current to flow).
(*c*) **Resistance** (that which regulates the flow of current).

2. Current. The Ampere. A current of water in a pipe is measured by the amount of water which flows through the pipe in a unit time of one second: as, 1 gal. per sec., 8 gal. per sec., etc. Likewise, a current of electricity is measured by the amount of electricity that flows in a wire in one second: as, 1 coulomb per sec., 8 coulombs per sec., etc. The coulomb is merely a quantity of electricity, just as the gallon is a quantity of liquid.

We see that current is always measured as a **rate of flow** and its magnitude is expressed as the quantity which flows in one second. Fortunately we have a special name for the **rate of flow** of one coulomb per second, which is **one ampere.** This name for the **rate of flow** makes it unnecessary to say **per second** in referring to current, because the name **ampere** always means quantity **per second.** Thus 8 coulombs per sec. is just 8 amperes; 25 coulombs per sec. **is 25 amperes,** etc.

Since we are generally interested in the **rate of flow** of electricity rather than the **quantity**, the coulomb is rarely used except as a means of defining some other unit.

We have no special name for a unit rate of flow of water, so we always have to use a cumbersome expression like "gallons per second" or "cubic feet per second."

3. Pressure. The Volt. The rate of flow of water through a pipe depends, to a large extent, on the **pressure** under which it flows. This water pressure is measured in pounds per square inch. Likewise, the number of amperes (coulombs per second) of electric current flowing in a wire depends in part on the **pressure** under which the electricity flows. The unit of electrical **pressure** is the **volt.** A **volt** means the same thing in speaking of electric current that a **pound-per-square-inch** pressure does in speaking of a current of water. Just as a higher pressure is required to force the same current of water through a small pipe than through a large pipe, so a higher electrical pressure is required to force the same current of electricity through a small wire than through a large wire of the same material. The **voltage** (pressure) between two points in an electric circuit is sometimes spoken of as the **difference in potential**, or the **drop in potential** or merely the "drop," between those two points. In the same way, the pressure between two points in a stream of water (as the top and bottom of a dam), is often spoken of as the **difference in level** or the **drop in level,** or merely the "drop," between those two points.

The distinction between **amperes** and **volts** should now be clear. The **amperes** represent the amount of electricity per second which flows in a circuit; the **volts** represent the pressure causing it to flow.

4. Resistance. The Ohm. In the case of both water and electricity, there may be a great pressure and yet be no current. If the path of the water is blocked by a valve turned the wrong way, there will be no current, yet there may be a high pressure. If the path of the electricity is blocked by a switch thrown the wrong way, there will be no current (amperes), though the pressure (volts) may be high.

There is, therefore, **something in addition** to the pressure, that determines the amount of the current, both of water and of electricity. This **something** is the **resistance** of the pipes and valves in the case of water, and the **resistance** of the wires and various devices in the case of electricity. The greater this resistance, the less the current under the same **pressure.**

There is no name given the unit to measure the **resistance** which a pipe offers to the flow of water. The electrical unit of **resistance** is the **ohm.**

We say a wire has **one ohm resistance** when a pressure of **one volt** forces a current of **one ampere** through it. If the resistance were 2 ohms, or twice as great, the current would be only half as large: that is, one-half ampere would flow.

5. Comparison of Electrical and Hydraulic Units.

	Electrical Units	Hydraulic Units
Quantity	Coulomb	Gallon
Current	Ampere = 1 coulomb per second	Gallon per sec.
Pressure	Volt	Pound per sq. in.
Resistance	Ohm	

6. Diagrams of Electric Circuits. In order that the diagrams used throughout the text may give no trouble, a few of the symbols in common use are shown in this and the following section.

To represent a battery cell, we use the symbol |▮, the longer line representing the positive (+) terminal, or the one from which current flows **out** of the cell.

An electric current always flows from a higher to a lower pressure level except through a generator or battery cell, just as water always flows from a higher to a lower pressure level except through a pump. In drawing diagrams, we mark the higher level + (positive), and the lower level − (negative) to denote in what direction the current will flow. Sometimes arrowheads are put on the wires to indicate the directions of flow, as in Fig. 1. Outside of a generator or of a battery, the current always flows from the positive (+) to the negative (−). A given point is then + to all points below its level and − to all points above its level.

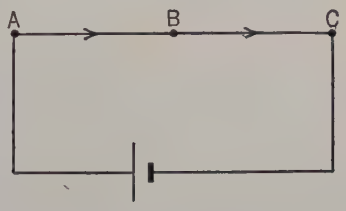


Fig. 1. A simple electric circuit. The point *B* is positive with respect to *C* but negative with respect to *A*.

Thus in Fig. 1, the current flow is from *A* to *C*. If we are considering points *A* and *B*, then *B* is negative (−) **with respect to *A*,**

since the flow is toward B. But if we are considering points B and C, the current flows away from B toward C. The point B is hence positive (+) **with respect to** C.

7. Use of Conventional Symbols. In the photograph of Fig. 2 is shown a direct-current generator supplying current to a group of incandescent lamps and a motor. The same circuit is repre-

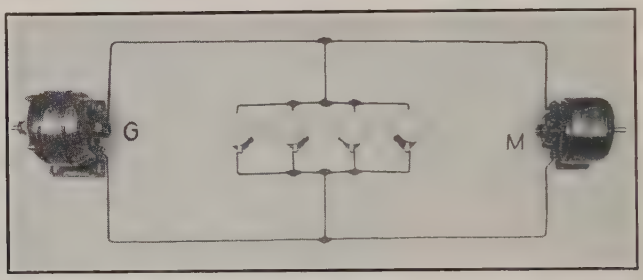


FIG. 2. A generator supplying a motor and a group of lamps.

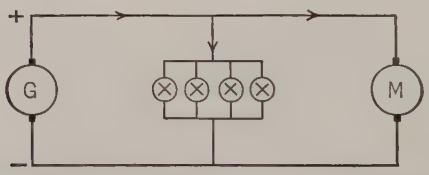


FIG. 3. The conventional diagram of the arrangement of Fig. 2. The arrowheads indicate that the current outside the generator flows from + to −.

sented in Fig. 3 by conventional symbols. Note the saving in time and labor which results when symbols are used to represent the vari-

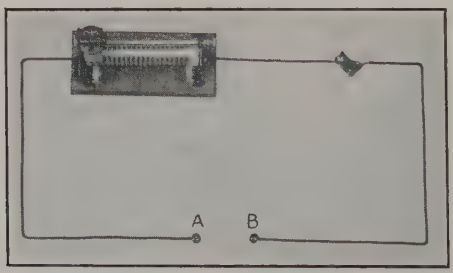


FIG. 4. An electric circuit consisting of an incandescent lamp and resistance. The current enters at A and leaves at B.

ous devices in the circuit. In drawing more complicated circuits these symbols are even more useful in gaining clearness and simplicity.

Sometimes the generator is omitted in the diagram and only the terminals of the power source are shown, as *A* and *B* in Fig. 4. Figure 5 is the conventional diagram for this arrangement. The current enters at the positive terminal *A*, goes through the re-

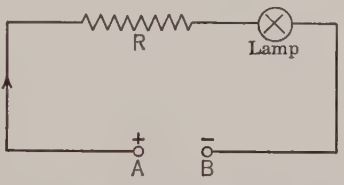
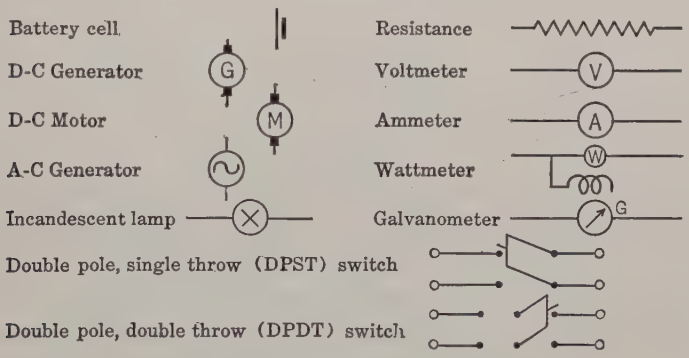


FIG. 5. The conventional diagram of the circuit in Fig. 4.

sistance *R*, then the lamp, and finally leaves at the negative terminal *B*.

The following table shows many of the symbols commonly used in modern representations of electric circuits.



8. Pressure, the Essential Factor. When we desire to have water piped to a house, the one thing above all others which we must have, in order to secure any flow at all, is **pressure.** The water company agrees to furnish this water pressure. The flow of water into your house will then depend entirely upon your wishes. You have the desired pressure always at hand and can allow it to force a great quantity of water through your pipes or only a small amount. You pay for the amount of water that you allow this pressure to force through your pipes. If it had been necessary to maintain **twice** as great a pressure in order to force this same amount of water through your pipes, the company might charge you **twice** as much for the water used. In other words, they would charge you for the **work done** in pumping so much water into your house. At a given

pressure, this amount of **work done** would be proportional to the amount of water which you allowed to flow into your house.

In the same way, if you desire to have electricity brought to your house, the one thing above all others that is necessary, in order to get a flow of electricity, is that an electric pressure be maintained. The electric company agrees to maintain this electric pressure. The flow of the current into your house depends entirely upon your wishes. The pressure is always at hand. You may allow it to cause a large current to flow to the house or only a small current. You pay for the amount of electricity you use, at that pressure. If the electric company maintained twice as much pressure and you used the same amount of electricity, you would have to pay twice as much for this quantity. In other words, they would charge you for the **work done** in forcing so much electricity into your house. At a given pressure, the amount of work done would be proportional to the amount of electricity which you allowed to flow into your house.

The starting point of all hydraulic and electric propositions is thus the **pressure.** There can be no current either of water or electricity without pressure. The amount of work done in a stated time by a given current depends upon **pressure.** It is necessary, then, to study this feature of **pressure** a little more in detail.

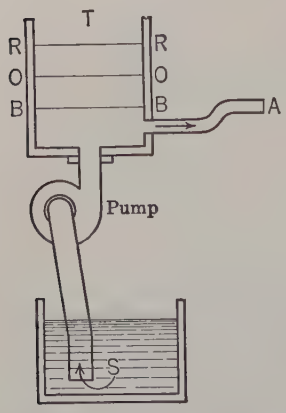


Fig. 6. The pump raises water to the tank T. When the level in the tank is higher than the point A, the water flows out at A.

9. Potential. In order to exert a pressure tending to cause water to flow toward a given point, it is merely necessary to raise the water to a potential or a level **above** that point. That is, we give it potential energy with regard to that point, by the raising of it to a point which has a higher level or higher **potential** than that point. Thus if we wish to cause water to flow out of tap A (Fig. 6), we may connect it to a tank T. As long as the water in the tank is kept above the level B, the water will flow from T to A, because it is at a higher level or potential in T than at A. This **difference** in potential, then, between two points is what causes a pressure between these two points. The greater this difference of potential the greater the

pressure. When the **difference** of **potential** between the level of the water in T and A is only OB, the **pressure is less** than when the difference of potential is RB.

10. Electrical Potential. Similarly, if we wish to cause a current of electricity to flow from one point to another, we must raise the potential of the first point above that of the second. Then a pressure is set up proportional to their **difference** in potential. This difference of potential tends to send a current from the higher potential to the lower, as in the case of water.

A battery or generator may then be thought of as a pump which pumps the electricity up from the lower level to the higher and keeps one side of the line at a higher potential than the other, thus setting up a pressure between these two points. Accordingly, the electric-power company runs two wires to your house and simply agrees to keep the difference in the potential between them up to a certain value; providing, of course, you do not allow so much electricity to flow that the generator is unable to pump fast enough to keep up this difference of potential. In such a case it would be just as if the water in Fig. 6 were allowed to flow out at A so fast that the pump could not keep the level in the tank above the line ABB.

In referring to Fig. 1, we said that the point A was at a higher level than B, and B higher than C. We may now say that A is at a higher potential than B, and B than C. A current therefore flows from A to B and from B to C, due to these differences in potential. The battery acts as a pump to keep the left-hand side continually at a higher potential than the right-hand side. The difference of potential between A and B is sometimes spoken of as the **fall** of potential from A to B, or as the **drop** in potential from A to B. The same applies to B and C, or any other two points. The difference in potential between two points in an electric circuit is called the **drop** in potential for that part of the circuit contained between those two points, and is the cause of any current flowing between these two points.

In Fig. 4 and 5 the terminal A is attached to the high-potential side of the generator, since it is marked $(+)$ and B is attached to the low-potential side of the generator, since it is marked $(-)$. The current therefore flows from A through R, then through the lamp to B, and back to the low-potential side of the generator. The generator must continually raise electricity up to the high-potential side, to make up for that which flows away from that side to the low-potential side. If no electricity is allowed to flow away

from A to B, then the generator has to raise no more electricity up to the potential of A. It merely has to keep the pressure up.

11. Fall of Potential along a Uniform Wire. In Fig. 7, the generator G is feeding a lamp bank situated 500 feet away. This necessitates two feed wires AF and HJ, each 500 feet long. Let us assume these wires to be uniform in cross-section. The current goes out from the generator along the top wire from A to F and back along the lower wire from J to H.

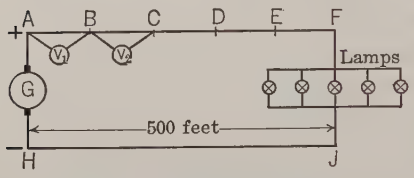


FIG. 7. The voltmeter V_1 shows the voltage from A to B. Voltmeter V_2 shows the voltage from B to C.

Let us mark the upper wire into 100-foot lengths and measure the difference of potential between any two points with a voltmeter. The voltmeter V_1 measures the difference in potential or voltage from A to B, and the voltmeter V_2 the difference in potential from B to C. Similarly, voltmeters might be connected to measure the difference of potential between C and D, and D and E, and E and F. If this were done, we would find all the voltmeters to indicate the same. This shows that the potential drop across equal distances along a uniform wire is the same.

Similarly, the drop across AC (twice the length of AB) would be just twice the drop across AB, and the drop across AF (five times the length of AB) would be just five times the drop across AB. The drop is thus proportional to the length. Since the resistance of a wire of uniform cross-section is proportional to its length, we may say:

The drop in potential across any part of an electric circuit is proportional to the resistance of that part, if the same current flows through each part.

Example 1. The voltage across 6 ohms resistance is 20 volts. What will be the drop across 30 ohms, if the same current is flowing through each resistance?

Solution.

Let V = drop across the 30 ohms.

Since the drops are proportional to the resistances, the current being the same,

$$\frac{V}{20} = \frac{30}{6}, \text{ hence } V = 100 \text{ volts.}$$

Prob. 1–1. If the drop along 650 ft. of the line wire in a certain circuit is 0.030 volt, what is the drop per mile?

Prob. 2–1. The drop across 60 ft. of wire is 0.08 volt. The drop across an unknown length of the same wire when carrying the same current is 14.4 volts. How great is the unknown length?

Prob. 3–1. Two resistances A and B are so connected that the same current flows in each. The resistance of A is 0.35 ohm, B is unknown. The drop across A is 0.48 volt, across B is 4.62 volts. What is the resistance of B?

12. Ohm's Law. Current. We have seen that if a certain current of electricity flows in a circuit, it flows because a certain pressure forces it to flow, and that the amount of the current is limited by the resistance of the circuit.

In fact, the amount of current depends upon the amount of the pressure and of the resistance. This fact was discovered by Georg Simon Ohm and is expressed by Ohm's Law, which is a fundamental equation of electrical science.

Ohm's Law states that:

The current in an electric circuit equals the quotient of the pressure divided by the resistance.

This may be expressed as follows:

$$\text{Current} = \frac{\text{Pressure}}{\text{Resistance}},$$

or

$$\text{Amperes} = \frac{\text{Volts}}{\text{Ohms}}.$$

In symbols $$I = \frac{E}{R}.$$

Where I = current in amperes.
E = pressure in volts.
R = resistance in ohms.*

Example 2. How much current flows through a lamp which has 16 ohms resistance when a pressure of 80 volts is applied to its terminals?

Solution.

$$\text{Amperes} = \frac{\text{volts}}{\text{ohms}},$$

*I stands for Intensity of current in amperes.
E stands for Electromotive force in volts.
R stands for Resistance in ohms.

or
$$I = \frac{E}{R}$$
$$= \frac{80}{16} = 5 \text{ amperes.}$$

Prob. 4–1. What current can 45 volts force through 6 ohms?

Prob. 5–1. A dry cell has a terminal voltage of 1.2 volts when a wire of 0.2 ohm resistance is connected across its terminals. What current flows in the wire?

Prob. 6–1. A certain tungsten lamp has a resistance of 420 ohms when heated to incandescence. What steady current flows through the lamp if it operates at incandescence on a 115-volt circuit?

13. Measurement of Current. Ammeter. If we wish to measure the current of water which is flowing through a pipe, we insert a flowmeter into the pipe as (*A*) in Fig. 8. The water flowing through the pipe also flows through the flowmeter and causes it to indicate the number of gallons per second which pass through it. We cannot attach an instrument to the outside of the pipe and find the current through the inside. We must open up the pipe and insert the instrument into the line so that the current which we wish to measure flows directly through the instrument.

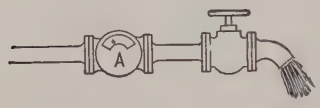


Fig. 8. The water meter *A* measures the flow of water through the pipe.

In the same way, if we wish to measure the current of electricity which is flowing through an electric circuit we insert a current meter into the circuit so that all the current which we wish to measure flows through the meter. Since an instrument which measures an electric current must read in amperes, such a current meter is called an **ammeter** (a contraction of the term ampere-meter). Figure 9 represents an ammeter *A* inserted to measure the current flowing through an incandescent lamp. Note that all the current which goes through the lamp must go through the ammeter. The ammeter then must be of very low resistance in order not to hinder the current. Such an instrument is very delicate and must be handled carefully. Sufficient practice in the use of this instrument should be gained by the student before proceeding with other work.

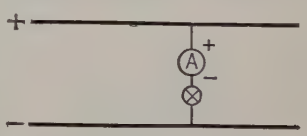


Fig. 9. The ammeter *A* measures the flow of electricity through the lamp.

Note that the terminals of the ammeter are connected so that the current enters the meter at the positive (+) and leaves at the negative (−) terminal.

14. Ohm's Law. Voltage. Sometimes it is desired to find the pressure which will be required to force a certain current through a certain resistance. In order to do this Ohm's Law may be used in the following form:

$$\text{Pressure} = \text{current} \times \text{resistance},$$

or

$$\text{Volts} = \text{amperes} \times \text{ohms}.$$

In symbols

$$E = IR.$$

Example 3. The hot resistance of an incandescent carbon lamp is 220 ohms. It requires $\frac{1}{2}$ ampere to cause it to glow. What voltage must be impressed across it?

Solution. $\text{Voltage} = \text{amperes} \times \text{ohms},$

or $E = IR$

$\text{Voltage} = \frac{1}{2} \times 220 = 110 \text{ volts}.$

Prob. 7–1. What voltage will produce a current of 6 amperes through a resistance of 18 ohms?

Prob. 8–1. What pressure is needed for an incandescent lamp of 80 ohms resistance, through which must flow a current of 1.2 amperes?

Prob. 9–1. What pressure will produce a current of 0.118 ampere through a resistance of 1000 ohms?

Prob. 10–1. An electromagnet has a resistance of 62.5 ohms. It will lift a certain piece of iron when the current through it is 3.8 amperes, but will not drop the load until the current is reduced to 2.5 amperes. What voltage is required (*a*) to lift, and (*b*) to release the load?

Prob. 11–1. The resistance of a telephone receiver is 180 ohms. The current required is 0.003 ampere. What voltage must be impressed across the receiver?

Prob. 12–1. A miniature incandescent lamp requires 0.4 ampere to make it glow. The hot resistance is 32 ohms. What pressure is required?

15. Measurement of Pressure. Voltmeter. When we wish to measure the water pressure in a pipe, we tap a pressure gauge to the pipe line, as *G* in Fig. 10. Note that no current flows through the gauge. It is merely tapped to the pipe at the point at which we wish to measure the pressure so that the pressure can get

at it and cause it to indicate. The pipe and the current flowing through the pipe are not disturbed. Similarly, when we wish to measure the electric pressure causing an electric current to flow through, say a lamp, we do not disturb the circuit nor the current flowing through it. We just **tap** the terminals of a **voltmeter** to the line, as in Fig. 11. We want to measure the pressure forcing the current through the lamp, so we tap the voltmeter leads across the terminals of the lamp. Note that the current which flows through the lamp does not go through the voltmeter. The voltmeter is not made to register current but pressure; there is no need for the current to go through it, but it must be placed so that the pressure across the lamp is also across the voltmeter, the (+) side of the voltmeter being connected to the (+) side of the lamp. This pressure causes it to indicate the **volts** across the lamp and not the current through the lamp.

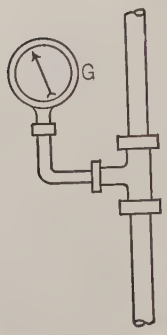


FIG. 10. The pressure gauge is attached to the pipe to measure the pressure within the pipe.

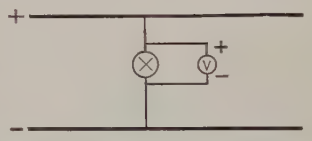


FIG. 11. The voltmeter V measures the electric pressure across the lamp.

Note that the method of connecting a voltmeter is quite different from the method of connecting an ammeter. An ammeter is **inserted into** the circuit, and becomes a part of the circuit, and carries the full current of the circuit. A voltmeter is **merely tapped on** to the circuit, and does not become a part of the circuit, nor does it carry the current which is flowing through the circuit. Great care must be used **not to tap on** an ammeter by mistake. An ammeter **tapped on** to a line may be ruined by the excessive current.

16. Ohm's Law. Resistance. At times we wish to limit the current in a piece of apparatus to a certain number of amperes. If the pressure is known, it then becomes necessary to compute the resistance which will allow just this desired amount of current to flow, when the pressure is applied. For this purpose we can use Ohm's Law in still another form:

$$\text{Resistance} = \frac{\text{pressure}}{\text{current}},$$

or

$$\text{Ohms} = \frac{\text{volts}}{\text{amperes}}.$$

In symbols

$$R = \frac{E}{I},$$

Example 4. What resistance must an electric car heater have if it is to be used on 550 volts and is to take 4 amperes?

Solution.

$$\text{Resistance} = \frac{\text{pressure}}{\text{current}},$$

or

$$R = \frac{E}{I}.$$

$$\text{Resistance} = \frac{550}{4} = 137.5 \text{ ohms.}$$

Prob. 13–1. An incandescent lamp uses 0.62 ampere on a 110-volt circuit. What is the resistance of the lamp when burning?

Prob. 14–1. Through what resistance will 180 volts force 16 amperes?

Prob. 15–1. An electric soldering iron takes 1.05 amperes when used on a 110-volt circuit. What is the resistance of the iron?

Prob. 16–1. What resistance must a 115-volt incandescent lamp have in order to take $\frac{1}{4}$ ampere?

Prob. 17–1. The voltage across a certain piece of apparatus is 90 volts. The current is found to be 6.2 amperes. What is the resistance?

17. Measurement of Resistance. Voltmeter and Ammeter Method. It can be seen from the above discussion that when we wish to find the resistance of an electrical piece we have merely to measure the current which a known voltage is forcing through it and divide the voltage by the current. Thus, to find the resistance of the lamp (L) in Fig. 12, an ammeter is inserted in the circuit with it and a voltmeter is tapped on across it. The voltmeter reading is divided by the ammeter reading and the result is the resistance of the lamp. This is one of the simplest methods of obtaining the resistance of any electrical appliance when it is in use on a circuit.

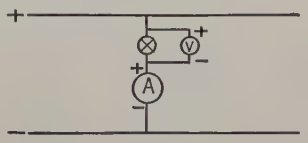


FIG. 12. The voltmeter V measures the pressure across the lamp and the ammeter A the current through it.

Prob. 18–1. In Fig. 13 a pressure of 110 volts is maintained between the points A and B. An electrical measuring instrument marked (X) is connected across A and B.

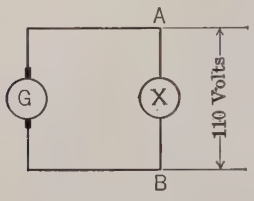


FIG. 13. The instrument X is subjected to a pressure of 110 volts.

(a) If the instrument X is a voltmeter of 150,000 ohms resistance, how much current will flow through it?

(b) If the instrument X were an ammeter of 0.005 ohm resistance, how much current would flow through it? Assume that the pressure of 110 volts is maintained across the points A and B. From this result it can be seen why an ammeter should never be tapped on across a circuit.

18. Ohm's Law in Its Three Forms. Ohm's Law states the relations which exist among the three electrical quantities, Current, Pressure and Resistance. By means of this law we can find any one of the three quantities if we know the other two.

Thus, we use the law in the three forms given below. The student will save himself much work if he will learn the law thoroughly in its **three** forms.

To find the **Current:**

$$I = \frac{E}{R}$$

$$\text{Amperes} = \frac{\text{volts}}{\text{ohms}}.$$

To find the **Pressure:**

$$E = IR$$

$$\text{Volts} = \text{amperes} \times \text{ohms}.$$

To find the **Resistance:**

$$R = \frac{E}{I}$$

$$\text{Ohms} = \frac{\text{volts}}{\text{amperes}}.$$

Prob. 19–1. What current is produced through a resistance of 3.5 ohms by an electromotive force of 60 volts?

Prob. 20–1. Through what resistance will an electromotive force of 40 volts force a current of 16 amperes?

Prob. 21–1. What voltage will produce a current of 11 amperes through a resistance of 24 ohms?

Prob. 22–1. What electromotive force will produce a current of 0.009 ampere through a resistance of 2500 ohms?

Prob. 23–1. Through what resistance will 900 volts produce 49.5 amperes?

Prob. 24–1. What current is produced by 30 volts acting across 0.082 ohm?

Prob. 25–1. A relay having a resistance of 3200 ohms requires 0.02 ampere to operate. At what voltage will the relay close its contacts?

19. Ohm's Law in Circuits. Ohm's Law can be applied to an electric circuit as a whole, or it can be applied to any part of it. Once the principle is firmly grasped, the student is able to attack intelligently a wide range of electrical problems. It is lack of thoroughness in studying the principle that accounts for many of the mistakes which are made in using it.

Many difficulties will be cleared up if in his work the student will keep in mind the **two** following statements of the law.

When applying the law to an **entire** circuit, state the law as follows:

(1) **The current in the entire circuit equals the voltage across the entire circuit divided by the resistance of the entire circuit.**

Notice that the word **entire** applies to **current, voltage,** and **resistance** alike. Not to **one** only, or to **two** only, but to all **three** factors of the equation.

When applying the law to but a **part** of the circuit, state the law as follows:

(2) **The current in a certain part of a circuit equals the voltage across that part, divided by the resistance of that part.**

Notice here again that the values for the **three** factors, **current, voltage,** and **resistance,** are all taken from the **same part** of the circuit. By far the greatest number of mistakes in applying Ohm's Law comes from dividing the voltage across **one part** of a circuit by the resistance of **another part** and expecting to find the current in some part or other.

A few examples will make the correct use plain.

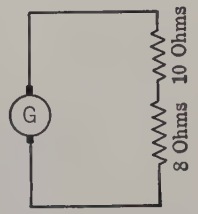


FIG. 14. A generator G is sending current through two resistances.

Example 5. The generator G, Fig. 14, has a resistance of 2 ohms and generates 150 volts pressure. What current flows through a circuit which has two resistances placed in series with the generator, if one resistance is 8 ohms and the other 10 ohms?

Solution. Since the pressure generated by the generator is the **voltage** of the **entire** system, the **resistance** of the **entire** system must be used in the equation:

$$\text{Current (in entire system)} = \frac{\text{voltage (across entire system)}}{\text{resistance (of entire system)}}.$$

Voltage (across entire system) = 150 volts;
Resistance (of entire system) = 2 + 8 + 10 = 20 ohms;

$$\text{Current (in entire system)} = \frac{150}{20} = 7.5 \text{ amperes.}$$

That is, $I = \frac{E}{R}$, applied to the **entire** circuit. Having found the current flowing in the circuit, it is now possible to compute the voltage necessary to force it through the different parts by applying Ohm's Law to each part separately.

Suppose it is desired to find the voltage across the 8-ohm resistance alone.

Voltage (across 8 ohms) =
current (in 8 ohms) × resistance (of 8 ohms).
Current (through 8 ohms) = 7.5 amperes.
Resistance (of 8 ohms) = 8 ohms.
Voltage (across 8 ohms) = 7.5 × 8 = 60 volts.

That is,

$E = IR$ applied to one part of circuit only.

Across the 10-ohm resistance, in the same way,

$$E = IR = 7.5 \times 10 = 75 \text{ volts.}$$

Across the resistance of generator,

$$E = 7.5 \times 2 = 15 \text{ volts.}$$

Total voltage = 60 + 75 + 15 = 150 volts, which result checks with the value given as the voltage generated.

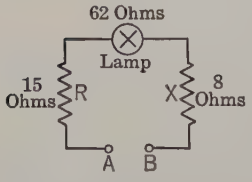


FIG. 15. A resistance of 8 ohms and one of 15 ohms are in series with a lamp across AB.

Prob. 26–1. In the circuit of Fig. 15, 2.7 amperes are flowing through each piece.

(*a*) What is the voltage across AB?

(*b*) What is the voltage across each piece?

Prob. 27–1. If the voltage across AB in Fig. 15, were 95 volts:

(*a*) What current would flow through the circuit?

(*b*) What would be the voltage across R?

Prob. 28–1. If the voltage across the resistance X of Fig. 15, were 27 volts what would be the voltage across AB?

20. Brush Potential or Terminal Voltage. We may say then, that in the above example 75 volts of the 150 are used to force the

current through the 10 ohms resistance of Fig. 14; 60 volts are used to force it through the 8 ohms resistance, and the remaining 15 volts are used to force it through the generator itself.

The pressure used to force the current through the two resistances of 8 ohms and 10 ohms would be 75 volts plus the 60 volts, or 135 volts. Since this 135 volts is used entirely in forcing the current from one brush through the **outside** circuit and back to the other brush, it is called the **brush potential** or **terminal voltage** of the generator for the given current. A voltmeter placed across brushes of the generator would indicate the **terminal voltage,** since that is all the voltage available for forcing a given current from brush to brush, through the external resistance. This **brush potential** of a generator must always be distinguished carefully from the total voltage generated by the generator, which is called the generated voltage of the generator. This is also called the electromotive force of the generator and is generally abbreviated to **e.m.f.**

Thus in the above example,

the e.m.f. of the generator = 150 volts;
the terminal voltage of the generator = 135 volts.

Example 6. The heater H, Fig. 16, requires 5 amperes to operate it properly and has a resistance of 16 ohms. There is a resistance A of 4 ohms in series with H.

(*a*) What voltage is required to operate the heater?

(*b*) What e.m.f. must a generator of 2 ohms internal resistance develop in order to force 5 amperes through the heater and the resistance A of 4 ohms?

(*c*) What is voltage across A alone?

(*d*) What is the brush potential of the generator?

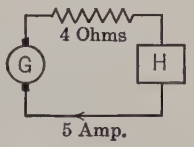


FIG. 16. The generator G supplies current to a heater H in series with a 4-ohm resistance.

Solution. (*a*) Voltage across heater.

$I = 5$ amperes (current through heater);
$R = 16$ ohms (resistance of heater);
$E = IR = 5 \times 16 = 80$ volts (pressure across heater).

(*b*) E.m.f. generated by generator G (that is, of course, voltage of **entire** circuit).

$I = 5$ amperes (current in **entire** circuit);
$R = 16 + 4 + 2 = 22$ ohms (resistance of **entire** circuit);
$E = IR = 5 \times 22 = 110$ volts (e.m.f. generated in **entire** circuit).

(*c*) Voltage across A.

$I = 5$ amperes (current in A);
$R = 4$ ohms (resistance of A);
$E = IR = 5 \times 4 = 20$ volts (voltage across A).

(*d*) Brush potential. Brush potential is voltage across all that part of the circuit which is outside of the generator.

$I = 5$ amperes (current through circuit outside of generator);
$R = 16 + 4 = 20$ ohms (resistance outside of generator);
$E = IR = 20 \times 5 = 100$ volts (pressure outside of generator or **brush potential**).

Notice that in each case, no matter what the problem, Ohm's Law was used, care being taken that the resistance, current and voltage used in each equation were in just that part of the circuit being considered at the time.

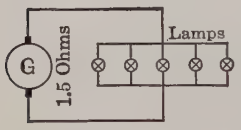


FIG. 17. The generator G with a 1.5-ohm internal resistance feeds the group of five lamps.

Prob. 29–1. In Fig. 17, the generator is supplying 4.5 amperes to the lamp bank. If the generated voltage is 117 volts, what is the terminal voltage of the generator?

Prob. 30–1. A certain storage battery has an internal resistance of 0.01 ohm and an e.m.f. of 2.00 volts. What terminal voltage will it have when delivering 48 amperes?

Prob. 31–1. When a certain dry cell is delivering 1.8 amperes it has a terminal voltage of 1.05 volts. If the internal resistance is 0.25 ohm, what is the e.m.f. of the cell at the time?

21. Series and Parallel Circuits. There are two ways of connecting two or more pieces of electrical apparatus together.

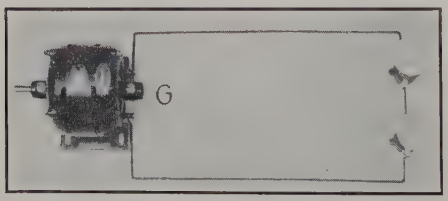


FIG. 18. The two lamps are in series.

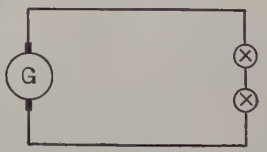


FIG. 19. A conventional diagram of the circuit in Fig. 18.

(1) **Series.** When the pieces are connected in **tandem** they are said to be in **series.** The lamps in Fig. 18, are in series. Figure 19 is a conventional diagram of this series circuit.

(2) **Parallel.** When the pieces are connected side by side so that the current is divided among them, they are said to be in **parallel** with one another. **Multiple** and **shunt** are other names for this

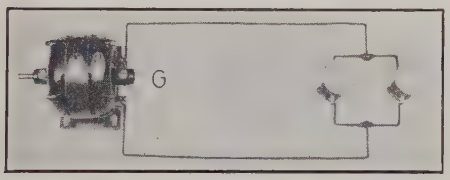


FIG. 20. The two lamps are in parallel.

FIG. 21. A conventional diagram of the circuit in Fig. 20.

combination. The lamps in Fig. 20 are in **parallel** with each other. Figure 21 is a conventional diagram of this parallel circuit.

The two combinations may exist in the same circuit, as in Fig. 22, where the parallel combination C and D is in series with the series combination A and B.

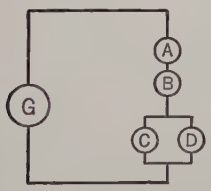


FIG. 22. The parallel combination of C and D is in series with the series combination of A and B.

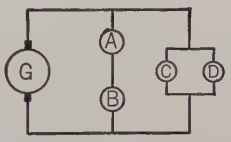


FIG. 23. The parallel combination of C and D is in parallel with the series combination of A and B.

Also as in Fig. 23, where the parallel combination C and D is in parallel with the series combination A and B.

22. Series Circuit — Resistance, Voltage, and Current Relations.

(1) **Resistance.** The resistance of a series combination equals the sum of the resistances of the separate parts. So in Fig. 22 and 23, the resistance of the series combination of lamps A and B equals the resistance of lamp A plus the resistance of lamp B.

(2) **Voltage.** The voltage, also, of a series combination equals the sum of the voltages across the separate resistances. Thus the voltage across both lamps A and B, Fig. 22 and 23, equals the voltage across A plus the voltage across B.

(3) **Current.** The current in every part of a series circuit is the same. If we join four pipes A, B, C and D, Fig. 24, of unequal di-

ameters in series, and force a current of water through them, the water cannot go in at M in any greater quantity per second than it comes out at N. There must be the same current (gallons per second) flowing through each pipe no matter what its size, because no more or less can go through one than goes through all the others.

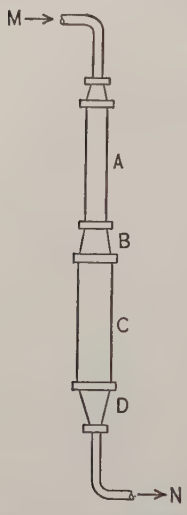


FIG. 24. The pipes A, B, C and D are in series.

Similarly, we may join in series several electrical pieces, as in Fig. 25, a resistance coil (R) of 10 ohms, a telephone receiver of 90 ohms, and an incandescent lamp of 200 ohms. Although the resistance of the coil R is much lower than that of the receiver or the lamp, still no greater current can flow through the coil than can get through the receiver and lamp. No more electric current can enter at one end of an electric circuit than can get out at the other end, any more than can a greater current of water enter at one end of a pipe line than can go out at the other.

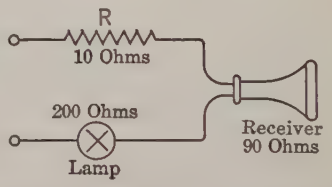


FIG. 25. The 10-ohm resistance is in series with the lamp and the telephone receiver.

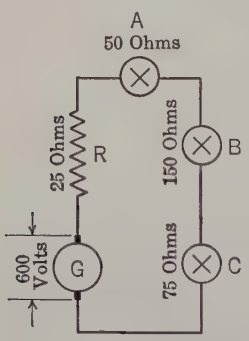


FIG. 26. The lamps A, B, and C are in series with the 25-ohm resistance.

Example 7. Series combination (Fig. 26):

Resistance A equals 50 ohms.
" B " 150 "
" C " 75 "
" R " 25 "

Voltage across the combination equals 600 volts.

Find:

(1) Combined resistance of A, B, C, R.
(2) Current through each.
(3) Voltage across each.

Solution.

(1) **Resistance** of combination:

$$50 + 150 + 75 + 25 = 300 \text{ ohms.}$$

(2) **Current** through each:

Apply Ohm's Law to the combination:

$$I = \frac{E}{R};$$

$$\text{Current (through combination)} = \frac{\text{Voltage (across combination)}}{\text{Resistance (of combination)}}.$$

$$I = \frac{600}{300} = 2 \text{ amperes.}$$

There are, therefore, 2 amperes flowing through each part of the combination.

(3) **Voltage** across each.

Apply Ohm's Law to each part of the combination successively, to lamp A, for instance:

$$E = IR.$$

That is,

Voltage (across A) = **current** (through A) × **resistance** (of A).

$E_A = 2 \times 50 = 100$ volts. (Voltage across A).
$E_B = 2 \times 150 = 300$ volts. (Voltage across B).
$E_C = 2 \times 75 = 150$ volts. (Voltage across C).
$E_R = 2 \times 25 = 50$ volts. (Voltage across R).
E (sum) $= 600$ volts. (Voltage across the combination).

Prob. 32–1. Three resistors of 6 ohms, 15 ohms and 25 ohms, respectively, are joined in series across a 110-volt line.

(*a*) What current flows through the series circuit?
(*b*) What is the voltage across each resistor?

Prob. 33–1. In Fig. 27 a lamp of 50 ohms, a resistor of 40 ohms and another lamp of 30 ohms are connected in series across the mains A and B. The 50-ohm lamp has a pressure of 120 volts across it.

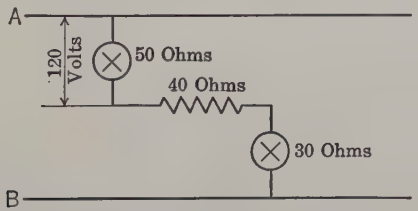


FIG. 27. The series arrangement of the two lamps and the 40-ohm resistance is across the line AB.

Find:

(*a*) Current through each piece.
(*b*) Voltage across each piece.
(*c*) Voltage across AB.

Prob. 34–1. Six lamps are joined in series; each has a resistance of 15 ohms. If the line wires have a total resistance of 4.5 ohms, how much voltage is required to send 6.5 amperes current through the lamps and line?

Prob. 35–1. (*a*) What is the voltage across each lamp in Prob. 34–1?

(*b*) What voltage is used to send the current through the line?

Prob. 36–1. (*a*) An electric heater of 21 ohms resistance is supplied over lines of 0.6 ohm resistance from a generator having an internal resistance of 0.5 ohm. If the heater requires a current of 5 amperes, what e.m.f. must the generator produce?

(*b*) What is the terminal voltage of the generator?

(*c*) What current would the heater carry if it were connected directly across the generator terminals?

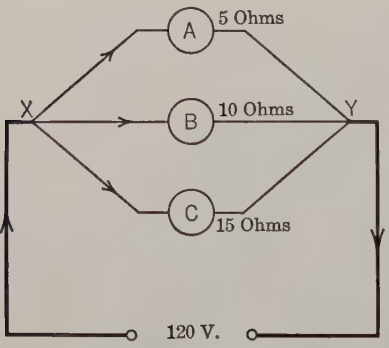


FIG. 28. The lamps A, B, and C are in parallel across a 120-volt circuit.

23. Parallel Circuits. Voltage, Current, and Resistance Relations.

(1) **Voltage.** The voltage across each branch of a parallel circuit is the same as the voltage across the combination. Consider Fig. 28. The voltage between the points X and Y is 120 volts, and the voltage across any piece of apparatus attached across X and Y must be 120 volts. Since the pieces A, B and C are each attached across X and Y, there must be 120 volts across A, across B and across C.

(2) **Current.** The total current in a parallel combination equals the sum of the currents in all the branches.

In Fig. 28, by Ohm's Law,

$$\text{the current through } A = \frac{\text{voltage across } A}{\text{resistance of } A} = \frac{120}{5} = 24 \text{ amps.}$$

$$\text{the current through } B = \frac{\text{voltage across } B}{\text{resistance of } B} = \frac{120}{10} = 12 \text{ amps.}$$

$$\text{the current through } C = \frac{\text{voltage across } C}{\text{resistance of } C} = \frac{120}{15} = 8 \text{ amps.}$$

Total current through the parallel combination = 44 amps.

(3) **Resistance.** The resistance of a parallel combination can be found by Ohm's Law.

In Fig. 28,

$$\text{Resistance (of combination)} = \frac{\text{voltage (across combination)}}{\text{current (through combination)}}$$

$$= \frac{120}{44} = 2.73 \text{ ohms.}$$

At first sight it may seem strange that the resistance of a combination of three pieces of 5, 10 and 15 ohms should be only 2.73 ohms. But the apparent difficulty disappears when we consider that the more paths we have in parallel for the current to flow along from one point to another, the more opportunity there is for electricity to flow, and the lower must be the resistance between those two points. Thus, if there had been only the 5-ohm path between the points X and Y then the resistance between these points would have been 5 ohms. But when another path of 10 ohms was placed between the same points X and Y, more current could flow and thus the resistance between the points became less than 5 ohms. And when a third path of 15 ohms resistance was added, the resistance became still smaller. Thus we may see that the resistance of any parallel combination is less than the resistance of the path of smallest resistance. The path having the smallest resistance in this case is the 5-ohm path, and the combined resistance of the three parallel paths amounts to but 2.73 ohms.

In case the voltage across the parallel combination is not given, one volt or any convenient voltage may be assumed. In case one volt is assumed, the procedure would be as follows in the above example.

Current per volt through $A = \frac{1}{5} = 0.20$ ampere per volt.
Current per volt through $B = \frac{1}{10} = 0.10$ ampere per volt.
Current per volt through $C = \frac{1}{15} = 0.0667$ ampere per volt.
Current per volt through combination $= 0.3667$ ampere per volt.

$$\text{Resistance of combination} = \frac{\text{volts across combination}}{\text{current through combination}}$$

$$= \frac{1}{0.3667} = 2.73 \text{ ohms.}$$

Note that the "**current per volt**" is in each case merely the reciprocal of the resistance. For this reason it is often called the **Conductance.*** If the resistance may be said to represent the **difficulty** with which an electric current is forced through a circuit, the conductance may be said to represent the **ease** with which an electric current may be forced through the same circuit.

* The name **mho** has been given to the unit of conductance. Thus a 5-ohm resistance would have $\frac{1}{5}$ or 0.2 mho conductance.

In terms of the conductance, the rule for finding the resistance of a parallel circuit is:

Add the **conductances** of the separate branches, which gives the **conductance** of the combination. Invert the **conductance** of the combination to get the **resistance** of the combination.

Thus the resistance of a parallel combination is often said to be the "**reciprocal of the sum of the reciprocals of the separate resistances.**"

Example 8. Resistors of 2 ohms, 3 ohms and 4 ohms are joined in parallel. What is the resistance of the combination?

Solution.

Current (per volt) through 2 ohms = $\frac{1}{2}$ = 0.5 ampere.
Current (per volt) through 3 ohms = $\frac{1}{3}$ = 0.333 ampere.
Current (per volt) through 4 ohms = $\frac{1}{4}$ = 0.25 ampere.
Combination = sum = 1.083 amperes.

$$\text{Resistance (of combination)} = \frac{\text{voltage (across combination)}}{\text{current (through combination)}}$$

$$= \frac{1}{1.083} = 0.923 \text{ ohm.}$$

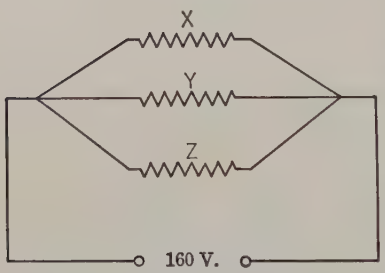


FIG. 29. The parallel arrangement of the resistances X, Y, and Z is across a 160-volt circuit.

Prob. 37–1. In Fig. 29, the resistance of X = 25 ohms; the resistance of Y = 50 ohms; the resistance of Z = 90 ohms. Find:

(*a*) Voltage across each branch.

(*b*) Current through each branch.

(*c*) Current through the combination.

(*d*) Resistance of parallel combination.

Prob. 38–1. A divided circuit has two branches of 2 ohms and 1 ohm respectively. What is the joint conductance of the two branches? What is the joint resistance?

Prob. 39–1. A circuit has three branches of 6, 10 and 14 ohms respectively. If 4 amperes flow in the circuit containing 14 ohms, what current will flow in each of the others?

Prob. 40–1. What pressure will be required to force 6 amperes through a parallel combination consisting of 14 ohms, 5 ohms and 8 ohms?

Prob. 41–1. Three lamps of equal resistance are placed in parallel across a 112-volt circuit, and the current taken by the combination is 3.6 amperes. What is the resistance of each lamp?

24. Simple Parallel Lighting Systems. Incandescent lamps are usually installed in parallel for interior lighting. The resistance of all the lamps of any make with the same rating is not exactly the same. Nor is the voltage across all the lamps when installed the same. Still for convenience in calculating the "line drop," efficiency of transmission, etc., each lamp is assumed to take the same current. The error introduced by this assumption is usually too small to be taken into account.

A general problem may be stated as follows:

Example 9. A lighting circuit is arranged with two groups of lamps as in Fig. 30. One group has two lamps, the other, three.

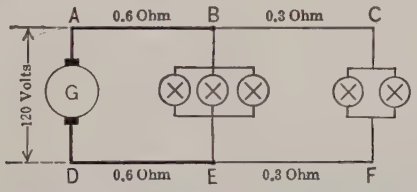


FIG. 30. Two groups of lamps attached at different places to a constant-voltage circuit.

Each lamp takes 2 amperes. The generator has a terminal voltage of 120 volts.

Resistance of line wires AB and DE = 0.60 ohm each.
" " BC and EF = 0.30 ohm each.

Find:

(*a*) Volts lost in line wires.
(*b*) Voltage across each group of lamps.

Solution.

First. Find the current distribution throughout circuit, assuming that each lamp takes 2 amperes.

Current through BC = 4 amperes
" " EF = 4 "
" " AB = 10 "
" " DE = 10 "

Second. By Ohm's Law, solve for the "drop" in line wire.

Drop (in AB) = current (through AB) × resistance (of AB).
= 0.6 × 10 = 6 volts.
Drop (in DE) = current (through DE) × resistance (of DE).
= 0.6 × 10 = 6 volts.

Total volts used in line between generator and first group of lamps then equals,

$$6 + 6 = 12 \text{ volts.}$$

Generator voltage − voltage lost in lead wires = voltage at lamps (BE).

$$120 - 12 = 108 \text{ volts across } BE.$$

Volts (lost in BC) = current (through BC) × resistance (of BC);
$= 4 \times 0.3 = 1.2$;
Volts (lost in EF) $= 4 \times 0.3 = 1.2$;

Total volts lost between two groups of lamps $= 1.2 + 1.2$
$= 2.4$ volts.
Voltage at first group − volts lost in connecting wires
= voltage at second group.

$$108 - 2.4 = 105.6 \text{ volts across } CF.$$

Thus:

The voltage across first group = 108 volts.
The voltage across second group = 105.6 volts.

The above simple method by which this general problem was solved, can be used in solving any problem in direct-current light and power distribution.

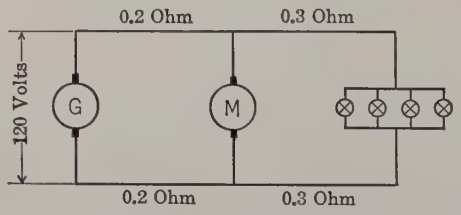


Fig. 31. The motor M and the group of lamps are attached to a constant-voltage circuit.

Prob. 42–1. Each lamp, Fig. 31, takes 2.2 amperes. Motor M takes 12 amperes. Find:

(*a*) Line drop in each section (both wires) of the line.
(*b*) Voltage across lamps.
(*c*) Voltage across motor.

Prob. 43–1. What is the resistance of each lamp in Prob. 42?

Prob. 44–1. Assume the voltage across the generator in Fig. 32 to be 600 volts.

Resistance of trolley between generator and car I = 0.6 ohm.
" " car I " II = 1.1 ohm.
" " car II " III = 0.35 ohm.

Resistance of track between generator and car I = 0.05 ohm.
" " car I " II = 0.1 ohm.
" " car II " III = 0.04 ohm.

Car I takes 50 amperes.
Car II takes 65 amperes.
Car III takes 42 amperes.

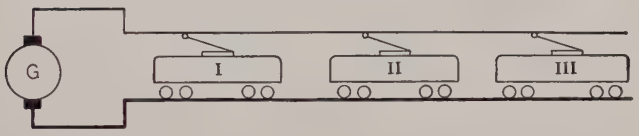


FIG. 32. An electric railway system with three cars in parallel.

Find: (*a*) Voltage drop in each section of the trolley wire.
(*b*) Voltage drop in each section of the track.
(*c*) Voltage across each car.

Prob. 45–1. Assume the voltage of the generator in Fig. 32 unknown but that the voltage across car II is 500 volts. Other data as in Prob. 44–1.

Find: (*a*) Voltage drop in each section of the trolley wire.
(*b*) Voltage drop in each section of the track.
(*c*) Voltage across generator and each car.

25. Standard Units. So far, only the fundamental ideas of volts, amperes and ohms have been set forth. It is essential, however, that the student know what the standard value of each is, and how it is obtained.

It is necessary to define independently but two of the three quantities. The third may be obtained by the application of Ohm's Law. At the present time, various international groups are studying the definitions in common use in order to select those which are best adapted to modern practice and also in order to eliminate certain small errors which appear in the older definitions. Internationally, we now define the **ampere** and the **ohm**, and then define the **volt** as that voltage which will force one **ampere** through one **ohm**. In this country, however, Congress has legally defined the **volt** and the **ampere**, and the **ohm** is derived from them. Finally, in actual practise, most standardization is carried out in terms of the **volt** and the **ohm**, and the **ampere** is determined as the current which

one **volt** forces through one **ohm.** This is done because it is easier as a practical matter to build and maintain standards of voltage and resistance than of current. For all practical purposes, the standards thus defined agree with each other and follow Ohm's Law. Furthermore, they agree with the absolute value of the **ampere** which is determined in terms of the fundamental units of mass, length and time. These definitions are summarized as follows:

The **standard volt** is defined as $\frac{1}{1.0183}$ of the voltage of a Standard Weston Cell under standard conditions.

The **standard ampere** is the current which when passed through a standard solution of silver nitrate deposits silver at the rate of 0.001118 gram per second.

The **standard ohm** is the resistance of a column of pure mercury 106.3 centimeters long, of uniform cross-section, and weighing 14.4521 grams at 0° C.

SUMMARY OF CHAPTER I

ELECTRICITY has been found to flow as a current along a conductor very much as water flows through a pipe.

THE CURRENT of electricity is measured in **AMPERES**, which state the **QUANTITY** passing through conductor **IN ONE SECOND.**

THE PRESSURE which causes the current to flow is measured in **VOLTS.** Corresponds to "pounds per square inch."

THE RESISTANCE which a conductor offers to the current is measured in **OHMS.** Corresponds to the **FRICTION** in a pipe.

THE DIRECTION of current flow is from a point of **HIGHER** potential to a point of **LOWER** potential, except in a generator or battery. On diagrams, a higher potential point is marked +, and a lower potential point is marked −.

OHM'S LAW states the relation which exists among current, pressure, and resistance. It is written in **THREE** forms.

1. Amperes $= \frac{\text{volts}}{\text{ohms}}$, that is, Current $= \frac{\text{pressure}}{\text{resistance}}$.
2. Volts = amperes × ohms, that is, Pressure = current × resistance.
3. Ohms $= \frac{\text{volts}}{\text{amperes}}$, that is, Resistance $= \frac{\text{pressure}}{\text{current}}$.

Using algebraic symbols:

1. $I = \frac{E}{R}$
2. $E = IR$
3. $R = \frac{E}{I}$

where

I = current in amperes
E = pressure in volts
R = resistance in ohms.

CURRENT is measured by INSERTING a low resistance ammeter INTO the circuit.

VOLTAGE is measured by TAPPING a high resistance voltmeter across two points in the circuit.

RESISTANCE is measured by dividing the VOLTMETER reading by the AMMETER reading according to Ohm's Law.

CAUTION. Be careful not to tap an ammeter to a circuit as you would a voltmeter. Always OPEN the circuit and INSERT the ammeter.

Electrical pieces connected in TANDEM are said to be in SERIES.

Electrical pieces connected SIDE BY SIDE are said to be in PARALLEL.

Series Combination

CURRENT through series combination is SAME as current through separate parts.

RESISTANCE of series combination is SUM of resistances of the separate parts.

VOLTAGE across series combination is SUM of voltages across separate parts.

Parallel Combination

VOLTAGE across parallel combination is SAME as voltage across each branch.

CURRENT through parallel combination is SUM of currents through all branches.

RESISTANCE of parallel combination is LESS than resistance of branch of smallest resistance. It is found by using Ohm's Law or by the conductance method.

Ohm's Law applied to any electric circuit should read:

The amperes through any PART of a circuit equal the volts across that same PART of the circuit, divided by the ohms of that same PART of the circuit.

Combinations of series and parallel arrangements can be separated into small parts and Ohm's Law applied to each part, in order to find the current, voltage and resistance distribution.

PARALLEL LIGHTING SYSTEMS. (*a*) When currents in lamps are given.

(First step.) Find the complete current distribution beginning at the point farthest from the generator.

(Second step.) Compute, by means of Ohm's Law, the line drops.

(Third step.) Combine line drops with known voltages according to the rules for series and parallel circuits.

(*b*) When resistances of lines and lamps are given.

(First step.) Combine resistances in series and parallel to obtain equivalent resistance across source of power.

(Second step.) Compute, by means of Ohm's Law, the total current supplied to the line.

(Third step.) Subtract line drop to group of lamps nearest generator to find voltage of that group.

(Fourth step.) Compute current in nearest group by Ohm's Law, and subtract from total current. The remainder is the line current to the next group.

(Fifth step.) Compute next line drop using the value of current just found and subtract from the voltage across first group of lamps. Repeat third and fourth steps until complete current and voltage distribution is found.

PROBLEMS ON CHAPTER I

Prob. 46–1. A certain 110-volt motor is to be adapted for use on a 220-volt line. How much resistance must be added in series with the field coils in order that the normal field current of 1.8 amperes is not exceeded?

Prob. 47–1. A 150-volt voltmeter has a resistance of 16,000 ohms. What current is required through the instrument to make it read 115 volts?

Prob. 48–1 Which has the greater resistance, the filament of a vacuum tube requiring 0.3 ampere at 6.3 volts, or an electric iron which takes 5.5 amperes from a 110-volt line?

Prob. 49–1. A 1400-ohm and a 2600-ohm resistor are connected in series across a 100-volt battery of negligible internal resistance. What voltage will be indicated by the voltmeter of Prob. 47 when connected across (*a*) the 1400-ohm resistance, and (*b*) the 2600-ohm resistance? What would the resistance of the voltmeter have to be in order that its indication of the voltage across the 1400-ohm resistance be within 1% of the true value?

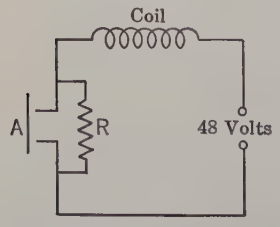


FIG. 33. The contacts at *A* open when the relay coil is energized.

Prob. 50–1. An electric relay is arranged as in Fig. 33 so that when its coil is energized the contacts at *A* open and the resistance *R* is inserted in series with the coil. The relay requires 0.26 ampere at 48 volts to operate the contacts, but it has been found that once energized the relay will "hold" in the energized position until the current is reduced to less than 0.11 ampere. What value of resistance should be used at *R* to keep the coil current at a minimum in the energized position?

Prob. 51–1. In Fig. 34 the resistance of R is 86.7 ohms. The ammeter (A) reads 1.26 amperes. What should the voltmeter (V) read?

Prob. 52–1. If 12 lamps of 260 ohms each are put in parallel across 110 volts, find:

(a) Resistance of the combination.
(b) Current through each lamp.
(c) Current through the combination.

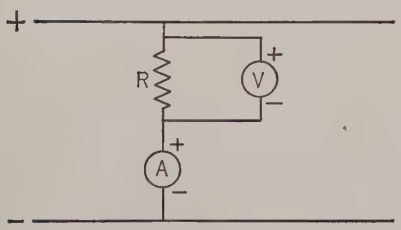


FIG. 34. The voltmeter indicates the voltage across the resistance R.

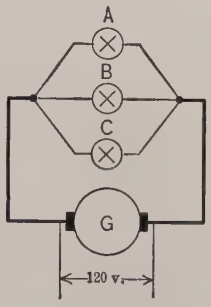


FIG. 35. Three lamps in parallel across the generator G.

Prob. 53–1. In Fig. 35 (parallel combination):

Resistance A = 25 ohms;
Resistance B = 80 ohms;
Resistance C = 45 ohms.
Voltage across combination = 120 volts.

Find:

(a) Voltage across each lamp.
(b) Current through each lamp.
(c) Current through combination.
(d) Resistance of combination.

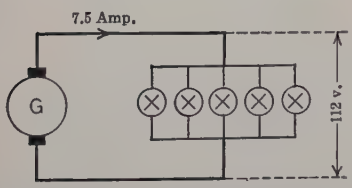


FIG. 36. A generator feeding a parallel group of five lamps.

Prob. 54–1. In Fig. 36, 7.5 amperes flow in the line. The lamps are alike and have 112 volts across them. Find:

(a) Current through each lamp.
(b) Resistance of each lamp.
(c) Voltage across each lamp.

Prob. 55–1. In a certain type of radio receiver, designed to operate from 110-volt, d-c lines, the heaters of the vacuum tubes are connected in series. A resistance is also placed in the series circuit to limit the current as shown in Fig. 37.

The ratings of the heaters of the tubes shown are given by the table below.

Type	*Volts*	*Amperes*
25Z5	25.0	0.3
43	25.0	0.3
6A7	6.3	0.3
77	6.3	0.3
78	6.3	0.3

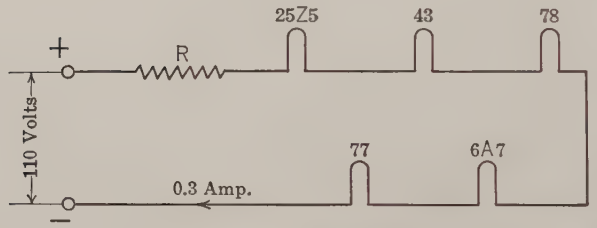


FIG. 37. The current through the series-connected radio-tube heaters is adjusted by the resistance *R*.

(*a*) What resistance is required to adjust the heater currents to their rated values?

(*b*) What potential differences exist between the negative (−) line and the positive (+) terminals of each heater?

Prob. 56–1. In the circuit of Fig. 38, when the ammeter (*A*) reads 1.0 ampere, the voltmeter V_1 reads 6.0 volts and V_2 reads 8.0 volts. What are the resistances of *A*, *B* and *C*?

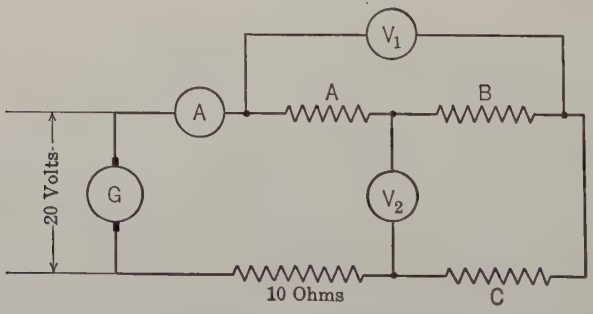


FIG. 38. Ohm's Law may be applied to entire circuits or parts of circuits.

Prob. 57–1. Two resistances, each of 5000 ohms, are connected in series across a 100-volt source of potential. If a voltmeter of 10,000 ohms resistance is connected across one of the resistors, what will it read? With the first voltmeter still connected, a second voltmeter is connected across the remaining resistor and is found to read 52 volts. What is the resistance of the second voltmeter?

Prob. 58–1. What pressure will be required to force 12 amperes through a parallel combination consisting of 16 ohms, 20 ohms and 8 ohms?

Prob. 59–1. Three pieces, one of 16 ohms, one of 22 ohms, and a third of 28 ohms are placed in series across a 110-volt line. It is found that a current of 1.60 amperes flows. What must be the resistance of the wires used to connect the pieces?

Prob. 60–1. What is the voltage across each piece in Prob. 59?

Prob. 61–1. A certain tungsten lamp requires 6.2 amperes at 90 volts pressure to operate at rated candle power. What resistance must be placed in series with this lamp when it is used on a 112-volt line?

Prob. 62–1. If three of the lamps of Prob. 61 were connected in parallel, what resistance would be required in series with the parallel combination?

Prob. 63–1. The resistance of a parallel circuit of two branches is 4 ohms. The resistance of one of the branches is 20 ohms. What is the resistance of the other?

Prob. 64–1. If 6 amperes are sent through the 20-ohm branch of Prob. 63,

(*a*) What voltage will be required?
(*b*) What current will flow in the other branch?

Prob. 65–1. The average resistance of the human body is 10,000 ohms. About 0.1 ampere through the body is usually fatal. What is the lowest voltage which would ordinarily kill a person?

Prob. 66–1. (*a*) What is the voltage across *AD*, Fig. 39?

(*b*) What is the voltage across *AB*?

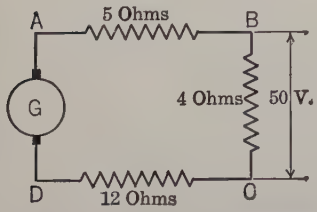


FIG. 39. The three resistances are in series.

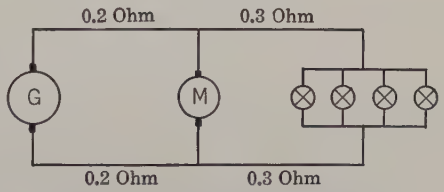


FIG. 40. A motor and a group of four lamps on a constant-potential line.

Prob. 67–1. Each lamp, Fig. 40, takes 1.5 amperes. Motor *M* takes 16 amperes. Generator voltage is 115 volts. Find:

(*a*) Line drop in each section (both wires) of the line.
(*b*) Voltage across lamps.
(*c*) Voltage across motor.

Prob. 68–1. Each lamp in Fig. 41 takes 1.6 amperes. The generator voltage is 120 volts; terminal A is positive. Find:

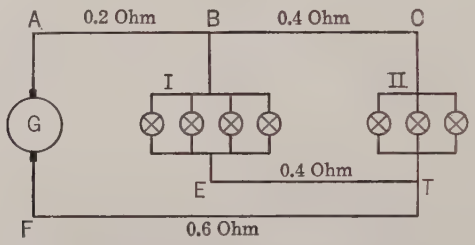


FIG. 41. A method of making the voltages across two groups of lamps more nearly equal.

(*a*) The amount and direction of the current in the following sections of the line: FT, ET, BC, AB.
(*b*) The voltage drop in each of the above sections of the line.
(*c*) The voltage across each group of lamps.

Prob. 69–1. Each lamp in Fig. 42 takes 1.4 amperes. Voltage across the switch = 112 volts. Resistance of each section of line (both wires) between the lamps, and from the switch to the lamps, is 0.2 ohm.

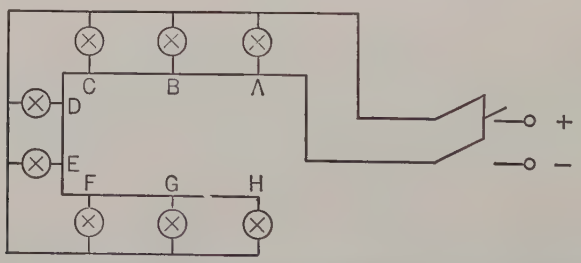


FIG. 42. The usual method of connecting lamps to a constant-potential circuit.

Find:

(*a*) The amount and direction of the current flowing in each section of the wire.
(*b*) Voltage drop in each section of wire.
(*c*) Voltage across each lamp.

Prob. 70–1. If the wiring of Prob. 69 were arranged as in Fig. 43, and all the data were the same as in Prob. 69, find:

(*a*) Amount and direction of current in each section.
(*b*) Volts drop in each section.
(*c*) Voltage across each lamp.
(*d*) Which is the better method of making connections, this or that of Prob. 69?

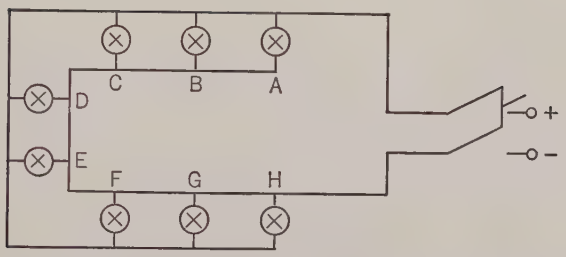


FIG. 43. The voltages across the lamps are nearer equal in this scheme of connections.

Prob. 71–1. If the wiring of Prob. 69 were arranged as in Fig. 44, and all the data were the same as in Prob. 69, find:

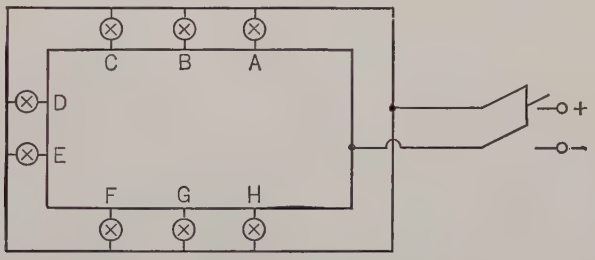


FIG. 44. The lamp feeders are connected to the supply at both ends in order to maintain nearly equal voltages across all lamps.

(*a*) Amount and direction of current in each section.
(*b*) Volts drop in each section.
(*c*) Voltage across each lamp.
(*d*) Compare the connections of Prob. 69 and 70 with these. Which is the best method of connecting?

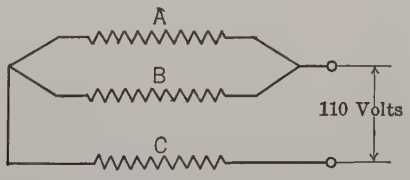


FIG. 45. The parallel combination of A and B is in series with C.

Prob. 72–1. In Fig. 45:

Resistance of A = 140 ohms.
Resistance of B = 160 ohms.
Resistance of C = 100 ohms.

Find:

(*a*) Current through each resistance.
(*b*) Resistance of parallel combination (*A* and *B*).
(*c*) Combined resistance of system.
(*d*) Voltage across each resistance.

Prob. 73–1. In Fig. 46 voltage from *A* to *B* is 60 volts. Current through resistance *X* is 3.2 amperes.

Resistance $Y = 5$ ohms.
Resistance $H = 4$ ohms.

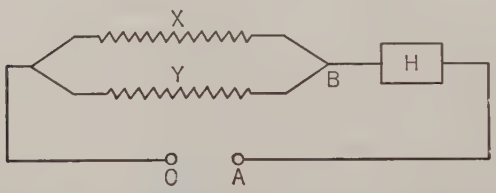


Fig. 46. The parallel arrangement of *X* and *Y* is in series with the heater *H*.

Find:

(*a*) Current through *Y*.
(*b*) Current through *H*.
(*c*) Resistance of *X*.
(*d*) Voltage from *B* to *C*.

Prob. 74–1. In Fig. 47 voltage across group *A* = 116 volts. Resistance of each lamp in group *A* = 180 ohms. Find:

(*a*) Voltage across group *B*.
(*b*) Average resistance of each lamp in group *B*.
(*c*) Line drop between generator and group *A*.
(*d*) Line loss in volts between *A* and *B*.
(*e*) Current through each lamp in *A* and each lamp in *B*.

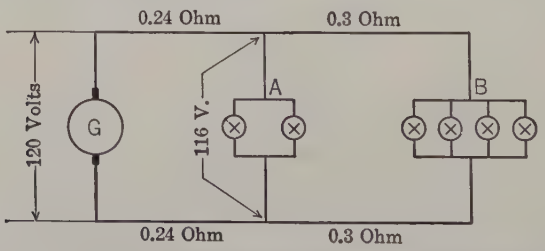


Fig. 47. Two groups of lamps on a constant-potential line.

Prob. 75–1. Two groups of lamps are connected to a generator as in Fig. 47. Each lamp of group *A* has a resistance of 65 ohms, and each lamp of group *B* has a resistance of 46 ohms. The line resistances are as in Fig. 47. If the generator voltage is 110 volts, find:

(*a*) Voltage across *A* and *B*.
(*b*) Current through *A* and *B*.
(*c*) Line drop between generator and *A*.
(*d*) Line drop between *A* and *B*.

Prob. 76–1. Three resistances, one of 15 ohms, one of 17 ohms and a third unknown are placed in parallel. The resistance of the combination is 4.85 ohms. What is the third resistance?

Prob. 77–1. If the 17-ohm resistance of Prob. 76 is carrying 3.6 amperes how much current is each of the other two resistances carrying?

CHAPTER II

ELECTRICAL POWER AND ENERGY

In stating the rating of an electrical apparatus it is customary to give not only the voltage at which it operates, but also the rate at which it produces or uses electrical energy. This rate of producing or using energy is called the **power**, and is measured in watts and kilowatts. Thus, a tungsten lamp may be rated 50 watts at 110 volts, a generator 1000 kilowatts at 600 volts.

26. Unit of Power. Watt. The flow of an electric current has been likened to the flow of water through a pipe. A current of water is measured by the number of gallons, or cubic feet, flowing per minute; a current of electricity, by the number of amperes (coulombs per second). The **power** required to keep a continuous current of water flowing is the product of the current in **cubic feet per minute** by the pressure expressed in **pounds per square foot.** This gives the power in **foot pounds per minute.** To reduce to horsepower, it is necessary merely to divide by 33,000; i.e.,

$$\frac{\text{(cubic ft. per min.)} \times \text{(pounds per sq. ft.)}}{33{,}000} = \text{horsepower.}$$

In exactly the same way, the **power** required to keep a continuous current of electricity flowing is the product of the current in **amperes** by the pressure in **volts.** This gives the power in **watts.**

$$\textbf{Watts} = \textbf{Amperes} \times \textbf{Volts.}$$

In the form of an algebraic equation this becomes

$$P = IE$$

where

P = power in **watts**
I = current in **amperes**
E = pressure in **volts.**

The term **watt** is merely a unit of power and denotes the power used when one volt causes one ampere of current to flow. The watts involved when any given current flows under any pressure can always be found by multiplying the current in amperes by the pres-

sure in volts. Thus, if an incandescent lamp takes 0.5 ampere when operating on a 110-volt line, the power consumed equals

$$0.5 \times 110 = 55 \text{ watts.}$$

That is,

$$P = IE$$

$$\text{power} = \text{current} \times \text{pressure}$$

or

$$\text{watts} = \text{amperes} \times \text{volts.}$$

Example 1. What power is consumed by a motor which runs on a 220-volt circuit if it takes 4 amperes?

Solution.

$$\text{Watts} = \text{amperes} \times \text{volts}$$
$$= 4 \times 220$$
$$\text{Power} = 880 \text{ watts.}$$

Prob. 1–2. A 20-candle-power tungsten lamp takes 0.25 ampere when on a 110-volt line. What power is consumed?

Prob. 2–2. An arc lamp takes 5.5 amperes at 78 volts. What power is consumed?

Prob. 3–2. A 110-volt electric iron uses 4.8 amperes. What power does it use?

Prob. 4–2. What current does a motor take which uses 250 watts on a 110-volt line?

Prob. 5–2. A motor is rated to take 8 amperes and to consume 900 watts. For what voltage is it built?

27. Variations of The Power Equation. Just as Ohm's Law has the three forms, (1) $I = \frac{E}{R}$, (2) $E = IR$, (3) $R = \frac{E}{I}$, so this power equation $P = IE$ may have three forms, found as follows:

Since $E = IR$, we can substitute this in the equation $P = IE$, obtaining

$$P = I(IR) = I^2R.$$

Likewise, since $I = \frac{E}{R}$, we can substitute in the same equation and obtain.

$$P = \left(\frac{E}{R}\right)E = \frac{E^2}{R}.$$

Thus there are three forms for the power equation.

$$P = IE = I^2R = \frac{E^2}{R}.$$

A considerable amount of mathematical work will be saved by knowing these three forms so that in any particular problem the form directly applicable can be used. If the volts and amperes are known, use the form $P = IE$; when amperes and ohms are known, use the form $P = I^2R$; and when volts and ohms are known, use the form $P = \frac{E^2}{R}$.

The result is always the same as though we used Ohm's Law first to find the amperes and volts and then multiplied.

Example 2. Generator G, Fig. 48, furnishes 0.50 ampere to the line at 115 volts pressure. What power is consumed by the 110-volt lamp L?

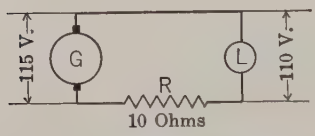


FIG. 48. A rheostat R in series with a lamp.

Solution.

(*a*) For the lamp L:

$E = 110$ volts;
$I = 0.5$ ampere;

(1) $P = EI = 0.5 \times 110 = 55$ watts.

Or find the resistance of the lamp.

$$R = \frac{E}{I} = \frac{110}{0.5} = 220 \text{ ohms.}$$

Then

$I = 0.5$ ampere;
$R = 220$ ohms;

(2) $P = I^2R = 0.5 \times 0.5 \times 220 = 55$ watts.

Or

$E = 110$ volts;
$R = 220$ ohms;

(3) $$P = \frac{E^2}{R} = \frac{110 \times 110}{220} = 55 \text{ watts.}$$

The result of any of the three methods is the same, 55 watts. Therefore it is usually best to use the easiest method for the data given. Since the voltage and the current in this case were known, it is best to use $P = EI$.

Prob. 6–2. What power is consumed by the resistance R in Fig. 48 if the lamp takes 0.50 ampere?

Prob. 7–2. What power does a car heater use if its resistance is 120 ohms and the pressure across it is 550 volts?

Prob. 8–2. What power does an electric flatiron use on a 115-volt circuit if the resistance is 30 ohms?

Prob. 9–2. A 60-volt arc lamp requires 400 watts to operate properly. What current does such a lamp take?

Prob. 10–2. How many watts are consumed in the lead wires in Fig. 49 if their resistance is 2 ohms each and the motor takes 1760 watts at 220 volts?

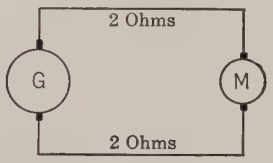


FIG. 49. A motor M fed by a generator G over a 4-ohm line.

28. To Determine the Power in an Electric Circuit. If we wish to know the power that is being consumed in a certain part of an electric circuit, we merely have to insert an ammeter to measure the current in that part of the circuit, and apply a voltmeter to measure the voltage across that part of the circuit, and multiply the ammeter reading by the voltmeter reading. This gives us our power directly in watts. This is stated as an equation as follows:

Watts (used in any part of circuit) = Volts (across that part of circuit) × Amperes (through that part of circuit).

The same precautions must be observed in the use of this equation as in the use of Ohm's Law. That is, the voltage and the current must be measured for the same part of the circuit at the same time, and their product is the power consumed in that part of the circuit alone. The following example illustrates the use of the equation.

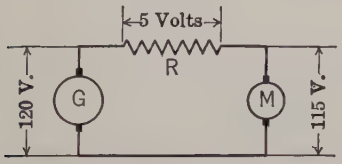


FIG. 50. The resistance is in series with the motor M.

Example 3. A generator G, Fig. 50, is furnishing a current of 4 amperes to the line at a pressure of 120 volts. There is in the circuit a resistance R, which requires 5 volts to force the current through it, and a motor M, which requires 115 volts. How much power does the resistance R consume, and how much does the motor M consume?

Solution.

The resistance R consumes:

$$P_R = I_R E_R$$

or

P (watts in R) = I (current through R) × E (voltage across R).

P = 4 (amperes) × 5 (volts) = 20 watts.

Motor M consumes:

$$P_M = I_M E_M$$

or

P (watts in M) = I (current through M) × E (voltage across M).

P = 4 (amperes) × 115 (volts) = 460 watts.

The total power consumed by R and $M = 460 + 20 = 480$ watts. This checks with the total power delivered to the circuit as follows:

$$P_C = I_C M_C$$

or

P (watts in circuit) = I (current in circuit) $\times$ E (voltage across circuit).

$P = 4$ (amperes) $\times$ 120 (volts) = 480 watts.

29. Kilowatt and Horsepower. Since the watt is a unit of power too small to express conveniently the output of modern electrical machinery, a unit called the **kilowatt**, equal to 1000 watts, is generally used.

Thus:

$$2500 \text{ watts would be } \frac{1}{1000} \text{ of } 2500 \text{ or } 2.5 \text{ kilowatts}$$

and

$$450 \text{ watts would be } \frac{1}{1000} \text{ of } 450 \text{ or } 0.45 \text{ kilowatt.}$$

Example 4. What power does a motor consume which takes 20 amperes at 220 volts?

Solution.

$$\text{Watts} = 20 \times 220$$
$$= 4400 \text{ watts}$$
$$\text{Kilowatts} = \frac{4400}{1000} = 4.4 \text{ kilowatts.}$$

Since a kilowatt is a unit of power it can be reduced to horsepower.

1 kilowatt = 1.34 horsepower or (approx.) $1\frac{1}{3}$ horsepower.

1 horsepower = 0.746 kilowatt or (approx.) $\frac{3}{4}$ kilowatt.

Example 5. What horsepower does the motor of the above example consume?

Solution.

1 kilowatt = 1.34 horsepower.

4.4 kilowatts = 4.4×1.34 horsepower.

= 5.90 horsepower.

Example 6. The power of a 10-horsepower motor could be rated as how many kilowatts?

Solution.

1 horsepower = 0.746 kilowatt.

10 horsepower = 10×0.746 kilowatt.

= 7.46 kilowatts.

Prob. 11–2. What electric power is required to drive a pump which must raise water 96 feet and supply 2600 gallons per hour? (1 gallon of water weighs 8.3 pounds.)

Prob. 12–2. What is the horsepower output of a generator which must deliver 24 amperes at 240 volts?

Prob. 13–2. What horsepower do 14 lamps consume if each takes 0.25 ampere at 110 volts?

Prob. 14–2. A steam engine is rated as a 70-horsepower engine. What would be its kilowatt rating?

30. Use of the Wattmeter. Instead of using two separate instruments, an ammeter and a voltmeter, to measure the power consumed in a certain part of a circuit, we may use a single instrument called a **wattmeter.** This instrument is a combination of an ammeter and a voltmeter and thus has an ammeter part of **low resistance** to carry the current, and a voltmeter part of **high resistance** to take the voltage. The indicator shows the product of the volts times the amperes, that is, the **watts.**

A wattmeter, therefore, usually has **four** terminals, two for the ammeter leads and two for the voltmeter leads. The utmost care must always be exercised to use the proper terminals. An error in this respect will ruin a very expensive instrument.

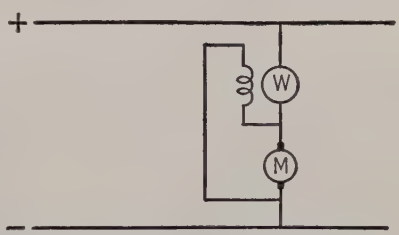


FIG. 51. The wattmeter *W* reads the power taken by the motor. The current coil is in series with the motor and the potential coil is across the motor.

Figure 51 shows the correct method of connecting a wattmeter in the circuit to measure the power taken by the motor *M*. A current coil is placed in **series** with the motor, and a voltage coil is connected in **shunt** across the motor.

31. Work. Commercial Units. In order to compute the amount of work done by a given engine, it is necessary to know the time it has been running and the power it has been supplying, that is, its rate of doing work. If the power is measured in **horsepower** and the time in **hours,** the work done is measured in **horsepower-hours** and is the product of the **horsepower** by the **hours.**

Similarly, if the power is measured in **kilowatts** and the time in **hours,** the work done is measured in **kilowatt-hours** and is merely the product of the **kilowatts** by the **hours.**

The Horsepower-hour and the Kilowatt-hour are the commercial units of work.

When a man buys mechanical energy to run his shop he has to pay not only according to the horsepower he uses but also according to the number of hours he uses the power. For instance, he may use 40 horsepower for 1 hour and pay $1.20 for it, that is, at the rate of 3 cents for each horsepower-hour. If he uses 40 horsepower for 2 hours he will have to pay twice as much because he has used twice as much energy or work. Another way of stating the same fact is to say that he used twice as many horsepower-hours. For in the first instance he used 40×1, or 40 horsepower-hours, and in the second 40×2, or 80 horsepower-hours. In other words, he did twice as much work in the second case as he did in the first, or received twice as much energy. The unit of work or energy then is the **horsepower-hour** and is the work done in one hour by a 1-horsepower machine.

$$1 \text{ horsepower-hour} = 0.746 \text{ kilowatt-hour.}$$
$$1 \text{ kilowatt-hour} = 1.34 \text{ horsepower-hour.}$$

The use of these units is illustrated by the following examples:

Example 7. How much work is done by a machine delivering 15 horsepower when it is run for 8 hours?

Solution.

1 horsepower in 1 hour does 1 horsepower-hour,
15 horsepower in 1 hour do 15 horsepower-hours,
15 horsepower in 8 hours do 8×15, or 120 horsepower-hours,

or

$$\text{work} = \text{horsepower} \times \text{hours},$$
$$= 15 \times 8 = 120 \text{ horsepower-hours.}$$

Example 8. How much work is done in one day of 8 hours by a 150-kilowatt generator running at full load?

Solution.

$$150 \times 8 = 1200 \text{ kilowatt-hours,}$$
$$1200 \times 1.34 = 1608 \text{ horsepower-hours.}$$

Example 9. At 6 cents per kilowatt-hour, what is the cost of burning 100 lamps for 8 hours if each lamp consumes 50 watts?

Solution.

$$\text{Power consumed in 100 lamps} = 100 \times 50$$
$$= 5000 \text{ watts} = 5 \text{ kilowatts.}$$
$$\text{Energy} = 8 \times 5 = 40 \text{ kilowatt-hours.}$$
$$40 \times 6 \text{ cents} = \$2.40$$

Prob. 15–2. What does it cost to run a 75-horsepower engine at full load for 180 hours at $1\frac{1}{2}$ cents per horsepower-hour?

Prob. 16–2. A man's power bill for 44 hours was $50.00. If he paid at the rate of 2 cents per horsepower-hour, what average power was delivered to him?

32. Work. Small or Laboratory Units. The commercial units of **work,** the **horsepower-hour** and the **kilowatt-hour,** are too large to be used conveniently in some problems. Accordingly use is made of two smaller units. The mechanical unit so used is the **foot pound.** The electrical unit is the **watt-second** or **joule.**

1 horsepower-hour = 1,980,000 foot pounds,
1 kilowatt-hour = 3,600,000 watt-seconds or joules,
1 watt-second = 0.738 foot pound.

Example 10. How much work is done when a 25-watt lamp is burned for 2 minutes?

Solution. $25 \times 2 \times 60 = 3000$ watt-seconds, or joules.

Prob. 17–2. How much will it cost at 5 cents per kilowatt-hour to run a 110-volt motor for 24 hours? The motor takes 30 amperes.

Prob. 18–2. If the work in Prob. 17 were paid for at the rate of 5 cents per horsepower-hour how much would it cost?

Prob. 19–2. What work is done in maintaining for 24 hours a current of 100 amperes in a wire of 1.8 ohms resistance?

Prob. 20–2. A bill for electric energy was $16.40 for 120 hours. If the price was 8 cents per kilowatt-hour, what was the average power used?

Prob. 21–2. When 120 incandescent lamps had been burned on a 110-volt circuit for 6 hours, a bill of $1.73 was presented, computed on the rate of 6 cents per kilowatt-hour. What was the average current taken by each lamp?

Prob. 22–2. At 5 cents per kilowatt-hour how much does it cost per year for transmission losses in a line which carries a current of 250 amperes and has a resistance of 0.63 ohm. The line is used two hours each day.

Prob. 23–2. By how many foot pounds are 1,000,000 watt-seconds greater or less than 0.40 horsepower-hour?

33. Electrical Energy and Heat Energy. An electric current may be used in one part of a circuit to produce mechanical motion as in a motor; in another part of the circuit to produce electrolytic action as in an electroplating vat; in another part to produce light as in an electric lamp. In any part of the circuit where it is doing no one of these things, all the energy consumed goes into producing heat. It even produces some heat in the portions of the circuit

where it is also producing some other form of energy. A motor never gives out in mechanical energy all that it receives in the form of electrical energy. Some of the electrical energy is turned into heat energy. This heat is produced in overcoming the electrical **resistance,** just as heat is produced in a machine in overcoming mechanical **friction.**

It is safe to say that in any part of an electric circuit where there is no transformation to other forms of energy, the whole of the electrical energy consumed is turned into heat. Thus in line wires all the electric energy consumed in forcing a current through them goes into heat.

The resistance of an electric circuit is similar to the friction of a machine. Just as the power used to overcome the resistance of the wire appears in the heat generated in the wire, so the power used to overcome the friction of a bearing appears in the heat generated in the bearing. The mechanical engineer strives to reduce the amount of power wasted in heat by reducing the friction of the bearings. The electrical engineer may reduce the power wasted in heat by reducing the resistance of the wire used to transmit a given current. In designing electric machinery a great deal of care is given to provide sufficient surface for radiation so that the heat generated may not cause an excessive rise in temperature.

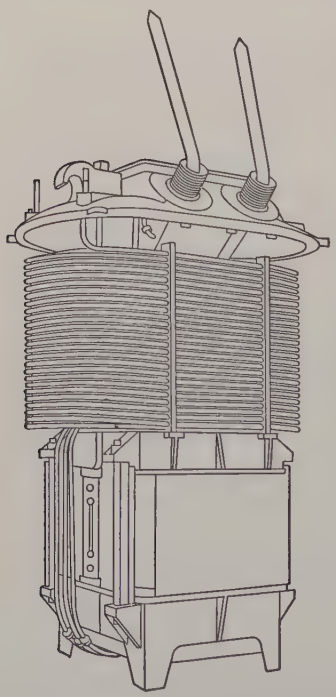


FIG. 52. The interior of a transformer showing the cooling coils.

This is well illustrated in Fig. 52, which shows a transformer, a device in very general use for raising or lowering the voltage of an alternating current line. Power is brought to the transformer by one set of terminals and delivered by the transformer through another set. But a transformer never delivers all the power brought to it. Some of it is used up inside the transformer. Since there is no mechanical motion of the parts of an ordinary transformer, all the energy used

up in it must be turned into heat. Unless special precautions are taken in the design to get rid of this heat produced, the transformer will be ruined by the high temperature to which parts of it will rise.

Notice the coil of pipes at the top of Fig. 52. Water is circulated through these pipes in order to carry off the heat generated in the core and windings.

The following examples show how this power lost in heat may be computed.

Example 11. If the transformer of Fig. 52 receives 100 kilowatts and delivers 98.5 kilowatts, how much power is used in heating the transformer?

Solution. $100 - 98.5 = 1.5$ kilowatts $= 1500$ watts.

Example 12. The generator G in Fig. 53 maintains a pressure of 240 volts across the line. The line wires have a total resistance of 4 ohms. The motor M requires 5 amperes at 220 volts. How much electrical power is lost in heat in the line wire?

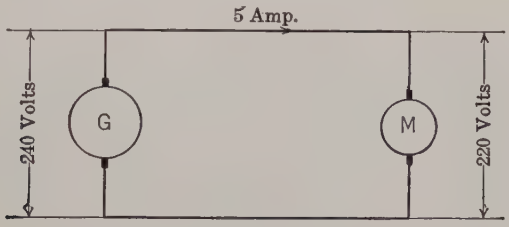


FIG. 53. The motor M takes 5 amperes.

Solution. Since the line wire is delivering no other form of power, all the electrical power consumed in it must be used in overcoming its resistance and must go into heating it. The power used in the line wire is found as follows:

$$I = 5 \text{ amperes}$$
$$R = 4 \text{ ohms}$$
$$P = I^2R = 5 \times 5 \times 4 = 100 \text{ watts.}$$

Thus 100 watt-seconds or joules of electrical energy are turned into heat each second. If this heat is not allowed to radiate, the wire will eventually become hot enough to destroy the insulation and finally fuse the copper itself. Thus is seen the necessity for arranging wires in the coils of a transformer and in the field and armature coils of motors in such a way as to allow for the dissipation of the heat generated in overcoming the resistance of the coils.

Referring again to Fig. 53, it is seen that the motor armature consumes 220×5 or 1100 watts. The greater part of this energy is used in doing the mechanical work of turning the armature. But

a considerable share of even this energy is used in overcoming the electrical resistance of the motor armature.

Assuming the armature resistance of the motor is 2 ohms, the power consumed in overcoming its resistance, and thus in heating it would be:

$$P = I^2R$$
$$= 5 \times 5 \times 2 = 50 \text{ watts.}$$

Of the total 1100 watts taken by the motor armature not more than 1050 watts are turned into mechanical energy, 50 watts being used to overcome the electrical resistance of the armature. These 50 watts are called the **I^2R loss** or sometimes the **copper loss,** since the electrical conductors are generally of copper.

Note particularly that the watts lost in heat cannot be found by the equation $P = IE$ or $\frac{E^2}{R}$ **unless the E is that part of the voltage which is used in forcing the current (I) through the resistance (R).**

Thus in the above example if we used the total voltage across the motor, 220 volts, in the equation, $P = IE$, we would obtain $P = 5 \times 220 = 1100$ watts, which is the total power put into the armature because the 220 volts is the total voltage across the armature, and 5 amperes the current through the armature.

If we wish to use this equation, $P = IE$, for the heat loss in the armature we must use E_R or volts used in forcing the current through the resistance of the armature. We find this E as follows:

$$E_R = I_R R_R,$$

where E_R = volts used in forcing current I_R through R_R
I_R = current through armature
R_R = resistance of armature
$E_R = 5 \times 2 = 10$
$P_R = I_R E_R$
$= 5 \times 10$
$= 50$ watts.

This is the same as the result we obtained by using I^2R. Using the same precaution that E be made E_R we may also find the heat loss by the power equation $P = \frac{E^2}{R}$

$$= \frac{10 \times 10}{2}$$
$$= 50 \text{ watts.}$$

Prob. 24–2. How much power (copper loss) is consumed in heating the armature of a generator which delivers 20 kilowatts at 440 volts? Armature resistance is 0.8 ohm.

Prob. 25–2. How many kilowatt-hours of electricity are used up in a week in heating the line wires of a transmission system carrying an average current of 70 amperes, 24 hours per day every day of the week? Total resistance of line wires is 0.3 ohm.

Prob. 26–2. A 220-volt motor requires 15 amperes. The resistance of the armature is 0.50 ohm.

(*a*) How many horsepower does the motor take?

(*b*) How many horsepower are used up in heating the armature wires (I^2R loss)?

34. Electrical Equivalent of Heat. Since electrical energy can be transformed into heat energy as well as into mechanical energy, electrical units of energy can be reduced to heat units of energy as well as to mechanical. The common unit of heat energy is the **calorie.***

One calorie is the quantity of heat necessary to raise one gram of water 1° C.†

If the heat generated by passing an electric current through a wire be measured by allowing it to raise the temperature of a known weight of water, it is found that one watt-sec. of electrical energy is the equivalent of 0.24 calorie. That is, one ampere flowing through an ohm resistance for 1 sec. will produce heat sufficient to raise 1 gm. of water 0.24° C. This is generally stated in the form of an equation:

$$H = 0.24\, I^2RT,$$

where

H = Heat in calories;
I = Current in amperes;
R = Resistance in ohms;
T = Time in seconds.

Since

$$I^2R = \text{watts},$$

then

$$I^2RT = \text{watt-secs. or joules};$$

$\therefore$ $H = 0.24$ watt-sec. or joule.

The factor (0.24) is called the **heat equivalent of electricity.**

* The English heat unit is the British Thermal Unit (B.t.u.) and is the heat required to raise the temperature of one pound of water one degree Fahrenheit. 1 B.t.u. = 252 calories.

† The calorie thus defined is sometimes referred to as a gram-calorie, because it is based on the heat required to raise the temperature of one gram of water 1° C. Occasional reference is also found to the kilogram-calorie which is the heat required to raise a kilogram of water 1° C. The kilogram calorie is hence 1000 times as large as the gram-calorie.

Example 13. How many heat units are generated when 50 amperes flow for 10 minutes through a 2-ohm wire?

Solution.

$$\begin{aligned} \text{Heat} &= 0.24\ I^2RT \\ &= 0.24 \times 50 \times 50 \times 2 \times 10 \times 60 \\ &= 720{,}000 \text{ calories.} \end{aligned}$$

Prob. 27–2. How much heat is generated per hour in an electric iron using 5.4 amperes at 110 volts?

Prob. 28–2. If the price of the electric energy used in Prob. 27 is 8 cents per kw-hr., what is the price per million calories?

Prob. 29–2. How many calories per hour are given off by a car heater which takes 4 amperes at 550 volts?

Prob. 30–2. In ordinary house wiring, a drop of two volts is usually allowed in the wires from the meter to farthest lamp. Assuming that an average current of 1.5 amperes flows in these wires for 24 hours per day, how much heat is generated per day in the wires?

35. Power Units and Work Units. We have considered the Power units and the Work units in common use in electrical, mechanical and heat measurements. It is well to take some time to get the relations among them firmly fixed in the mind.

First, it must be clearly understood which are the power units and which are the work or energy units.

The **Power** units are:

(*a*) Electrical { Watt; Kilowatt = 1000 watts

(*b*) Mechanical { Foot pounds per minute; Horsepower = 33,000 foot pounds per minute

(*c*) Heat { Calories per second; B.t.u. per second = 252 calories per second

Note that in the case of electrical, mechanical and heat power there are two units each, one the small unit, the other the large unit. Since all are units of power, a quantity of power expressed in any unit may be expressed in any of the other units of power. Thus, 5000 watts may be expressed as a number of kilowatts, a number of foot pounds per minute, a number of horsepower, or a number of B.t.u. per second. It is merely necessary to know the heat and the mechanical equivalents of the electrical unit.

The common units of **Work** or **Energy** are:

(*a*) Electrical	Watt-second or Joule Kilowatt-hour
(*b*) Mechanical	Foot pound Horsepower-hour
(*c*) Heat	Calorie British thermal unit

Note that there are defined two electrical, two mechanical, and two heat units of work, a large and a small one, just as in the case of power units.

A quantity of energy expressed in any one of these units can be expressed in any of the other units of energy. Again it is merely necessary to know the heat and mechanical equivalents of the electrical unit.

It is not necessary to memorize the electrical, mechanical and heat units of both power and energy equivalents. If one work equivalent and one power equivalent are learned the rest can be derived as desired. The three most useful equivalents are:

$$1 \text{ kilowatt} = 1.34 \text{ horsepower}$$
$$1 \text{ watt-second} = 0.24 \text{ cal.}$$
$$1 \text{ B.t.u.} = 252 \text{ cal.}$$

Caution. There are no equivalents between power units and energy units. They are different things and therefore cannot be reduced one to the other. Thus, British thermal units which are **work** units cannot be reduced to an equivalent number of watts or horsepower because watts and horsepower are units of **power** and not of work. The greatest care is necessary not to confuse the **work** with the **power** units. Keep these clearly defined and no difficulty will be met in a change from one system of units to another. Remember that **work** is always found by multiplying **power** by **time.** Thus a kilowatt-hour is a unit of **work,** found by multiplying a unit of **power,** the kilowatt, by a unit of **time,** the hour. Expressed in symbols this becomes

$$\text{Work} = \text{Power} \times \text{Time},$$

or

$$W = Pt.$$

The following problem is one of the best tests of one's knowledge of the relations between units.

Prob. 31–2. From the three values given above fill in the values of the equivalents in the following table.

	Power			Work or Energy		
Electrical	1 watt	=	ft. lb. per min.	1 watt-sec.	=	B.t.u.
					=	ft. lb.
					=	calorie
	1 kw.	=	watts	1 kw.-hr.	=	watt-sec.
		=	hp.		=	hp-hr.
					=	B.t.u.
Mechanical	1 ft. lb. per min.	=	watt	1 ft. lb.	=	watt-sec.
					=	B.t.u.
	1 hp.	=	ft. lb. per min.		=	calorie
				1 hp.-hr.	=	ft. lb.
		=	kw.		=	kw.-hr.
					=	B.t.u.
					=	calories
Heat	1 B.t.u. per sec.	=	watts	1 B.t.u.	=	watt-sec.
		=	ft. lb. per min.		=	ft. lb.
					=	hp.-hr.
		=	hp.		=	kw.-hr.
		=	kw.			
	1 calorie per sec.	=	watts	1 calorie	=	watt-sec.

36. Efficiency of Electrical Appliances. Since all pieces of electrical apparatus have resistance, there must always be some heat generated when a current is sent through them. Unless the apparatus is to be used for heating purposes, this energy is wasted, just as the energy is wasted which goes toward overcoming mechanical friction. Thus no electrical machine gives out all the power it receives. The percentage which it does give out is called its **efficiency.** Accordingly a motor that gives out 9 kilowatts for every 10 kilowatts it receives is said to have an efficiency of 90 per cent. If it gives out only 8 kilowatts for every 10 kilowatts it receives, it has an efficiency of only 80 per cent.

The power a machine receives is called the **input.** The power it gives out is called the **output.** The efficiency is defined as the ratio of the output to the input, or

$$\text{efficiency} = \frac{\text{output}}{\text{input}}.$$

Since the output is always smaller than the input, the fraction $\frac{\text{output}}{\text{input}}$ is always less than unity, and is stated as a percentage, as 75 per cent,

90 per cent, etc. Another way of stating the same fact is: The efficiency of any device is always less than 100 per cent. Of course, the output and the input must always be stated in the same units. We cannot compare the input of a motor in kilowatts with the output in horsepower. They must both be expressed either in kilowatts or in horsepower.

Example 14. A 3-horsepower motor requires 2.4 kilowatts to drive it. What is its efficiency?

Solution.

$$\text{Output} = 3.0 \text{ horsepower}$$
$$\text{Input} = 2.4 \times 1.34 = 3.2 \text{ horsepower}$$
$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{3.0}{3.2} = 94\%.$$

For a discussion of the efficiency of motors and generators see Chapter IX.

Prob. 32–2. What efficiency has a 25-horsepower motor which requires 90 amperes at 230 volts?

Prob. 33–2. A generator has an efficiency of 92%. What current can it deliver at a pressure of 230 volts when it receives 75 horsepower from the driving engine?

Prob. 34–2. What is the efficiency of a motor which does 1,200,000 foot pounds of work in 15 minutes if the input is 2.5 kilowatts?

37. Efficiency of Transmission. When energy is transmitted from one place to another, some energy is always lost in the transmission line. In mechanical transmission of energy there is the pulley and belt friction, etc., to be overcome. In electrical transmission, the resistance of the wires must be overcome. That method of transmission is most efficient which loses the smallest percentage of energy in the line. The efficiency of transmission is defined as that percentage which the useful output of the line is of the total input.

Example 15. The lamp bank *A*, Fig. 54, uses 12 amperes. What is the efficiency of the transmission; that is, what percentage of the total power delivered to the line is used by the lamps?

Solution.

$$\text{Total power} = (IE) = 120 \times 12 = 1440 \text{ watts.}$$
$$\text{Watts lost in line} = (I^2R) = 12 \times 12 \times 0.4 = 57.6 \text{ watts.}$$
$$\text{Watts left for lamps} = 1440 - 57.6 = 1382.4 \text{ watts.}$$
$$\text{Efficiency of transmission} = \frac{1382}{1440} = 96\%;$$

or

$$\text{Volts lost in line} = (IR) = 12 \times 0.4 = 4.8 \text{ volts.}$$
$$\text{Volts across lamps} = 120 - 4.8 = 115.2 \text{ volts.}$$
$$\text{Watts delivered to lamps} = (IE) = 115.2 \times 12 = 1382 \text{ watts.}$$
$$\text{Efficiency} = \frac{1382}{1440} = 96\%.$$

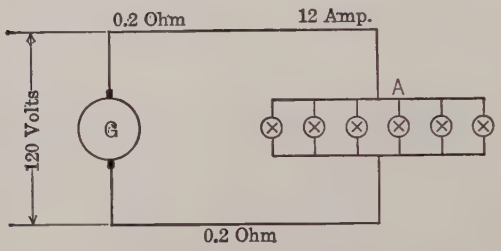


Fig. 54. The line current heats the wires.

Prob. 35–2. What is the efficiency of transmission for a circuit which receives 18 kilowatts from a generator at one end and delivers 15.8 kilowatts to a motor at the other end?

Prob. 36–2. Each conductor of a line has a resistance of 1.2 ohms. The load consists of 6 lamps in parallel each taking 0.5 ampere and each having a hot resistance of 220 ohms. What is the efficiency of transmission?

Prob. 37–2. Each lamp in Fig. 55 takes 0.4 ampere. What is the efficiency of transmission?

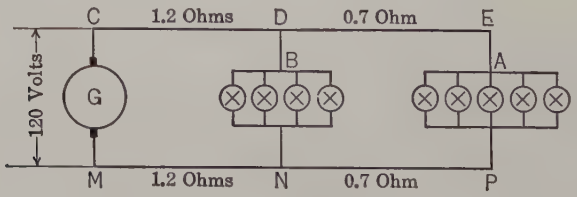


Fig. 55. Two groups of lamps on a constant-potential line.

Prob. 38–2. If each lamp in Group *B*, Fig. 55, takes 0.8 ampere and other data remain as in Prob. 37, what is the efficiency of transmission?

38. Electric Heating. There are many electric devices which make use of the heat generated when an electric current is sent through a resistance. A common example is the fuse, Fig. 56.

Inside the fibre case is a wire or link of lead alloy which melts when a given current passes through it. The fuse is connected in series with the appliance to be protected from an excess of current.

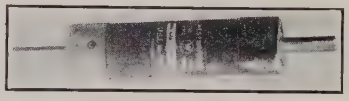


FIG. 56 A cartridge fuse for protecting the line from too great currents. *Westinghouse Electric & Mfg. Co.*

Electric heaters of which Fig. 57 is an example are in common use for auxiliary sources of heat. Even whole houses may be economically heated by electricity when the price of electric energy is $\frac{1}{2}$ cent per kw-hr. or less and the cost of coal $12.00 to $15.00 per ton delivered.* As heaters, electric appliances are practically 100 per cent efficient.

FIG. 57. An electric heater. *Westinghouse Electric & Mfg. Co.*

Figure 58 illustrates an application of electric heating in extending the length of a growing season and accelerating plant growth. Resistance elements are buried in the soil, and by means of a thermostat the current flow is regulated so as to produce the temperature desired.

Prob. 39–2. In one test on heating a house by electricity it was found that it required 3.6 kw-hr. per year to heat one cubic foot of space. At $\frac{1}{2}$ cent per kw-hr. what would be the yearly cost of heating by electricity a house containing 11,300 cubic feet?

Prob. 40–2. An immersion heater is a resistance element so constructed that it will operate under water without damage. Assuming that no heat is lost to the surrounding air, how long would it take a 100-watt immersion heater to boil 2 liters of water taken from a faucet at 50°F? At the rate of 5 cents per kilowatt-hour how much would this energy cost?

Prob. 41–2. At 3.5 cents per kilowatt-hour, what is the monthly bill for using a flatiron which takes 5.5 amperes at 110 volts? The iron is in use an average of 18 hours per month.

* Bulletins University of Washington, Nos. 15–20.

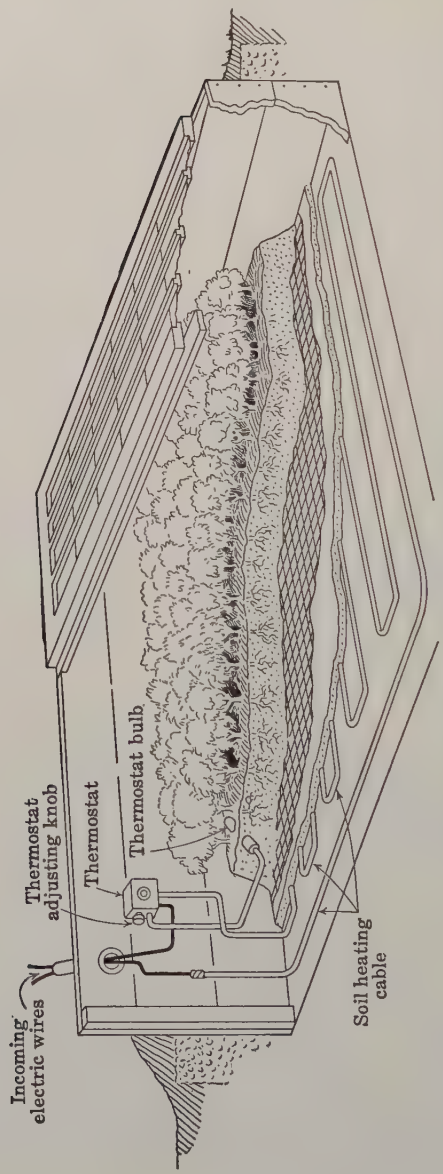


FIG. 58. An electric hot bed. Heating cables are buried in the soil and controlled by a thermostat. *General Electric Co.*

SUMMARY OF CHAPTER II

USE OF WATTMETER. Measures power; is a combination of ammeter and voltmeter. Current coil is connected in series with circuit and potential coil across circuit. Great care must be used not to get current coil across the circuit.

COMPUTATION OF ELECTRICAL POWER

Watts (power) = amperes $\times$ volts

This should be understood to mean that the Watts consumed in any part of a circuit equal the product of the Amperes flowing through that same part of the circuit times the Volts across that same part. Just as Ohm's Law may be written in three ways, so the power equation may be written in three ways, thus:

$$P = IE = I^2R = \frac{E^2}{R}.$$

ELECTRICAL EQUIVALENT OF HEAT ENERGY

1 B.t.u. = 1050 watt-seconds.
1 Calorie = 4.17 watt-seconds.
$H = .24\ I^2Rt$ calories.
$= \frac{I^2Rt}{1050}$ B.t.u.

MECHANICAL EQUIVALENT OF ELECTRIC POWER

1 Kilowatt = 1.34 horsepower = $1\frac{1}{3}$ horsepower (approx.)
1 Horsepower = 746 watts = $\frac{3}{4}$ kilowatt (approx.)

EFFICIENCY OF ELECTRIC MACHINES. The commercial efficiency of an electric machine, as of any machine, is defined as that percentage which the useful energy given out by the machine is of the energy put into the machine. The same is true of electric transmission systems.

ELECTRIC HEATING APPLIANCES which use the heating effect of an electric current are rapidly coming into use as the price of electricity is getting lower.

PROBLEMS ON CHAPTER II

Prob. 42–2. The temperature in a florist's forcing bed is maintained by means of a soil heater of the type shown in Fig. 58. It is found that in order to maintain the desired temperature the heater regularly is turned on for 7 minutes and off for 21 minutes, and takes 410 watts at 115 volts during the time it is on. At $4\frac{1}{2}$ cents per kilowatt-hour, what is the cost of heating the bed for a 30-day month? How many calories are produced per hour (average)?

Prob. 43–2. A generator delivers a current of 80 amperes at a pressure of 112 volts. What power does it supply in kilowatts? How many horsepower?

Prob. 44–2. The generator in Prob. 43 has a shunt-wound field with a resistance of 210 ohms. How many watts are absorbed in the field?

Prob. 45–2. The armature of generator in Prob. 43 has a resistance of 0.02 ohm and a current of 80.5 amperes. What power is lost in the armature conductors?

Prob. 46–2. If the generator of Prob. 43 requires 13.5 horsepower to drive it, what is its efficiency?

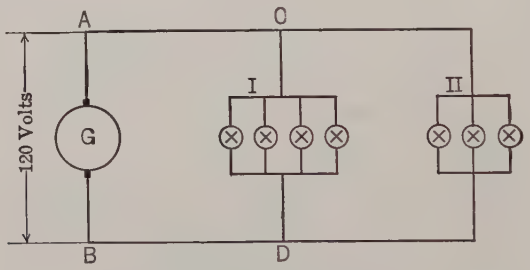


Fig. 59. A generator feeding two groups of lamps.

Prob. 47–2. In Fig. 59 the lamp bank at *I* takes 300 watts at 115 volts; that at *II* takes 160 watts at 113 volts. Internal resistance of generator *G* is 4 ohms. Find:

(1) Resistance of $AC + BD$.
(2) Volts lost between *G* and Group I.
(3) E.m.f. of generator.
(4) Efficiency of transmission.
(5) Average resistance per lamp of Group I.
(6) Average resistance per lamp of Group II.

Prob. 48–2. Assuming an efficiency of 50 per cent for the whole arrangement, what current will be used by a 230-volt electric hoist when raising 2500 pounds 200 feet per minute?

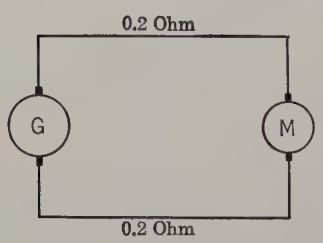


Fig. 60. A generator feeding a motor.

Prob. 49–2. The generator in Fig. 60 delivers 55 kilowatts to the line. Motor uses 50.2 kilowatts. Find:

(*a*) Watts lost in line.
(*b*) Voltage and current of motor.
(*c*) Brush potential of generator.

Prob. 50–2. Assume internal resistance of generator in Prob. 49 to be 0.22 ohm. The owner of the motor pays the owner of the generator 3 cents per kilowatt-hour for energy delivered to the motor. If it costs

the owner of the generator 0.9 cent per kilowatt-hour to deliver electric energy to the line, how much does he earn or lose in a month of 250 hours?

Prob. 51–2. At $3\frac{1}{2}$ cents per kilowatt-hour, how much will it cost, per week of 60 hours, to run a motor having an average load of 40 horsepower and an average efficiency of 90%?

Prob. 52–2. A current of 0.5 ampere flowing through a lamp generates 0.5 B.t.u. of heat in 12 seconds. (*a*) What is the resistance of the lamp? (*b*) What power is expended in the lamp? Express in watts and in horsepower.

Prob. 53–2. Each lamp, Fig. 55, takes 1.5 amperes. Find:

(*a*) Voltage across group *A* and across *B*.
(*b*) Volts lost in *CD* and *MN* and in *DE* and *NP*.
(*c*) Watts delivered to each group.
(*d*) Watts lost in line.
(*e*) Efficiency of transmission.

Prob. 54–2. The generator in Fig. 61 supplies 100 watts at 50 volts to the group of resistors. If R_1 has the same value as R_3 and the voltmeter across R_2 reads 20 volts, find:

(*a*) Size of each resistor.
(*b*) Power used in each resistor.

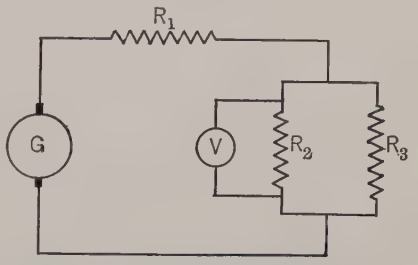


Fig. 61. The generator supplies resistances connected in series-parallel.

Prob. 55–2. Electrical energy is supplied at 4 cents per kilowatt-hour for driving a 10-horsepower motor. The efficiency of the motor at full load is 85%. Find the cost of operating the motor for 100 hours.

Prob. 56–2. At what rate in calories per second is heat supplied to a radio tube heater which draws 2 amperes at 2.5 volts?

Prob. 57–2. A certain battery has an e.m.f. of 6.2 volts and an internal resistance of 0.02 ohm. At what terminal voltage will it deliver 400 watts?

Prob. 58–2. Two generators, *A* and *B*, are connected in series to supply a load of 10 kilowatts. Generator *A* has an e.m.f. of 118 volts and an internal resistance of 0.16 ohm; generator *B* develops an e.m.f. of 130 volts and its internal resistance is 0.25 ohm.

(*a*) What is the voltage across the load?
(*b*) What is the brush potential of each generator?
(*c*) What power does each generator deliver to the line?
(*d*) What is the loss in the armature of each generator?

Prob. 59–2. What is the internal loss in the battery of Prob. 57?

Prob. 60–2. Assuming that 50% of the energy stored in a weight raised above the earth's surface can be converted to electrical energy, how far must a weight of 100 pounds be raised in order to store sufficient energy to light a 100-watt lamp one hour?

CHAPTER III

CONCERNING WIRE

In designing an electric appliance it is often necessary to be able to predetermine two things, — the resistance of the electric circuit, and the current-carrying capacity for a given temperature rise. The first of these is usually computed, the second obtained from tables.

39. Circular Measure. The Circular Mil. Inasmuch as most wire is drawn **round,** it is inconvenient always to measure the cross-sectional area in **square inches.** The cross-section is circular, not square; so we use a circular measure, and not a square measure. In circular measure we make use of a **circular unit of area,** instead of a square unit of area. This circular unit of area is a **circular mil**; it is the **area** of a circle whose diameter is 1 mil in length.

The term " mil " always means $\frac{1}{1000}$. As in our coinage 1 mill = $\frac{1}{1000}$ of a dollar, so in distance, 1 mil = $\frac{1}{1000}$ of 1 inch.

One **circular mil,** then, is the area of a **circle,** the diameter of which is **one mil** or **one-thousandth** of an inch. Just as a **square mil** is the area of a **square** whose sides are 1 mil or one-thousandth of an inch.

The area of a square whose sides measure 5 mils is 5×5 or 25 sq. mils. Similarly the area of a circle whose diameter is 5 mils is 5×5 or 25 circular mils. To find the number of **circular mils** in a **circle** of a given diameter, we have merely to square the number of mils in the diameter.

This may be demonstrated as follows:

Let d equal diameter of a circle in thousandths of an inch or mils. Then in square measure

$$\text{Area of circle} = 0.7854\, d^2 \text{ square mils.}$$

In square measure:

$$\text{Area of a circular mil} = 0.7854 \times 1 \times 1 = 0.7854 \text{ square mils.}$$

There are, therefore, as many circular mils in a circle having an area of 0.7854 d^2 sq. mils as 0.7854 is contained in 0.7854 d^2 or

$$\text{circular mils} = \frac{0.7854\ d^2}{0.7854} = d^2.$$

Prob. 1–3. What is the circular mil area of a wire 0.047 inch in diameter?

Prob. 2–3. What is the circular mil area of a wire $1\frac{1}{8}$ inch in diameter?

Prob. 3–3. Find the circular mil area of a square 0.624 inch on each side.

Prob. 4–3. What is the diameter of a wire containing 1024 circular mils?

Prob. 5–3. What area in circular mils will a circle have which has an area of 2.65 sq. in.?

Prob. 6–3. What is the diameter of a wire containing 26,250 square mils?

40. Unit Wire. Mil-Foot. We may therefore take for our wire of unit cross-section, a wire with a cross-section area of one circular mil. Any wire may then be thought of as composed of a bundle of parallel wires of unit area bound together. The number of circular mils in any given wire is thus merely the number of circular-mil wires which go to make up its cross-section.

We know that the resistance of any parallel bundle of similar wires is equal to the resistance of any one of the wires divided by the number of wires in the bundle. Thus, if we know the resistance of a circular-mil wire of a certain length and material, we can find the resistance of a wire of the same material and length, whatever its diameter. We have merely to divide the resistance of the wire of one circular mil area by the number of circular mils in the area of the given wire.

Of course the resistance varies directly as the length; a wire 3 times as long as another of same cross-section would have 3 times the resistance.

We take for our unit wire, **1 mil-foot.** A wire **one foot long,** and having a cross-section area of **one circular mil** is called a **mil-foot wire.** In order, then, to find the resistance of a certain number of feet of wire of given cross-section, we merely have to multiply the resistance of a mil-foot by the length in feet and divide by the sectional area in circular mils. This is evident from the fact that

the resistance of a wire **increases** as the length increases, but **decreases** as the section area increases.

This is generally expressed in the form of an equation:

$$R = \frac{Kl}{d^2};$$

where R = resistance of wire in ohms;
K = resistivity (resistance of 1 mil-ft. in ohms);
l = length in feet;
d = diameter in mils;

or

d^2 = section area in circular mils.

K for copper = 10.4 ohms at 20° C.

Prob. 7–3. What is the resistance of 10 miles of copper wire $\frac{1}{2}$ inch in diameter?

Prob. 8–3. How many miles of copper wire $\frac{1}{4}$ inch in diameter will it take to make 5 ohms resistance?

Prob. 9–3. Ordinary fixture wire has a diameter of 0.064 inch. What is the resistance per 1000 feet?

Prob. 10–3. Annunciator wire generally has a diameter of 0.040 inch. How many feet does it take to make a resistance of 1 ohm?

Prob. 11–3. What line drop is there in a $1\frac{1}{2}$-mile copper trolley wire carrying 250 amperes, if the wire is 0.525 inch in diameter?

Prob. 12–3. What current can be transmitted over a copper wire 1000 feet long, 0.052 inch in diameter, with but 7 volts "line drop"?

Prob. 13–3. What will be the line drop in voltage and loss in watts per mile in transmitting 12 kilowatts at 550 volts, if a copper wire is used having a diameter of 0.842 inch?

Prob. 14–3. A group of incandescent lamps takes 12 amperes. The line drop is not to exceed 2.6 volts. What must be the size of the copper wire to be used if the lamps are 8000 feet from the generator?

41. Specific Resistance of Metals other than Copper. Although copper, on account of its low resistivity, is the metal most widely used for electrical conductors, aluminum and even galvanized iron are sometimes used. The resistivity of aluminum is 17.0 ohms per mil-foot at 20° C., about 1.6 times that of copper. But its low specific gravity more than counterbalances this, so that for equal lengths and weights aluminum wire has less resistance than copper, and for this reason is coming into more general use.

The resistivity of iron and steel is about seven times that of copper. These materials, therefore, can be used only where a conductor of a large cross-section can be installed, as in the case of a third rail, or where very little current is to be transmitted, as in the case of the telegraph. Some alloys of copper, nickel, zinc, manganese, chromium, etc., have a resistivity of more than 600 ohms per mil-foot. For the resistivity of various pure metals and alloys, see Appendix, Table I.

Prob. 15–3. What is the resistance of a mile of aluminum wire 0.20 inch in diameter?

Prob. 16–3. What cross-section must a steel rail have if the resistance per mile of the rail is to equal the resistance per mile of a copper wire 0.525 inch in diameter? Specific resistance of rail steel may be taken as 90 ohms per mil-foot.

Prob. 17–3. How many feet of constantan wire 0.023 inch in diameter are required in a coil which is to have a resistance of 6.31 ohms?

42. Temperature Coefficient of Resistance. It will be noticed that when the resistance of a mil-ft. of copper wire was given as 10.4 ohms, and of aluminum as 17.0 ohms, the metal was assumed to be at a temperature of 20° C. The reason for stating the temperature is that the resistance of all pure metals **increases** as the temperature **rises.** The amount that the resistance increases per degree rise of temperature, for each ohm of the resistance at the standard temperature, is called the **temperature coefficient of resistance.** For all pure metals this coefficient is nearly the same, lying between the values 0.003 and 0.006, and depends upon what is taken as the standard or initial temperature.

Taking the resistance at 0° C. as the standard, copper increases 0.00426 ohm per degree for each ohm at 0°. Thus if the resistance of a copper wire is 80 ohms at 0° C., it will be increased by 80×0.00426 or 0.341 ohm for each degree the temperature rises above 0° C. At 50° C. the resistance would have increased 50×0.341 or 17.1 ohms. The resistance then at 50° C. would be the resistance at 0° C. plus the increase or, $80 + 17.1 = 97.1$ ohms. Other metals increase at a slightly different rate. See Appendix, Table I.

If we plot a curve between the resistance and the temperature of a coil of Standard Annealed copper wire, we shall obtain a curve similar to that of Fig. 62. It is a straight line except at very low temperatures. If we extend the straight part of the curve, it cuts

the horizontal axis at P or $-234.5°$ C., which may be called the **absolute resistance zero of copper.** The point A on the curve represents a resistance of R_1 ohms and a temperature of t_1 above zero C. or $(234.5° + t_1°)$ above absolute zero. The point B represents a resistance of R_2 ohms and a temperature of t_2 above zero C. or $(234.5° + t_2°)$ above absolute zero. Note that the triangles PAS and PBV are similar. The sides are therefore proportional.

Thus $$\frac{R_1}{R_2} = \frac{234.5 + t_1}{234.5 + t_2}.$$

This is the most workable equation expressing the relation between resistance and temperature. Note that the constant 234.5

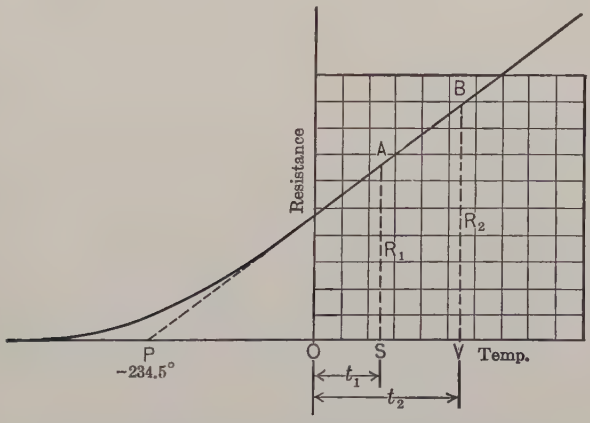


FIG. 62. A curve showing the increase of resistance with temperature.

is merely the reciprocal of the temperature coefficient 0.00426 at zero Centigrade.

To use this equation for any other metal merely replace the constant 234.5 with the reciprocal of the constant for that metal in Table I in the Appendix.

Example 1. The resistance of a coil of copper wire is 48 ohms at 25° C. What is the resistance at 75° C.?

Solution.
$$\frac{R_1}{R_2} = \frac{234.5 + t_1}{234.5 + t_2}.$$
$$\frac{48}{R_2} = \frac{234.5 + 25}{234.5 + 75}.$$
$$R_2 = \frac{309.5 \times 48}{259.5} = 57.2 \text{ ohms.}$$

Example 2. If the coil in the above example had been made of commercial iron wire, what would the resistance at 75° C. have been, assuming the resistance at 25° C. to be 48 ohms? The temperature coefficient of iron at 0° C. is 0.00618.

Solution. The reciprocal of 0.00618 = 162.

$$\frac{R_1}{R_2} = \frac{162 + t_1}{162 + t_2}.$$

$$R_2 = \frac{(162 + t_2)R_1}{162 + t_1}$$

$$= \frac{237 \times 48}{187}$$

$$= 60.8 \text{ ohms.}$$

Prob. 18–3. The resistance of a copper wire is 6.4 ohms at 10° C. What is it at 75° C.?

Prob. 19–3. The resistance of the field coils of a generator is 218 ohms at 20° C. When the coils become heated to 75° C. what will the resistance be?

Prob. 20–3. What will the resistance of a coil of copper wire become at −8° C. if the resistance is 200 ohms at 20° C.?

Prob. 21–3. What will the resistance of a coil of copper wire become at −8° C., if the resistance is 200 ohms at 0° C.?

Prob. 22–3. A nickel coil has a resistance of 12.5 ohms at 20° C. How high will the temperature be when the resistance reaches 14.7 ohms?

Prob. 23–3. The resistance of a field coil of copper is 130 ohms at 12° C. What will it be at 105° C.?

Prob. 24–3. What will be the resistance of a silver wire at 10° C. if the resistance at 45° C. is 1.56 ohms?

43. Temperature Change Measured by Change in Resistance. Electrical machines are generally sold under a guarantee that the wire in the coils, etc., will not rise more than a given number of degrees when running under a specified load for a specified time.

By measuring the resistance of the coils when at room temperature (20° C. or 68° F.) and then again at the end of the test, and applying the equation for temperature change, the average temperature rise can easily be found.

An instrument called a **resistance pyrometer** is based on this rise of resistance with rise of temperature. It is used for accurate temperature measurements from −250° C. to +1000° C. See Art. 54 for the description of one type of resistance pyrometer.

Example 3. The primary coils of a transformer have a resistance of 5.48 ohms at 20° C. After a run of 2 hours the resistance has risen to 6.32 ohms. What is the average temperature rise of the coil?

Solution.

$$\frac{R_2}{R_1} = \frac{234.5 + t_2}{234.5 + t_1}.$$

$$t_2 = \frac{(234.5 + t_1)R_2}{R_1} - 234.5$$

$$= \frac{(234.5 + 20)\ 6.32}{5.48} - 234.5$$

$$= 59.0°.$$

$$\text{Temperature rise} = t_2 - t_1$$

$$= 59.0 - 20$$

$$= 39.0°.$$

Prob. 25–3. The cold (20° C.) resistance of an armature was 3.64 ohms. The hot resistance was 4.86 ohms. What was the temperature rise?

Prob. 26–3. It is generally specified that the temperature of the field coils of a dynamo must not rise more than 55° on full load. The resistance of a set of field coils before running was 80 ohms at 20° C. After run of 3 hours at rated load the resistance became 92.4 ohms. Did machine meet the usual specifications?

Prob. 27–3. The resistance of certain coils in a machine was found to be 7.46 ohms at a temperature of 40° C. It was specified that running continuously under full load the temperature of these coils should not exceed 95° C. (the usual limit). The resistance of the coils, measured after a long run, was found to be 9.59 ohms. Did the machine meet this specification?

44. Temperature Coefficient of Alloys, etc. It has been stated that the temperature coefficient of resistance for all pure metals is nearly the same, that is, somewhere about 0.4 per cent. Alloys, though of a much higher resistance per mil-foot, usually have much lower coefficients, some having practically zero and even negative coefficients at certain temperatures.

" Manganin," for instance, an alloy consisting of copper, nickel and iron-manganese, has a resistance per mil-foot of from 250 to 450 ohms, according to the proportions of the different metals used, and a temperature coefficient so low as to be practically negligible for most purposes.

Certain substances, notably carbon, porcelain, glass, electrolytes, etc., **decrease** in resistance very rapidly when heated. The cold resistance of a carbon lamp filament is about twice as great as the " hot " resistance. The porcelain " glower " of a Nernst lamp when cold is a good insulator, but when heated to incandescence it be-

comes a conductor. The filaments of tungsten lamps are pure metal and accordingly have a positive coefficient which is about 0.0049 at 0° C.

Prob. 28–3. In order to replace a burned-out electric heating element, a sample of the original wire was obtained for measurement. The sample was 18 inches long and 0.010 inch in diameter. Its resistance was measured at 20° C. and at 100° C. and found to be 9.00 ohms and 9.31 ohms, respectively. Using data from Appendix, Table I, what material was used in the heater?

45. Copper-Wire Tables. Tables have been prepared by the Bureau of Standards and adopted by the A. I. E. E. which give the resistance of 1000 feet of Standard Annealed copper wire of different standard sizes and at several temperatures. The sizes are designated by gauge numbers, diameter in mils, and section area in circular mils, etc. There are several standard wire gauges, B. & S. (Brown & Sharpe) in general use in America, B. W. G. (Birmingham Wire Gauge) in general use in Great Britain. Table No. II gives the complete data for the American, or Brown & Sharpe, gauge numbers. By means of these tables it is easy to find the resistance of any length of wire of a given standard section area, etc.

If the following simple facts concerning this table are memorized, the gauge number, resistance and size of any wire can be roughly estimated without reference to the table.

No. 10 wire is practically $\frac{1}{10}$ inch (100 mils) in diameter, 10,000 circular mils in area and has practically 1 ohm per 1000 feet. As the wires grow smaller, every third gauge number **halves** the area and **doubles** the resistance. For instance, No. 13 has about 5000 circular mils area and 2 ohms resistance; No. 16 has 2500 circular mils area, and 4 ohms per 1000 feet, etc. As the wires increase in size, every third gauge number **doubles** the circular mils area and **halves** the resistance; No. 7, for instance, has practically 20,000 circular mils, and 0.5 ohm per 1000 feet, etc.

Example 4. What copper wire (B. & S. gauge) should be used to transmit electric power 2 miles (out and back); resistance not to exceed 2.7 ohms; temperature to be assumed, 20° C.?

Solution.

$$2 \text{ miles} = 2 \times 5280 = 10{,}560 \text{ feet.}$$

$$2.7 \text{ ohms for 2 miles} = \frac{2.7}{10.56} \text{ ohm per thousand feet}$$

$$= 0.256 \text{ ohm per thousand feet.}$$

From wire table:

No. 5 = 0.3133 ohm per thousand feet;
No. 4 = 0.2485 ohm per thousand feet.

No. 4 must be used in order not to exceed limit of 0.256 ohm per thousand feet.

Prob. 29–3. What size wire (B. & S.) will have a resistance of practically 1 ohm per 1000 feet?

Prob. 30–3. Sixty 50-watt, 110-volt lamps are to be used in a building so situated that it requires 300 feet of feeder wires (each side of line) from the generator to the distributing point. What size wire should be run in order that there shall not be more than 2 volts drop in the feeders?

Prob. 31–3. How far can 40 amperes be transmitted through a No. 8 wire, B. & S. gauge, with 4 volts line drop?

Prob. 32–3. What wire, B. & S., has about 2 ohms per mile?

Prob. 33–3. How many miles of No. 00 wire will it take to make 8 ohms?

Prob. 34–3. A coil for an electromagnet has 1800 turns of No. 32 B. & S. copper wire. The average length of a turn is 12.5 inches. What is the resistance of the coil?

Prob. 35–3. It is desired to construct a coil of not more than 150 ohms resistance. The coil must have 200 turns of about 16 inches average length. What size wire, B. & S., should be used?

46. Aluminum Wire. We have noted that although the resistance of aluminum wire is 17.0 ohms per mil-foot (practically 1.6 times that of copper) its weight is only 0.3 that of copper. For this reason some transmission lines are strung with aluminum wire. While this necessitates a larger wire for the same resistance per 1000 feet, the weight of such a wire will be less than that of a copper conductor. However, such a line possesses one disadvantage over a copper line in that aluminum melts at a lower temperature than copper. A short circuit which would burn up but a few inches of a copper wire is likely to burn out long sections of an aluminum line. Aluminum line wires also offer greater surface to the wind and will accumulate a greater weight of sleet per foot than will copper conductors. For this reason, aluminum wire with a central steel core is now extensively used.

Example 5. What size will an aluminum wire be which has the same resistance as a No. 4 copper wire?

Solution. No. 4 copper wire has a resistance per 1000 feet of 0.2485 ohm. The resistance of an aluminum wire is found from the equation

$$R = \frac{17.0\,l}{d^2}.$$

Thus,

$$0.2485 = \frac{17.0 \times 1000}{d^2}.$$

Or,

$$d^2 = \frac{17.0 \times 1000}{0.2485}$$
$$= 68{,}400 \text{ circular mils.}$$

This would require an aluminum wire of gauge No. 1, of 83,690 circular mils. No. 2 has a cross-section of only 66,370 circular mils. A No. 1 aluminum wire would be called the **equivalent** of No. 4 copper wire.

Example 6. How would the weight per 1000 feet of the equivalent aluminum wire in the above example compare with the weight of the copper wire?

Solution.

Weight of 1000 feet of No. 4 copper wire = 126 pounds.
Weight of 1000 feet of No. 1 copper wire = 253 pounds.
Weight of 1000 feet of No. 1 aluminum wire = 0.30×253
= 75.9 pounds.

Thus the weight of the aluminum equivalent is only $\frac{75.9}{126}$ or 0.602 of that of the copper wire.

Prob. 36–3. What size aluminum wire will have the same resistance per mile as a No. 8 B. & S. copper wire?

Prob. 37–3. What size copper wire is equivalent to a No. 00 aluminum wire in resistance per 1000 feet?

Prob. 38–3. An aluminum cable which will be equivalent to a No. 0 solid copper wire is to be made up of 19 strands. What size must the strands be?

47. Safe Carrying Capacity for Copper Wires. In installing wire in buildings, it is necessary to take into account another factor besides the voltage drop when determining the size to be used. An electric current heats any conductor through which it passes. If heat is generated in the wire faster than it can be dissipated from the surface of the wire, the temperature will continue to rise as long as this condition exists. It is necessary, therefore, to select a wire which will dissipate the heat generated by the current at such a rate that the temperature never rises high enough to cause the insulation to deteriorate.

Accordingly the National Board of Fire Underwriters has issued a table of the safe current-carrying capacity of copper wire of the

sizes used in house wiring. Wherever local regulations do not specify otherwise, the currents carried by any interior wiring should not exceed the values given in this table. Table III, Appendix.

Example 7. It is desired to install a conductor to carry 40 amperes. What size copper wire should be used?

Solution.

From the table, No. 6, rubber insulated wire will carry 50 amperes and is the size to be used.

If weather proof wire can be used, No. 8 will do.

48. Determination of Right Sizes for Interior Wires. In deciding upon the wire sizes which should be used in the different parts of any interior distributing system it is necessary to take into consideration two factors:

First: The size in each section must be such that the current in no wire exceeds the amount given in the Underwriters' Table of safe carrying capacities for wires.

Therefore, it is necessary to determine accurately the current which each wire must carry and select its minimum size from the above table on this basis.

Second: The voltage drops throughout the system must then be computed in order to make certain that they do not exceed certain values. For if lamps are to be operated anywhere on the system, a variation of more than 5 per cent in the voltage at the lamps causes an unpleasant variation in the illumination. If the entire load consists of motors, heating appliances, etc., a drop of 10 per cent is usually allowable. Any greater drop than this, however, would have a bad effect upon the speed of the motors. Where excessive drops are found, larger wires must be used.

Example 8. The panel board in Fig. 63 is situated 150 feet from the main switch. From the board run three branch lines each 50 feet long. Each branch is supplied with 6 outlets for 100-watt, 110-volt lamps. What size must the mains be?

Solution. Each branch line carries 6×100 or 600 watts. This means $\frac{600}{110}$ or 5.45 amperes in each branch wire.

Branch wires. Size according to Table III.

Although, according to the table, No. 16 wire would carry this current safely, it will be noted that no size smaller than No. 14 can be installed in order that the wires may have sufficient mechanical strength.

Mains. Size according to Table III.

Each main must carry the current in all three branches or $3 \times 5.45 = 16.35$ amperes.

According to Table III, No. 12 wire must be used as the next size in common use, No. 14, can carry but 15 amperes.

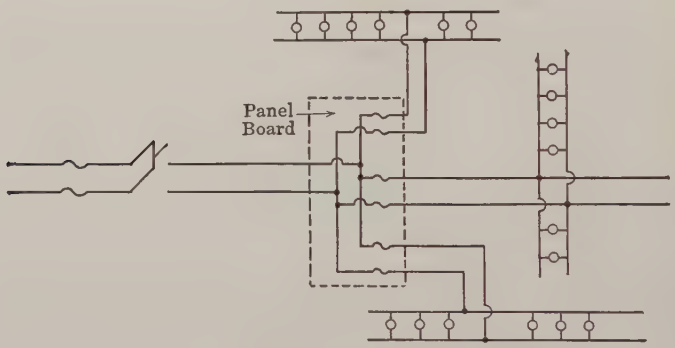


Fig. 63. A conventional wiring diagram.

Check on above sizes for voltage drop.

Distance from panel to load center* of branches	= 50 feet
Length of wire in each branch = 2×50	= 100 feet
Resistance of 1000 feet No. 14 (Table II)	= 2.525 ohms
Resistance of 100 feet of No. 14 = $\frac{1}{10}$ of 2.525	= 0.252 ohm
Voltage drop in each branch = 5.45×0.252	= 1.38 volts
Length of mains = 2×150 feet	= 300 feet
Resistance of 1000 feet of No. 12 (Table II)	= 1.588 ohms
Resistance of 300 feet No. 12 = 0.3 of 1.588	= 0.476 ohm
Voltage drop in mains = 16.35×0.476	= 7.78 volts
Total drop from main switch to lamps = $7.78 + 1.38$	= 9.16 volts

Percentage line drop $= \frac{9.16}{110} = 8.3\%$.

This is nearly twice as great a drop as is allowed in good practice because the brightness of the lamps would vary through wide ranges depending on how many were in use at one time. When only a few lamps were on, the voltage at these lamps would be about the same as that at the main switch, $110 + 9.16$ or about 119 volts. This would cause the lamps to glow far above their rated candle power and would either burn them out at once or shorten their life to a small per cent of the normal rating. It would, therefore, be necessary to install larger than No. 12 mains. Let us try No. 10.

* **Note.** The load center is that point on the branch line at which, for convenience in calculation, all the lamps may be considered to be concentrated.

Resistance of 300 feet of No. 10 = 0.3 of 0.9989 = 0.300 ohm
Drop in main = 0.300 × 16.35 = 4.91 volts
Total drop = 4.91 + 1.38 = 6.29 volts
Percentage drop = $\frac{6.29}{110}$ = 5.73%

This is still somewhat too large a drop. It is, therefore, necessary to use No. 8 mains.

Prob. 39–3. What size mains would be used in the above example if the panel board were situated 50 feet from the main switch and same loads were on the branch lines?

Prob. 40–3. An installation requires fifty 50-watt 110-volt lamps. The panel board is situated 280 feet from the main switch. What size main should be run? Note that the Underwriter rules do not ordinarily allow more than twelve 50-watt lamps on a single branch line.

Prob. 41–3. If the panel board in Prob. 40 could be placed 60 feet from the main switch what size mains might be used?

Prob. 42–3. If the load on the installation of Prob. 40 consisted of motors instead of lamps, but using the same total load, what size wire could be used for the mains?

SUMMARY OF CHAPTER III

MIL-FOOT. A wire one foot long and one circular mil cross-section area.

A circular mil is the area of a circle one mil, i.e. one thousandth of an inch, in diameter.

The area of a circle in circular mils equals the square of the diameter in mils. $A = d^2$.

SPECIFIC RESISTANCE, OR RESISTIVITY. The resistance of a mil-foot of any material. (Sometimes also taken as the resistance of a centimeter cube, from face to face opposite.)

The specific resistance of commercial copper at 20° C. equals 10.4 ohms. Symbol = K.

RESISTANCE OF WIRE. The resistance of a wire equals the specific resistance (resistance per mil-foot) multiplied by the length in feet and divided by the section area in circular mils.

$$R = \frac{Kl}{d^2}$$

For copper at 20° C.

$$R = \frac{10.4l}{d^2}.$$

TEMPERATURE COEFFICIENT OF RESISTANCE. For every degree centigrade rise in temperature, the resistance of a pure copper wire increases 0.426 of 1% of its resistance at 0° C. This

constant 0.00426 is called the Temperature Coefficient of Resistance. It has a much lower value in alloys and a negative value in carbon, porcelain, etc. Other pure metals have about the same coefficient as copper.

THE RISE IN TEMPERATURE of copper windings, etc., can be computed by measuring the cold and hot resistance and applying the equation

$$\frac{R_1}{R_2} = \frac{234.5 + t_1}{234.5 + t_2}.$$

AMERICAN or BROWN & SHARPE WIRE GAUGE. The usual sizes of copper wire have been standardized in America according to the gauge used by Brown & Sharpe.

ALUMINUM WIRE is sometimes used on account of its small weight and moderate specific resistance. It has 0.3 the weight of copper and 1.64 the resistance.

SAFE CARRYING CAPACITY OF COPPER WIRES. In interior wiring no wire must carry greater current than that specified for that wire in Table III prepared by the National Fire Underwriters.

TO DETERMINE THE PROPER WIRE SIZE in an interior installation. (1) Determine current to be carried by each section and select a wire size from the table of safe carrying capacities which is recommended for the current as determined. (2) Check the voltage drop in the line. This must not exceed 5% for lamp loads or 10% for motor loads.

PROBLEMS ON CHAPTER III

Prob. 43–3. A rough rule for the safe carrying capacity of copper is, " 1000 amperes per square inch cross-section." According to this rule what should be the diameter of a round wire capable of carrying 400 amperes?

Prob. 44–3. According to rule in Prob. 44, what should be safe carrying capacity of No. 00 B. & S. copper wire? Compare value with that in Table III.

Prob. 45–3. How many volts are required to send 8 amperes through 600 feet of copper wire 0.064 inch in diameter?

Prob. 46–3. How far will a pair of copper line wires transmit 45 amperes with a line drop of 8 volts, if the wire is 0.162 inch in diameter?

Prob. 47–3. What resistance will a one-mile copper wire have which is three-quarters of an inch in diameter?

Prob. 48–3. If the wire in Prob. 47 is of aluminum, what resistance will it have?

Prob. 49–3. If the wire in Prob. 47 is of iron, what resistance will it have? Use 75 as the mil-foot resistance of iron.

Prob. 50–3. Construct a graph from the data of Appendix Table III, plotting current-carrying capacity of rubber insulated wires (vertical) against cross-section area in circular mils. Only data for wires of 100,000 circular mils or greater need be used. Verify on this plot that for wires one inch in diameter or larger the current capacity is given by the rule "400 amperes per million circular mils plus 250 amperes." Why is this plot not a straight line (carrying capacity directly proportional to area) throughout the entire range of area?

Prob. 51–3. State a rule similar to that given in Prob. 50 which will give accurately the current-carrying capacity of wires from 500,000 to 1,000,000 circular mils in area.

Prob. 52–3. What would be the current-carrying capacity of No. 0000 B. & S. wire if computed from the rule of Prob. 50?

Prob. 53–3. What would be the current-carrying capacity of 1,800,000 circular-mil wire if computed from the rule of Prob. 51?

Prob. 54–3. A two-wire line (one "line" and one "return") of 600,000 circular-mil cable delivers 450 amperes at 440 volts at a distance of 1000 feet. If the voltage at input end of line is maintained constant, by what percentage will the voltage at the output end rise as the load is reduced from 450 amperes to zero?

Prob. 55–3. What must be the size (Underwriters' Table) of two (equal) smaller wires in parallel, to deliver the same load as the single large wire of Prob. 54? Rubber-insulated wire in both cases. Assuming the cost of conductor to be in direct proportion to weight of metal, what per cent is saved by this substitution?

Prob. 56–3. Calculate for the parallel feeders of Prob. 55 the per cent rise of voltage at output end of feeders from full load to zero load, voltage at input end being maintained constant. Compare this result with the corresponding figure for Prob. 54, and draw conclusions.

Prob. 57–3. What size iron wire will have the same resistance per mile as a No. 00 B. & S. aluminum wire? Use $K = 75$ for iron.

Prob. 58–3. What size iron wire will have the same resistance per mile as a No. 0 B. & S. copper wire? Use $K = 75$ for iron.

Prob. 59–3. To what size copper wire is a stranded aluminum cable equivalent which is made up of 19 strands each 0.045 inch in diameter?

Prob. 60–3. In a certain mercury thermometer, the diameter of the capillary bore is 0.002 inch and the scale is divided so that a $\frac{3}{4}$ inch length represents 10° C. What would be the resistance of the mercury column measured between the 0° C. mark and the top of the column when the thermometer reads 25° C.

Prob. 61–3. In Fig. 64 the wires used are No. 8 B. & S. copper.

AB = 1700 feet;
BC = HJ = 800 feet;
CD = HF = 800 feet;
EF = 100 feet.

Each lamp takes 1.4 amperes. Find:

(1) Volts lost in each section.
(2) Voltage across each group.
(3) Efficiency of transmission.

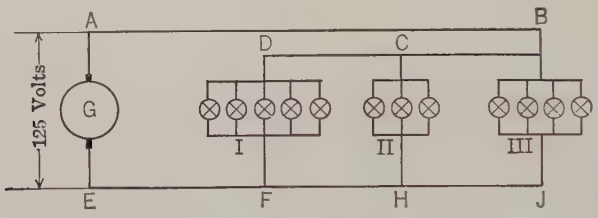


Fig. 64. A method of equalizing the voltage across lamps.

Prob. 62–3. What will be the drop per mile in a line wire consisting of No. 10 B. & S. copper carrying 25 amperes?

Prob. 63–3. What will be the line loss in voltage and in watts per mile in transmitting 10 kilowatts at 650 volts if a No. 00 copper wire is used?

Prob. 64–3. What voltage is required to send 50 amperes through 800 feet of copper wire 0.162 inch in diameter?

Prob. 65–3. A copper wire carries 175 amperes for 1600 feet. If its diameter is 0.464 inch, what is the voltage drop?

Prob. 66–3. A $\frac{1}{8}$-inch copper wire carries 45 amperes. What is the voltage drop per mile?

Prob. 67–3. What size copper wire must be used if 18 volts are to be used in forcing 40 amperes through one mile of wire?

Prob. 68–3. How many amperes can be forced through one thousand feet of copper wire $\frac{1}{8}$ inch in diameter with a voltage drop of 6.5 volts?

Prob. 69–3. A trolley wire consists of No. 0 hard-drawn copper. What will be the drop per mile on a day when the temperature of the wire is $-10°$ C., if the wire carries 150 amperes?

Prob. 70–3. A generator is supplying 10 amperes to 50 arc lamps in series. Drop across each lamp = 45 volts. Twelve miles of No. 10 wire are used for line. Find brush potential of generator if temperature of wire is 0° C.

Prob. 71–3. If the generator of Prob. 70 has 25 ohms resistance and line wire is at temperature of −5° C., what e.m.f. and brush potential must be used in Prob. 70?

Prob. 72–3. What will be the line loss in voltage and in watts per mile in transmitting 10 kilowatts at 220 volts if a No. 00 copper wire is used? What conclusion do you draw from a comparison of this result and that of Prob. 63–3?

Prob. 73–3. A coil of No. 32 wire is found to have a resistance of 98.7 ohms. How many feet are there in the coil?

Prob. 74–3. An electric soldering iron has a resistance composed of commercial iron wire of 80 ohms when cold (20° C.). When hot the resistance rises to 150 ohms. What temperature is reached, assuming a constant temperature coefficient?

Prob. 75–3. A building situated 450 feet from a 115-volt generator is to be supplied with sufficient current from the generator to light 500 lamps each taking 0.52 ampere. The efficiency of transmission must be 95%. What size copper wire must be used?

Prob. 76–3. A No. 0 two-wire feeder was originally installed to carry a load of 120 amperes, but the load grew to 220 amperes and the station operator attempted to meet the situation by paralleling the line with a feeder of No. 1 size. Both feeders have rubber insulation. Do you think that together they are sufficient?

Prob. 77–3. If the feeder of No. 1 wires of Prob. 76 were carrying the largest current permitted for it by N. E. C. (Table III), what would be the total voltage drop along the wires? The feeders are 500 feet long. How many amperes must the No. 0 parallel feeder then be carrying, since the drop must be identically the same along both feeders? What ratio exists between current in each feeder and total current? How do these results bear on Prob. 76?

Prob. 78–3. A certain resistance unit contains 80 feet of No. 40 B. & S. German silver wire. What is its resistance at 40° C.?

Prob. 79–3. An alloy containing 73% copper, 24% manganese, and 3% nickel has a resistivity (mil-foot) of 289 and a temperature coefficient of resistivity of −0.00003. If the resistor of Prob. 78 had been made of this material what would its resistance have been at 40° C.? At 100° C.?

Prob. 80–3. A resistor is to be constructed of two parts connected in series. One part is to be made of German silver, the other of the alloy of Prob. 79. What percentage of each material should be used in order to make the overall temperature coefficient of resistance equal zero at 0° C.?

CHAPTER IV

MEASUREMENT OF RESISTANCE

In scientific work, precise methods of measuring resistance are indispensable aids to the control of experimental investigation and the interpretation of results. Likewise in industry, resistance measurement plays a large part in the manufacturing, testing, repair and maintenance of electrical equipment. The methods of measurement vary widely in their precision, speed and applicability; this chapter will discuss only those methods in general use.

49. Ammeter-Voltmeter Method. One of the simplest methods of determining the resistance of an appliance is to pass a current through it and measure the current with an ammeter, and the voltage drop with a voltmeter. The resistance is then given by Ohm's Law as

$$R = \frac{E}{I}.$$

If proper ranges of voltmeters and ammeters are available, this method is readily applied with precision sufficient for most commercial measurements. But simple as the method is, certain precautions must be taken in the use of the instruments and in computing resistance from the results secured.

Ohm's Law as stated above gives the correct value of resistance only when the voltage used is that existing **across** the resistance, and the current is that which at the same time flows **through** the resistance. We have seen, however, that the voltmeter itself requires current through it to operate it, and the ammeter likewise will cause a voltage drop because of its own internal resistance.

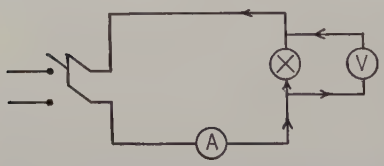


FIG. 65. One method of connecting a voltmeter and an ammeter to determine the resistance of a lamp.

Now suppose we are measuring the resistance of an ordinary incandescent lamp, and we have connected the instruments as in Fig. 65. In this connection, the voltmeter V indicates the voltage

across the lamp correctly, but the current indicated by the ammeter A is the sum of the current in the lamp and in the voltmeter. Hence this ammeter reading is too high by the amount of current taken by the voltmeter, and should not be used in Ohm's Law unless the error thus produced is within allowable limits.

In this example, an ammeter of 0.045 ohm and a voltmeter of 15,000 ohms might be typical. Assume both instruments are correctly calibrated and suppose the voltmeter reads 110 volts and the ammeter 0.500 ampere. According to these readings the resistance of the lamp $= \frac{110}{.500} = 220$ ohms. We know that 110 volts is the true voltage across the lamp and is also the voltage across the internal resistance of the voltmeter. Hence, the current through the voltmeter is, by Ohm's Law,

$$\frac{110}{15{,}000} = 0.0073 \text{ ampere.}$$

Since the ammeter reading is too high by this amount, the true current through the lamp must be

$$0.500 - 0.0073 = 0.493 \text{ ampere.}$$

True resistance of lamp $= \frac{110}{.493} = 223.1$ ohms. The percentage error in lamp resistance if the uncorrected ammeter reading were used would be $\frac{223.1 - 220}{223.1} = 1.39$ per cent. This error is altogether too large for any such simple measurement.

Suppose that we connect up the same instruments as in Fig. 66 with the voltmeter across both the lamp and ammeter.

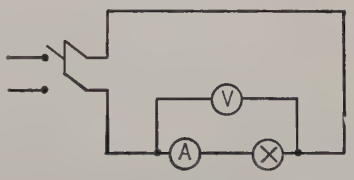


Fig. 66. Another method of connecting a voltmeter and an ammeter to determine the resistance of a lamp.

The ammeter now reads the current through the lamp only, or 0.493 ampere, and is therefore correct. But the voltmeter reads the voltage across both the lamp and ammeter and, therefore, reads too high, by the amount of the voltage across the ammeter.

Voltage across ammeter, by Ohm's Law, is

$$0.493 \times 0.045 = 0.0222 \text{ volt.}$$

The true voltage across the lamp is

$$110 - 0.0222 = 109.98 \text{ volts.}$$

The per cent error in the voltage reading is

$$\frac{0.0222}{109.98} \times 100 = 0.02 \quad \text{or} \quad \frac{1}{50} \text{ per cent}$$

This error would be considered allowable in any grade of commercial work and is, in fact, smaller than the error produced by normal inaccuracy in the meter readings. Hence, the readings obtained from the connections of Fig. 66 are sufficiently accurate for direct use in Ohm's Law.

It is apparent that when a rather large resistance is to be measured, the method of Fig. 66 should be used. In the above case, the lamp resistance is 223.1 ohms. This resistance is almost 5000 times that of the ammeter in series with it and the error in voltage reading due to the ammeter is hence only $\frac{1}{5000} \times 100 = \frac{1}{50}$ of 1 per cent. On the other hand, the voltmeter resistance is only 67 times that of the lamp and the error in ammeter reading produced by using the connections of Fig. 65 is $\frac{1}{67} \times 100 = 1.5$ per cent. If the lamp resistance had been one tenth as great, the ratio of voltmeter resistance to lamp resistance would have been 670, and the error in ammeter reading only 0.15 of 1 per cent, which is allowable for most work.

In general, the connection to be used depends upon the ratio of meter resistance to the resistance being measured. If the voltmeter resistance is large compared to that of the specimen, the connections of Fig. 65 can be used; if the specimen resistance is large compared to the ammeter resistance, the connections of Fig. 66 can be used. Sometimes either method can be used. If the lamp resistance above were 26 ohms, either connection would give a result within $\frac{1}{6}$ per cent of the true value, using these particular meters. Since even high-grade commercial instruments are precise to only about $\frac{1}{4}$ per cent, we cannot expect the results of measurement by this method to be any more precise.

This method of measuring resistance has the advantage that it is very simple and is done with instruments usually available in all laboratories and industrial plants. It has the disadvantage that the voltmeter must be accurately calibrated to indicate volts and the ammeter to indicate amperes. Furthermore, in order to determine the probable error one must know the resistance of each instrument.

Prob. 1–4. A low resistance is to be measured. A 3-volt voltmeter of 300 ohms resistance and an ammeter of 0.0009 ohm resistance are arranged as in Fig. 66. The voltmeter reads 2.15 volts and the ammeter 65.0 ampere. What error is made in this measurement?

Prob. 2–4. If the instruments of Prob. 1 are arranged as in Fig. 65, how great will the error be?

Prob. 3–4. It is desired to measure the resistance of an electric device. Instruments are arranged as in Fig. 65. The ammeter has a resistance of 0.045 ohm; the voltmeter has a resistance of 16,500 ohms. Ammeter reads 0.365 ampere. Voltmeter reads 112 volts. If instruments are correct, what is the percentage error in resistance measurement due to the arrangement of the instruments?

Prob. 4–4. If the instruments of Prob. 3 were arranged as in Fig. 66, (*a*) What would each read? (*b*) What would be the percentage error in resistance measurement for this arrangement?

50. Voltage Comparison Method. If a **known** or **standard** resistance is connected in series with a resistance to be measured and

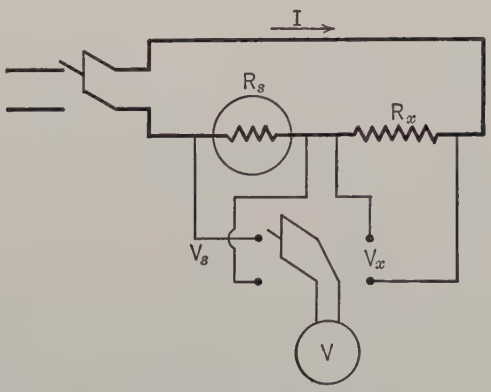


FIG. 67. The current through the standard and the unknown resistance is the same. The voltmeter measures the voltage drop across each.

a current is passed through the series circuit, the voltage drop across each resistor will be proportional to its resistance. By thus comparing the voltage drop across an unknown resistance with that across a known resistance we can compute the value of the unknown resistance. Figure 67 shows a convenient method of making these measurements. The unknown resistance is R_x, and the known resistance R_s. By means of the double-pole double-throw switch the voltmeter V can be quickly connected across each resistance in

turn. The two voltages will be

$$V_x = R_x I$$

and

$$V_s = R_s I.$$

Since the current I is the same in each resistance,

$$\frac{R_x}{R_s} = \frac{V_x}{V_s},$$

or the value of the unknown resistance is

$$R_x = R_s \frac{V_x}{V_s}.$$

In this equation, R_s, V_x and V_s are all known so that R_x is easily found. The resistance is simply proportional to the voltage drop; if the voltage drop across the unknown is twice that across the known resistance, the unknown resistance is twice that of the known, etc.

In deriving the above formula, it was assumed that the current I was the same in each resistance. Actually, it can be proved that the currents in the two resistances will not differ, even though the resistances are not equal, regardless of the shunting resistance of the voltmeter. It is relatively easy to keep the total applied voltage constant, and it can be shown that, in this case, the above formula is exact. Of course, the same instrument must be used, without change of scale, for both measurements.

(1) The current need not be known since it is not used in the computation.

(2) The voltmeter need not be accurately calibrated, provided its deflections are proportional to voltage. In other words, the method does not depend on accurate measurement of voltage or current, but simply on the accurate comparison of two voltages.

(3) A sufficiently accurate standard of resistance is easily made and is reliable over long periods of time. It does not require the fairly frequent calibration that must be given instruments.

(4) With constant applied voltage, no error is produced by using a low-resistance voltmeter.

Prob. 5–4. In a certain measurement of resistance by the voltage comparison method, the standard had a resistance of 106.8 ohms. A voltmeter of 100,000 ohms resistance gave readings as follows:

$$V_s = 52.3 \text{ volts.}$$
$$V_x = 98.6 \text{ volts.}$$

(*a*) What is the value of the unknown resistance?

(*b*) Assuming a constant applied voltage of 200 volts, what voltage readings would be obtained using a voltmeter of 1000 ohms resistance?

51. The Wheatstone Bridge. By far the most important device for precise measurement of resistance is the well-known Wheatstone bridge. It consists fundamentally of a loop of four resistances, R_x, R_1, R_2 and R_3, of which R_x is unknown; see Fig. 68. An electric current, generally from a battery, is sent into the loop at A, and divides into two parts, one part taking the path composed of the resistances R_1 and R_3, the other part taking the path made up of R_2 and R_x. The currents come together again at C and return to the battery. A galvanometer G is now "bridged" between the two branches from B to D.

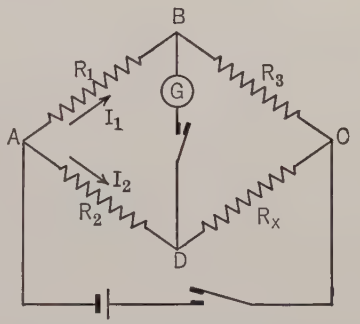


FIG. 68. The conventional diagram of a Wheatstone bridge.

R_x is the unknown resistance. R_1, R_2, R_3 are variable resistances and are now adjusted until, with the galvanometer key in the circuit closed, the galvanometer gives no deflection. This is called "balancing" the bridge. The points B and D are then at the same voltage level or potential. For we have seen that where there is any difference of potential between two points, a current always flows when they are joined by a conductor. Since the points B and D are joined through the galvanometer, and no current flows, they must be at the same potential.

Let us now consider the voltages across the separate resistances. We have said that the current divides into two branches, ABC and ADC. Now the current in each branch need not be the same, so let us designate the current in the branch ABC, by the letter I_1, and the current in the branch ADC, by the letter I_2.

Then the voltage across AB (that is, R_1) $= I_1R_1$;
Then the voltage across AD (that is, R_2) $= I_2R_2$.

But we have said that the points B and D are at the same level (since no current flows through the galvanometer) and, therefore, the voltage drop from A to B (I_1R_1) equals the voltage drop from

A to D (I_2R_2). That is,

$$I_1R_1 = I_2R_2. \tag{1}$$

For the same reasons, the voltage drop from C to B, (I_1R_3), must equal the voltage drop from C to D, (I_2R_x); that is,

$$I_1R_3 = I_2R_x. \tag{2}$$

Dividing equation (1) by equation (2) we get,

$$\frac{I_1R_1}{I_1R_3} = \frac{I_2R_2}{I_2R_x}.$$

Cancelling the I_1's and I_2's,

$$\frac{R_1}{R_3} = \frac{R_2}{R_x}.$$

This is the fundamental equation of the Wheatstone bridge.

Since R_x is the only unknown, the equation can be solved and the value of R_x found.

The ratio $\frac{R_1}{R_3}$ is generally made to equal some such fraction as $\frac{1}{10}$ and then it follows that $\frac{R_2}{R_x} = \frac{1}{10}$. This makes R_x 10 times the value of R_2. If R_3 is made the same as R_1, then R_x will be the same as R_2. R_2 is called the Rheostat arm.

In writing the equation for the bridge, notice that we started at A (a point where the battery line came in). For one side of our equation we read along one branch. That is, we said, "R_1 is to R_3" or $\frac{R_1}{R_3}$.

For the other side of the equation we went **back to the point A again** and read along the other branch, saying "R_2 is to R_x" or $\frac{R_2}{R_x}$. We did **not continue around** the loop and make the second side of the equation "R_x is to R_2." Had we done so, the two ratios would have been unequal and we could have formed no equation.

It is to be noticed, then, that in making the fundamental equation for a Wheatstone bridge, we read along one branch for one side of the equation, then go back to the same starting point and read along the other branch for the other side of the equation.

Prob. 6–4. In a Wheatstone bridge arranged as in Fig. 68, a balance is obtained when $R_1 = 10$ ohms, $R_2 = 18.3$ ohms, $R_3 = 100$. What is the value of R_x?

Prob. 7–4. In Fig. 68, if $\frac{R_1}{R_3} = 100$, and $R_x = 42.4$, what is the value of R_2?

Prob. 8–4. What value is a convenient value for R_3, if R_1 is 1000 ohms and R_2 is 58 ohms and it is desired to measure R_x, which is known to be about 600 ohms?

52. Construction of Two Common Types of Bridges. In many cases, standard resistance units are assembled as in Fig. 68 to form a Wheatstone bridge. Where only a limited amount of equipment is available, and the resistance need not be measured with too high precision, this method has the advantage that the bridge resistors can be used for general purposes when not required for resistance measurement. If resistance measurements are to be made at frequent intervals, it is preferable to use one of the various commercial bridges, but the student should be thoroughly familiar with the method so that he can assemble a bridge when necessary.

For the highest precision measurements, a plug type bridge is desirable, as shown in Fig. 69. In this type, the resistance wire is

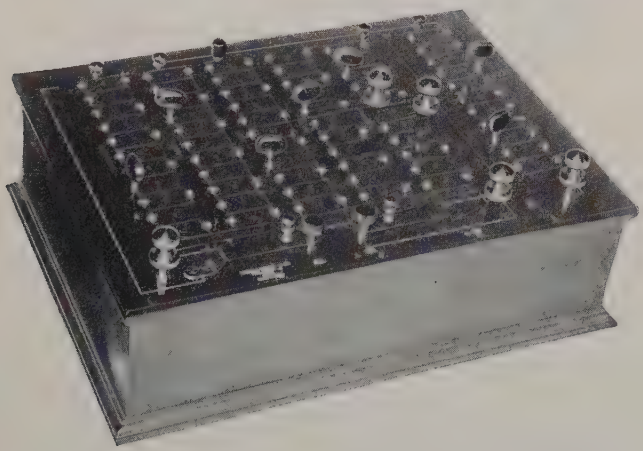


Fig. 69. Plug type Wheatstone bridge. *Leeds and Northrup Co.*

wound non-inductively on spools or cards, and the ends of the wire are connected between adjacent brass blocks, as shown in Fig. 70. Between each pair of blocks in the left-hand row, there is a 1000-ohm resistor, the next row to the right contains 100-ohm coils, etc. In using this bridge, the plugs marked "*a*" are normally left as shown in Fig. 70. By placing a plug in a hole to the left of any

block, a connection is made to the side strip and this places in the circuit a value of resistance equal to the number of thousands, hundreds, etc., indicated by the digit engraved on the block. Thus in Fig. 70, the " balancing " resistance R is shown set for 4726.9 ohms. This resistance corresponds to R_2 in Fig. 68. The " ratio

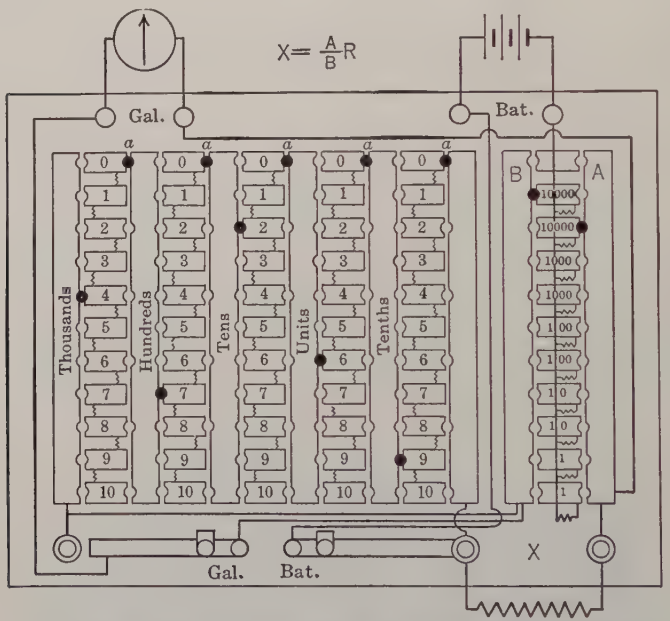


Fig. 70. Internal connections of the Wheatstone bridge of Fig. 69. *Leeds and Northrup Co.*

arms " A and B in Fig. 70, correspond to R_3 and R_1, respectively, in Fig. 68.

This bridge is particularly adapted to high precision measurement. By means of the massive contact blocks and carefully made taper plugs, the contact resistances are kept very small and the error due to such resistances is minimized.

The dial type of bridge is shown in Fig. 71 and its internal connections are given in Fig. 72. In this type, two arms of the bridge designated as the " ratio arms " are adjusted by means of plugs, the unknown resistance forms another arm, while the fourth or " balancing " arm is a variable resistance which can be adjusted by turning the dials. The dial bridge has the advantage of permitting much more rapid balancing than the plug type, but since the

switch contacts on the dials are not so tight as those obtained by plugs, the error due to contact resistance is somewhat greater. However, even in careful laboratory work, this error is usually negligible when using well constructed bridges which have been kept clean and free from injury. The individual resistors of the dial bridge are similar to those used in the plug type and in well designed bridges of either type, the resistance wire has a very small or zero temperature coefficient.

Of course, in comparing these two types of bridges, it must be remembered that the sensitivity of the galvanometer used will

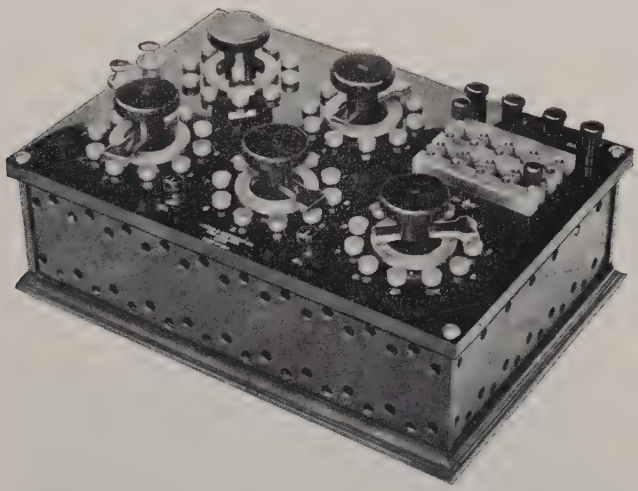


FIG. 71. Dial type Wheatstone bridge. *Leeds and Northrup Co.*

influence the precision of measurement; the comparison given above assumes that the same galvanometer is used for each bridge.

53. Ohmmeters. It is often convenient to have a simple, portable means for rapid measurement of resistance with moderate accuracy. Several types of ohmmeters have been developed for this purpose. The name ohmmeter is used because in this instrument the resistance value is obtained from a scale reading, instead of by computation as in the use of the Wheatstone bridge.

One type of ohmmeter is based on the modified Wheatstone bridge shown in Fig. 73. The resistances R_1 and R_2 of the standard bridge circuit (Fig. 68) are replaced by a single wire of relatively high resistance and uniform in cross-section. The point B is established by a sliding contact piece which can be moved along the wire AC

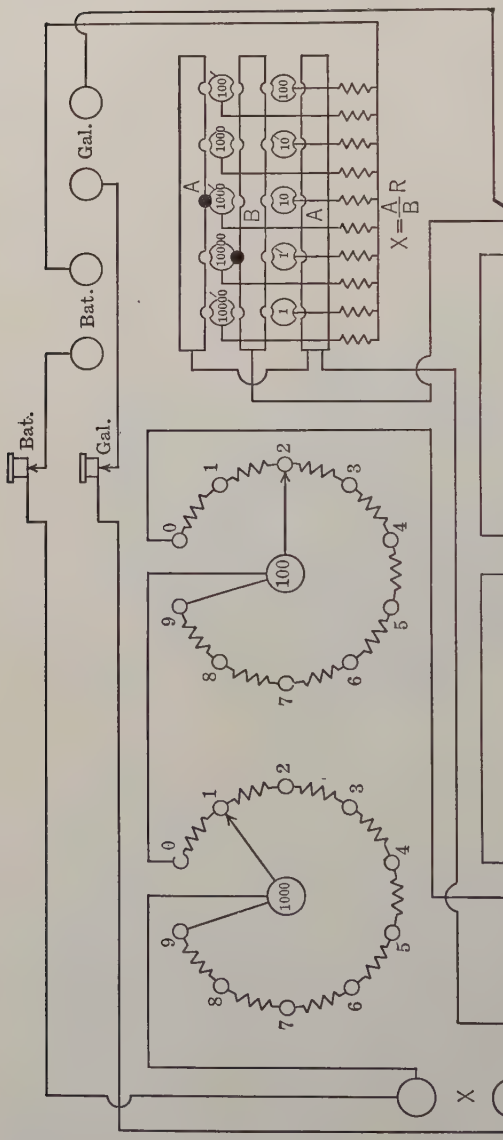


FIG. 72. Internal connections of the dial bridge of Fig. 71. *Leeds and Northrup Co.*

until a balance is obtained as shown by a zero deflection of the galvanometer.

From the bridge equations found in Art. 51, we can find the value of R_x, the unknown resistance, as

$$R_x = R_2 \frac{R_3}{R_1},$$

and since the resistance of any section of the wire AC (in Fig. 73) is proportional to length,

$$R_x = R_2 \frac{(\text{Length } BC)}{(\text{Length } AB)}.$$

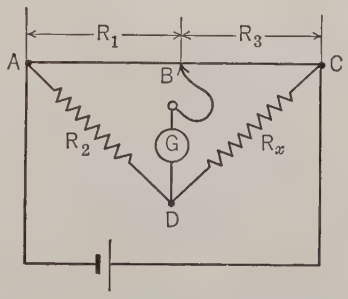


FIG. 73. Conventional diagram of a slide-wire Wheatstone bridge.

Thus, if a fixed value of R_2 is used, a scale can be marked off along the line AC such that when the bridge is balanced, it is merely necessary to read the scale position of the slider B in order to state the value of R_x.

A commercial form of this ohmmeter is shown in Fig. 74; its internal connections are given in Fig. 75. The instrument has a

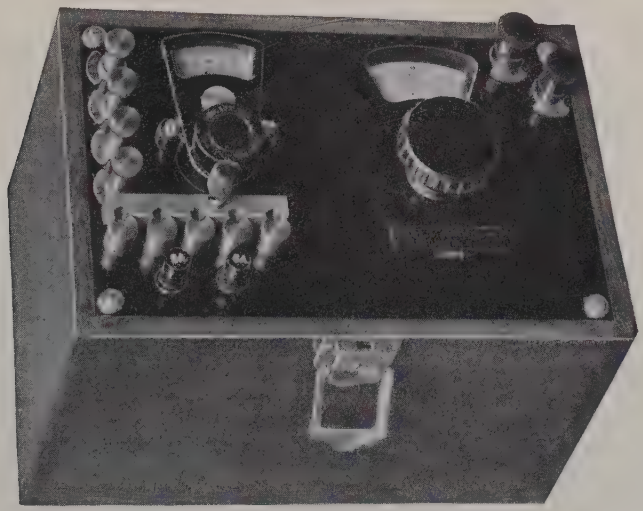


FIG. 74. An ohmmeter of the slide-wire type. *Leeds and Northrup Co.*

small, moderately sensitive built-in galvanometer to indicate balance, and current is supplied from a dry cell within the case. The slide

wire AC is constructed circular in shape and the slider is carried on a rotary arm. The amount of rotation is indicated by a calibrated dial carried on the same shaft as the contact. By means of a plug, the size of R_2 can be changed to one of several values, thus permit-

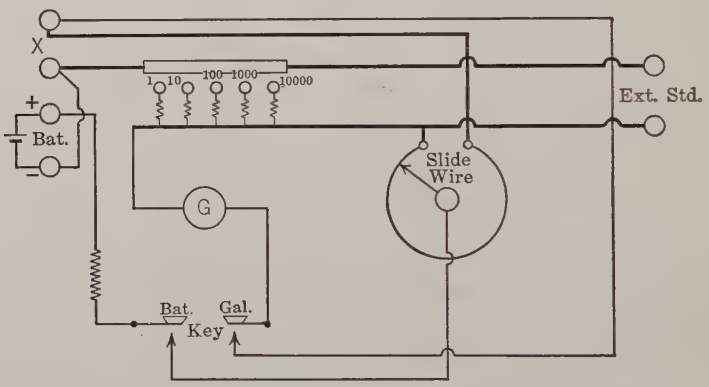


Fig. 75. Internal connections of the ohmmeter of Fig. 74. *Leeds and Northrup Co.*

ting resistance measurement over a wide range of values. In order to keep the numbering on the dial simple, it is customary to use values of R_2 such that the value of R_x is found by multiplying the dial reading by a factor such as 1, 10, 100, etc. The proper multiplier is marked opposite the position of the plug used to adjust R_2.

For less precise measurement, an ohmmeter using the circuit of Fig. 76 is often used. A small dry cell is in series with a fixed resistor and a specially calibrated ammeter. The terminals of this series combination are brought out to binding posts at a and b across which the unknown resistance is connected. The ammeter deflects due to the current which flows and its scale is marked in ohms instead of amperes, so that the resistance can be read directly. In order to allow for the decrease in battery voltage as it wears out, an adjustment is provided so that the sensitivity of the meter can be varied. Before using the meter, it should be short-circuited and the adjusting screw then turned until the pointer indicates zero resistance. If this adjustment is properly

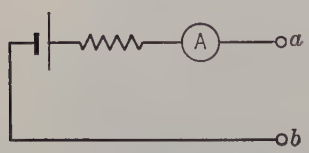


Fig. 76. A simple ohmmeter circuit. The resistance to be measured is connected across ab.

made, the markings on the scale will then be correct for all other resistances.

Figure 77 is a photograph of an ohmmeter of this type. The markings opposite the binding posts indicate the maximum values of resistance which can be measured with this instrument.

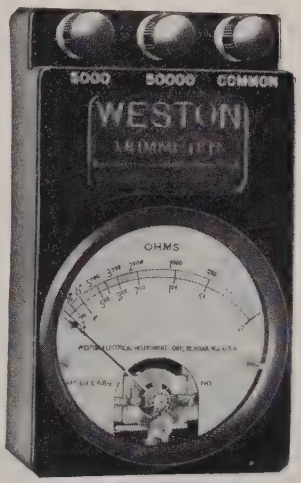


Fig. 77. An ohmmeter using the circuit of Fig. 76. *Weston Electrical Instrument Corp.*

Prob. 9–4. In a bridge arranged as in Fig. 73, AC is 100 centimeters long, R_2 is 50 ohms. If bridge balances when AB is 26.4 centimeters, what value is R_x?

Prob. 10–4. If in Fig. 73 $R_2 = 32$ and $R_x = 49$, and AC is 150 centimeters long, what are the lengths of AB and BC?

54. The Resistance Pyrometer.* A form of Wheatstone bridge is used in the Resistance Pyrometer, an instrument for measuring wide ranges of temperatures. The coil X in Fig. 78 is of extremely pure platinum and is encased in a glazed tube. The resistance of the coil is carefully measured at such known temperatures as those of solidifying carbon dioxide, melting ice, and boiling sulphur. The resistance of the coil is then measured at some unknown temperature, the value

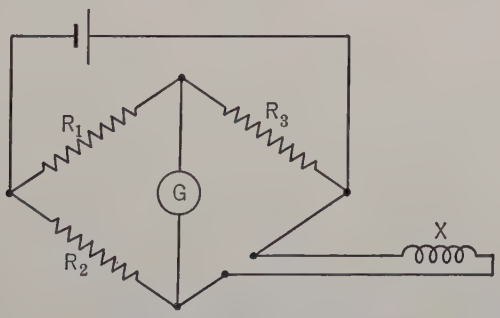


Fig. 78. The conventional diagram of a resistance pyrometer.

of which it is desired to ascertain. From the increase or decrease in resistance and the temperature coefficient of platinum, the rise

* See Law's "Electrical Measurements."

or fall in the temperature can be computed, and the unknown temperature ascertained. Since the temperature coefficient of platinum is not constant over wide ranges of temperature, it is usually necessary to apply a correction factor.

55. Location of Faults in Cables, etc. It sometimes happens that a telephone conductor becomes " grounded." It is then very desirable to locate the distance of this " fault " from one end or the other of the line before starting out from the office to repair it. Where the wire is laid underground, it would be too costly a proceeding to inspect any great length of cable, and thus methods have been worked out for locating such faults to a great degree of precision. There are two methods in common use, called respectively the Murray loop and the Varley loop. They are both modifications of the Wheatstone bridge, and, therefore, the underlying principles stated above apply to them.

56. Murray Loop. In order to apply a loop test to locate a " ground " it is necessary to have a spare ungrounded line available.

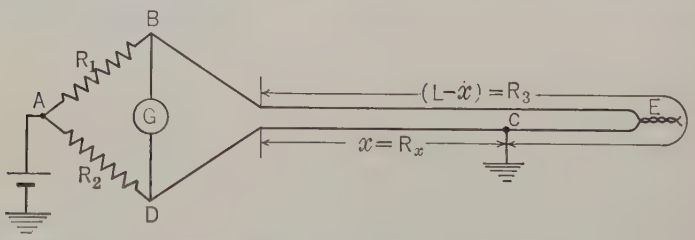


FIG. 79. The Murray Loop for locating the position of a "ground" in a conductor.

This is usually available, especially in telephone work where more than one circuit between two points is commonly found.

In Fig. 79, the lower line is grounded at C, at an unknown distance x from the measuring end. We wish to find out how far the ground is from the measuring end, and as a first step we " loop " the faulty conductor with a good one by connecting their ends at the distant point E. (If the lines are very long it will be necessary to have someone at E make this connection.) Both ends of the " loop " are now available, and with the resistors R_1 and R_2 we form a Wheatstone bridge by connecting the loop ends to B and D. Note that the battery is connected between A and C, using the ground itself as a conductor. The bridge can now be adjusted until the galvanometer G shows no deflection.

The regular Wheatstone bridge equation can now be written for this balanced condition.

$$\frac{R_1}{R_3} = \frac{R_2}{R_x} \quad \text{or} \quad \frac{R_2}{R_1} = \frac{R_x}{R_3}.$$

If we let L equal the length of the entire loop, which would be known, and let x equal the distance out to the fault, then the equation becomes,*

$$\frac{R_2}{R_1} = \frac{x}{L - x}.$$

Since x is the only unknown quantity, it can easily be found as follows:

$$x = \left(\frac{R_2}{R_1 + R_2}\right) L$$

If the "good" wire is not of the same size and material as the "faulty" wire, the resistance of the loop is first found in the usual way by a bridge. It is then connected as in Fig. 78, and worked out as above, except that L stands for the **resistance** of the entire loop instead of the **length** of it, and x for the **resistance** out to the fault. Knowing the size of the grounded wire, the distance out to the fault is readily computed.

57. Varley Loop. In the Varley loop, Fig. 80, instead of varying the ratio of R_1 to R_2 as in the Murray form of bridge, these

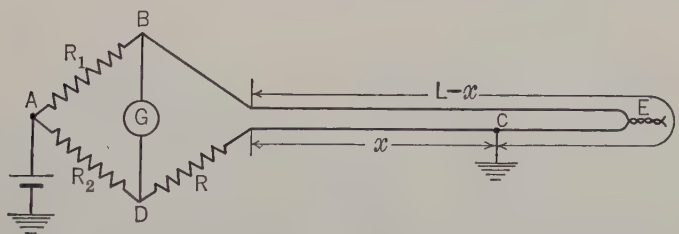


FIG. 80. The Varley Loop for locating a "ground" in a conductor.

resistances are left constant and a balance is obtained by means of a third set of resistance coils R inserted in series with the faulted cable. The resistance of the total length of the good and bad cables L, is either known or first found as in the Murray loop.

* It is assumed here that the "good" wire is of the same size and material as the "faulty" wire.

The value of x (resistance out to fault) is then found by the equation of the Wheatstone bridge:

$$\frac{R_2}{R_1} = \frac{R + x}{L - x};$$

$$x = \frac{R_2 L - R_1 R}{R_1 + R_2}.$$

As all the values in the right-hand member of the equation are known, the value of x may be found and the distance out to the fault C be computed from the resistance per foot of the cable.

If the two wires used to form the loop are of the same size and material, we can make $R_1 = R_2$ and the above equation becomes simply:

$$x = \tfrac{1}{2}(L - R).$$

Thus the resistance x is given as $\frac{1}{2}$ the difference between the total loop resistance L, and the resistance R required to secure balance. This gives a rapid method of finding the location of a ground in a telephone circuit.

Prob. 11–4. A Murray loop is set up as in Fig. 79 to locate a ground on a line between two cities 85 miles apart. The return line is identical with the grounded line. If the bridge balances at $R_1 =$ 1260 ohms and $R_2 = 954$ ohms, how far is the fault from the test end?

Prob. 12–4. In a Murray loop arranged as in Fig. 79, the resistance of the loop is 68.3 ohms. At balance, $R_1 = 76.3$ ohms and $R_2 = 20.0$ ohms. If the line wires are of No. 12 B. & S. copper, how far is the fault from the test end?

Prob. 13–4. A Varley loop in which the resistance of the twin cables is 56.2 ohms is set up as in Fig. 80. A balance is obtained when $R_1 = 10$ ohms, $R_2 = 100$ ohms, $R = 256$ ohms. If the faulted wire is copper, No. 14 B. & S., how far is the fault from the test end?

Prob. 14–4. In Prob. 13, with the fault located at the same point, what would be the value of R required to balance the bridge if $R_1 = R_2$?

Prob. 15–4. If the line wires in Prob. 13 are of iron measuring 24 ohms per mile, how far is the fault from the test end?

58. Voltmeter Method of Measuring High Resistances. It is often necessary to know whether a certain resistance is above or below a given value; as, for example, the insulation resistance between the commutator and the frame of a motor. If a voltmeter of known resistance is available, it may be used for this test.

Referring to Fig. 81, line B is connected to one of the commutator segments; the voltmeter V is connected between line A and the blade of a double throw switch. When this switch is thrown to the right, the voltmeter is connected across the supply line AB. With the switch to the left, the voltmeter is in series with the insulation resistance between the commutator and the motor frame F, and the series combination is across the line.

We now read the voltmeter with the switch in each position, and call the line voltage (switch to right) V. With the switch to the left, the voltmeter will read a lower value V_1, because the insulation resistance and the voltmeter resistance are in series, and the voltage

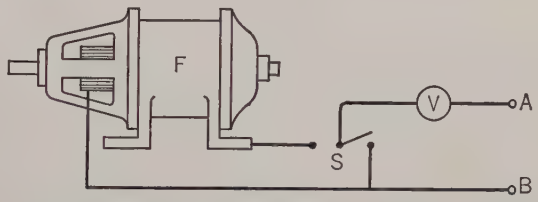


FIG. 81. Connections for the voltmeter method of measuring high resistances.

from AB is divided between motor and voltmeter in direct proportion to their separate resistances.

The voltmeter reads the voltage V_1 across its own resistance, so the voltage across the motor insulation then equals the line voltage V minus the drop through the voltmeter V_1. Hence, we can write

$$\frac{\text{Insulation resistance}}{\text{Voltmeter resistance}} = \frac{V - V_1 \text{ (volts across insulation)}}{V_1 \text{ (volts across voltmeter)}}$$

Let R = insulation resistance; R_v = voltmeter resistance. The equation then becomes:

$$\frac{R}{R_v} = \frac{V - V_1}{V_1}.$$

This equation can be solved for R, since all other values are known.

$$R = R_v\left(\frac{V}{V_1} - 1\right).$$

A deflection of 1 volt for V_1 would give R the following value:

$$R = R_v(V - 1).$$

If no deflection is noticed when the voltmeter is connected in series with motor-insulation, we know that the insulation is at least greater than $R_v(V - 1)$. Thus if the resistance of the voltmeter R_v is 15,000 ohms, and the voltage across AB, (V), is 111 volts, we would know, that in case of no deflection, we had at least 15,000 $\times$ 110 = 1,650,000 ohms insulation resistance between frame and commutator.

Measurements of insulation are also frequently made with an instrument known as a " megger," so called because it reads directly in megohms (1 megohm equals 1 million ohms). The " megger " is merely an ohmmeter of the type shown in Fig. 76 except that a hand-cranked generator is used instead of a battery. This generator delivers a moderately high voltage and is designed so that its voltage is fairly constant as long as it is being cranked rapidly.

59. Insulation Resistance of Covered Wire, etc. Fire insurance companies require that insulated wire of various types and grades have certain minimum values of insulation resistance per mile, after being soaked in salt water for a stated length of time. Usually this resistance is too high for precise measurement by the voltmeter method given above and the following method is one commonly used.

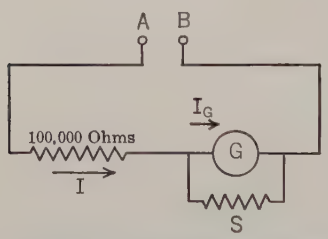


FIG. 82. A shunted galvanometer in series with 100,000 ohms.

The basic idea in this measurement is to compare the current which flows through the insulation resistance when it is connected to a constant-potential source of power, to the current which flows in a known value of resistance when it is connected to the same source. We first set up the circuit shown in Fig. 82 in which a 100,000-ohm resistor is placed in series with a shunted galvanometer across a constant-potential source AB. The deflection of the galvanometer is noted and then from the known values of galvanometer resistance and shunt resistance we compute what the deflection would have been if no shunt were present.

Suppose, for example, the following data were found:

Deflection = 80 scale divisions.
Resistance of G = 2000 ohms = R_G.
Resistance of S = 10 ohms = R_S.

The resistance of G and S in parallel is $\frac{R_G R_S}{R_G + R_S}$ and the drop due to the total current I flowing through the parallel combination is $I \frac{R_G R_S}{R_G + R_S}$. Now this is the voltage across the galvanometer, and the current through the galvanometer is hence

$$I_G = \frac{E_G}{R_G} = \frac{I \frac{R_G R_S}{R_G + R_S}}{R_G} = I \frac{R_S}{R_G + R_S}.$$

Putting the data given above in this equation, we see that the galvanometer current is only $\frac{1}{201}$ of the total current, and, therefore, the deflection of 80 is only $\frac{1}{201}$ (or $\frac{1}{201}$) as large as it would be if no shunt were present.

If no shunt had been present, the total current would have passed through the galvanometer and its deflection would have been $201 \times 80 = 16{,}080$ divisions. If the series resistance had been 1,000,000 ohms (1 megohm) instead of 100,000 ohms, the current would have been only $\frac{1}{10}$ as great and the deflection of the unshunted galvanometer would have been 1608 divisions. This deflection (for 1 megohm in series) is called the **galvanometer constant.**

The 100,000-ohm resistance is now replaced by the unknown insulation resistance as shown in Fig. 83. The wire under test is immersed in salt water and only its ends are brought out at X and Y. Since the resistance of the salt water is very small, the resistance between one end of the wire, say Y, and the tank is practically equal to the insulation resistance of the coil. The actual measuring circuit now starts at A, goes through the water and insulation to Y, through the galvanometer, and back to B.

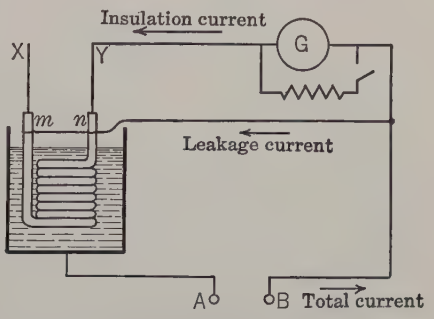


FIG. 83. Unshunted galvanometer in series with the insulation of the cable. Guard wire attached to prevent error due to surface leakage.

It is practically impossible in making this test to prevent some current from flowing along the surface of the insulation to the water and this current would also pass through the galvanometer. Hence, the galvanometer deflec-

tion would not be due to the current passing **through** the insulation but would be increased because of the leakage of current along the surface of the insulation. To prevent this, a **guard wire** (Fig. 83) is wrapped around the insulation at m and n and connected back to the source of power at B. We do not try to prevent the leakage current from flowing, but merely allow it to flow **around** the galvanometer instead of through it. There will still be some leakage between the cable conductor and the guard wire, but this will be due only to the voltage drop across the galvanometer and will not cause appreciable error.

Having set up the circuit as in Fig. 83, we connect it at AB to the same source as was used before and note the galvanometer deflection with its shunt removed. (Caution — do not remove the shunt until after the power is applied.) Suppose the galvanometer deflects 16 divisions. Since it was found that the galvanometer would deflect 1608 divisions when in series with 1 megohm, a deflection of only 16 must mean that the unknown resistance is much greater. The deflection is inversely proportional to the series resistance (for constant voltage), so the insulation resistance R_x must be

$$R_x = \frac{1608}{16} = 100.5 \text{ megohms.}$$

This is the insulation resistance of the coil.

If the coil were $\frac{1}{4}$ mile long, the insulation resistance per mile would be only $\frac{1}{4}$ as great, or 25.1 megohms. This is because the current would have 4 times as much area to leak through in a mile as in $\frac{1}{4}$ mile.

Notice that the galvanometer constant 1608 is not used as a **multiplier** but as a **dividend,** which is **divided** by the deflections.

Note that in this test the resistance of the battery and galvanometer are neglected because they are too small in comparison with the resistance of the insulation and standard box to affect the results appreciably. When the standard 100,000-ohm box above is in the line, the galvanometer is shunted so that it has less than 10 ohms resistance, which is one ten-thousandth of the total resistance of the circuit. When unshunted in the second part of the test, the resistance of the galvanometer is but one fifty-thousandth of the total line resistance. In fact, the standard resistance of 100,000 ohms might be left in the line in the second part as well, and its resistance neglected, so small is it in comparison with the insulation resistance.

Prob. 16–4. In a test, as in Paragraph 59, for the insulation of a coil of wire the following data were taken:

Resistance of Galvanometer = 3000 ohms;
Resistance of Shunt = 20 ohms.

Deflection of shunted galvanometer with 200,000 ohms in series = 38.6.

Deflection of unshunted galvanometer with the resistance of the insulation in series = 14.4.

Length of coil = 500 feet.

(*a*) What is the galvanometer " constant "?

(*b*) What is the insulation resistance per mile of cable?

SUMMARY OF CHAPTER IV

METHODS OF MEASURING RESISTANCE.

(1) Voltmeter-ammeter.
Makes use of instruments in general use, but which must be accurately calibrated.

(2) Voltage Comparison.
(a) Standard resistance and voltmeter which need not be accurately calibrated to read volts.
(b) Accurate method for measuring low resistance if voltmeter has high resistance or if a potentiometer is used.

(3) Wheatstone bridge.
(a) Three standard variable resistances and galvanometer. Has advantage of being a " no-deflection " method, thus galvanometer need not be calibrated. A slide-wire may take the place of two of the resistances.
(b) Modifications of this bridge are used in the Murray and Varley " loop " methods of locating faults in cables.
(c) One common type of " ohmmeter " uses a bridge circuit with a calibrated slide wire.

(4) Voltmeter, for high resistance.
A simple method of ascertaining whether or not insulation resistance between different parts of a machine exceeds a certain minimum.

(5) Insulation resistance of covered wire.
Galvanometer " constant " found by noting deflection when a standard resistance takes the place of the insulation resistance. Galvanometer deflections must be proportional to current through it. Source of power must be of constant potential. Wire must be soaked for specified number of hours.

PROBLEMS ON CHAPTER IV

Prob. 17–4. It is desired to measure the resistance of an appliance by the ammeter-voltmeter method. A 15-ampere ammeter connected in series with the appliance reads 11.85 amperes. A 150-volt voltmeter connected across the appliance reads 110 volts. (*a*) Compute the resistance of the appliance from these readings making allowance for the current taken by the voltmeter and the drop across the ammeter. (*b*) Is this the best arrangement of the instruments?

Assume the voltmeter has 100 ohms per volt of scale, and ammeter requires 0.045 volt for a full scale deflection.

Prob. 18–4. Figures 84 and 85 represent the Fisher modification of the Murray loop. Conductor *A* is grounded at *F*. There are two

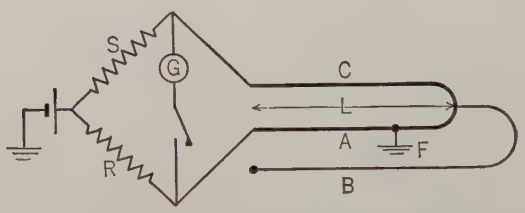


Fig. 84. The Fisher loop for locating a ground in a conductor.

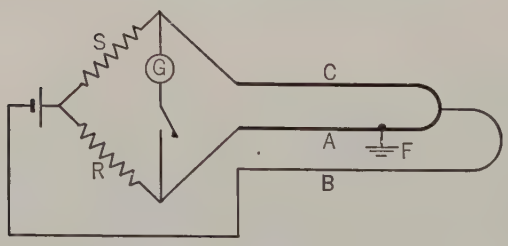


Fig. 85. The Fisher loop arranged for the second reading.

other conductors (*C* and *B*) which may be unlike each other and unlike *A*. The far ends of conductors *A*, *B* and *C* are joined. The battery is grounded as in Fig. 84 and resistances *R* and *S* are adjusted until galvanometer does not deflect. *R* is found to be 26 ohms and *S* to be 72 ohms.

The battery is then disconnected from the ground and connected to conductor *B*, as in Fig. 85, and *R* is readjusted until a balance is again obtained. The new value of *R* is found to be 62 ohms, while *S* remains unchanged. The length (*L*) of the faulted conductor is 6520 feet. How far out is the fault (*F*)?

Prob. 19–4. One terminal of the field windings of a motor was connected to the negative terminal of a 115-volt circuit. A voltmeter connected between the positive terminal of the circuit and the

frame of the motor indicated $\frac{1}{2}$ of a volt. The voltmeter had a 300-volt range and a resistance of 118 ohms per volt of scale. What was the value of the insulation resistance between the field coils and the frame of the motor?

Prob. 20–4. A 5-ampere ammeter having 0.010 ohm resistance is to be used with a 150-volt voltmeter of 16,000 ohms resistance to measure a resistance which is known to be about 35 ohms. The source of power is a 110-volt line. Show how you would connect up instruments to measure this resistance to the greatest degree of precision. Give readings of instruments, and percentage error in the value of resistance obtained by dividing the voltmeter reading by the ammeter reading.

Prob. 21–4. In Fig. 78, the ratio arm R_1 has the value 10 ohms, and R_2, 1000 ohms. With the platinum coil at a temperature of 100° the bridge balances with R_3 set at 132.3 ohms. What is the temperature of a furnace, if the bridge balances with R_3 at 412.5 ohms, after the coil has reached the furnace temperature? Assume the temperature coefficient of platinum at 0° C. to remain constant throughout this range.

Prob. 22–4. On a circuit diagram copied from Fig. 70, show the locations of R_1, R_2 and R_3 as in Fig. 68.

Prob. 23–4. A certain resistance is measured by the voltage comparison method, using a 1000-ohm standard. The voltmeter used has two ranges; a 150-volt range having a resistance of 16,000 ohms, and a 15-volt range having a resistance of 1600 ohms. In the test, the voltage across the standard resistance is found to be 93.4 volts, measured on the 150-volt scale. The voltage across the unknown resistance, measured on the 15-volt scale is 13.8 volts. Assuming that the line current is constant throughout the test, what is the true value of the unknown resistance?

Prob. 24–4. A building is supplied power at 115 volts, with the negative side of the line grounded. With the main supply switch open, a voltmeter of 24,000 ohms resistance is connected between the positive supply line and both sides of the building wiring. If the voltmeter reads 1.6 volts, what is the insulation resistance of the wiring system?

Prob. 25–4. Demonstrate that, in the voltage-comparison method for measuring resistance (Par. 50), the result is independent of the resistance of the voltmeter when the total applied voltage is kept constant.

CHAPTER V

MAGNETS AND MAGNETISM

In much of the apparatus used in the generation, transmission and utilization of the world's electric power, a magnet in some form or other plays an essential part. It is therefore necessary to be familiar with the laws of magnetic action if we are to develop an understanding of electrical apparatus.

60. Magnetic Lines. The best known form of a simple magnet is the bar magnet. If we place this bar magnet under a glass and sprinkle iron filings on the glass, the filings will arrange themselves in lines as in Fig. 86. These lines pass around the magnet from one end to the other end. The North pole* of an exploring compass

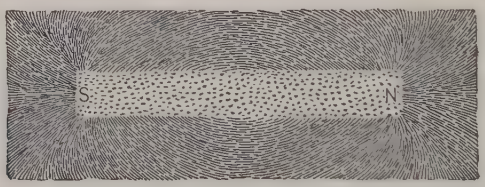


FIG. 86. The field about a bar magnet shown by iron filings.

placed anywhere along these lines will be found to point always **away** from one end of the bar magnet and **toward** the other end, showing that the lines nearly all come out of the region near one end and go into the region near the other end. The region **out** of which the lines come is said to be the **North** pole of the bar magnet, and the region where they **enter** the magnet is called the **South** pole.

It is important to note the fact that as far as electrical appliances are concerned, magnetic poles are not **points** but entire regions. In fact, wherever magnetic lines come out of the surface of an appliance, that part of the appliance is called a North pole, and

* The end of a compass which points to the earth's north is here called the North pole of the compass. A compass needle will come to rest in such a position that it lies along a line of force with the North pole pointing in the direction in which the line runs. Note that according to this commonly accepted convention the pole we designate as North on the earth's surface is really a South magnetic pole.

wherever they enter the appliance again, that part is called a South pole. This can always be tested very simply with a compass held near the appliance. Remember that the needle points in the direction in which the lines run. Thus if it points to any particular part of the appliance, that particular part of the appliance is a South pole, for the lines are entering there. If it points away from any particular part, that part is a North pole, because the lines are coming out there.

61. Flow of Magnetic Lines. A map of the magnetic field of a magnet presents a better idea of the lines than is given by the iron filings. Figure 87 represents such a map. Note that the lines come out of the North pole, go around to the South pole, enter the magnet there and go through the magnet to the North pole. **Each line is a complete loop.** The magnetic field is always thought of and spoken of as a **flux** or current of **magnetism** which goes around its **circuit** — out of the North pole around to the South pole, into the South pole and through the magnet back to the North pole. The **magnetic flux** going through a **magnetic circuit** is not as real as an **electric**

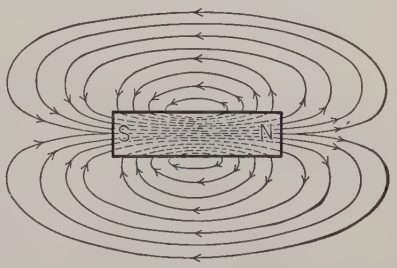


FIG. 87. A diagram of the field about a bar magnet.

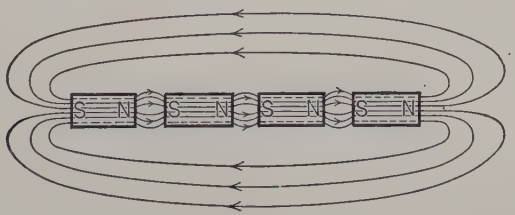


FIG. 88. When a bar magnet is broken, each piece becomes a magnet.

current flowing through an **electric circuit,** but, as will be shown later, can be treated in a similar way. The fact is that magnetic lines are merely a convenient means of representing the direction and amount of a force in space.

It is of the utmost importance that the student grasp the idea of the **magnetic flux** and the fact that it constitutes a **complete magnetic circuit.** Do not think of the lines as starting out from a North

pole and ending at a South pole, but think of them as continuing on through the magnet to the North pole again.

The fact that the lines actually run through this magnet and form a complete loop, and do not merely start at the North pole and end at the South pole, is proved by breaking a magnet into several pieces. Each piece becomes a separate magnet with a North pole and a South pole of its own as is shown in Fig. 88.

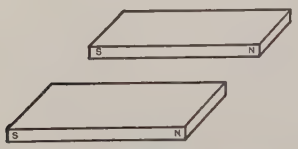


Fig. 89. Two bar magnets.

Prob. 1–5. Draw the magnetic field (external and internal) of two magnets placed side by side as in Fig. 89.

Prob. 2–5. Reverse one magnet in Fig. 89 and draw the magnetic field produced.

62. Ring Magnets. There are cases where a piece of iron may be strongly magnetized and still possess no poles, as in the ring, Fig. 90 (*a*). Since the lines nowhere come out of the iron, they cannot produce any poles or external field. If, however, we break the ring, as in Fig. 90 (*b*), we have the usual two poles, a North and a South, with an external field of great intensity, though small in area. The complete ring is used in some types of meters and also in transformers where no external field is desired. The broken ring or horseshoe is the fundamental form of the magnetic circuit of generators and motors where an intense external field is desired in a limited air space.

Fig. 90 (*a*). A solid ring magnet.

Fig. 90 (*b*). A ring magnet with an air gap.

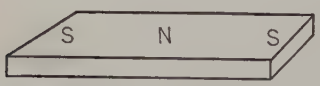


Fig. 91. A magnet with a consequent pole.

Prob. 3–5. Draw the magnetic field of a bar magnet shown in Fig. 91, having South poles at the ends and a North pole at the middle.

63. Magnetic Field of a Dynamo. The magnetic field of the two-pole dynamo in Fig. 92 may be considered as merely the field of a bar magnet bent like a horseshoe. Note that here, too, the lines come out of the North pole and enter the South pole, go through the yoke and back again to the North pole. The armature revolves

in the field between the North and South poles, and the wires on it cut the magnetic lines of the field and produce a voltage across the commutator, whence it is transmitted to the brushes. Figure 93 shows the field of a four-pole dynamo. Note that the lines come out of a North pole and go into a South pole, and that North poles and South poles **alternate** around the frame.

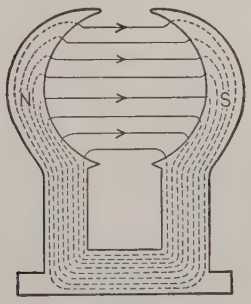


FIG. 92. The magnetic circuit of a two-pole motor or generator.

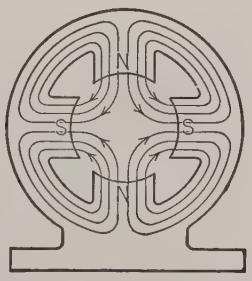


FIG. 93. The magnetic circuits in a four-pole motor.

Prob. 4–5. Draw a 6-pole motor frame showing the complete paths of the magnetic lines.

Prob. 5–5. Draw an 8-pole generator frame showing the complete paths of magnetic force lines.

64. Permanent Magnets. Most of the electrical measuring instruments, such as ammeters, voltmeters, galvanometers, etc., contain a permanent magnet. In order that the instrument retain its precision the magnet must remain of the same strength from month to month.

We shall see that hard steel makes the strongest permanent magnet. For short magnets the steel should be "glass hard," that is, tempered by being heated to a red heat and suddenly immersed in water, oil or mercury. A magnet made of steel treated in this way will be very strong at first, but will gradually lose some of its strength, especially if handled roughly. Soft iron and annealed steel lose practically all their magnetism as soon as the magnetizing force is removed.

It has also been found that thin magnets are stronger in proportion to their weight than thick ones, so a laminated magnet, that is, one made of thin magnets bound with like poles together, is

stronger than one made of a solid piece of steel. This arrangement is called a compound magnet.

Heat and rough treatment will ruin the best of magnets, hence the need of careful handling of electrical instruments.

Although steel is the basic material from which permanent magnets are made, it has been found that certain alloys of steel produce even more powerful permanent magnets. Tungsten steel is a commonly used alloy, and alloys containing cobalt are coming into greater use. A new alloy containing aluminum, nickel, and cobalt has recently been introduced, and exceptionally powerful magnets are made from it. These materials are all very hard and usually have to be cast to the desired shape.

65. Nature of Magnetic Lines. We have seen that we can picture the force directions and relations of a magnetic field by lines of force. We represent the strength of the field by how closely together we draw the lines, the more lines per unit area, the stronger the field. We may get an even clearer idea of the magnetic field if we think of the magnetic lines as very much like a bundle of **stretched rubber bands.** They therefore have two distinct actions.

(1) They represent a **tension** along their length and are always tending to **shorten,** just as a stretched rubber band does.

(2) They exert a **sidewise crowding** effect on one another, each always tending to push the others away sidewise, just as each band in a close bundle of rubber bands would tend to crowd the others sidewise.

By means of these two properties, the tension and the crowding, we can explain nearly all magnetic phenomena as illustrated below.

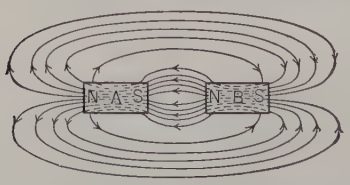


FIG. 94 (*a*). Two bar magnets with unlike poles near each other.

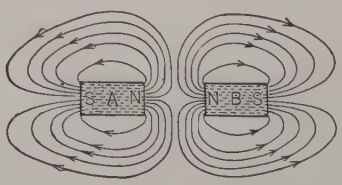


FIG. 94 (*b*). Two bar magnets with like poles near each other.

66. Attraction and Repulsion. When we place the North pole of one magnet near the South pole of another, the magnets attract each other. Figure 94 (*a*) shows this action. The lines coming out of *B* enter the South pole of *A*, go through this, and emerging from the North pole of *A*, return to the South pole of *B*. The tension in

the lines thus threading the two magnets tends to pull the magnets together, as so many loops of stretched rubber bands would.

If we place the North ends near each other as in Fig. 94 (*b*), the lines coming out of the North poles turn one another aside and their lateral crowding pressure tends to push the magnets apart.

The rule, then, is that **unlike poles attract,** while **like poles repel** one another.

67. Reluctance of a Magnetic Circuit. We have likened the magnetic flux in a magnetic circuit to the electric current in an electric circuit. It will be remembered that the flow of an electric current is opposed by the resistance of the circuit. In a similar manner the magnetic flux is opposed by the resistance of the magnetic circuit. This **magnetic resistance** is called **reluctance** to distinguish it from **electric resistance.**

To carry an electric current we generally use copper or aluminum circuits because the resistance of these metals is low compared with the resistance of other materials. Similarly to carry a magnetic flux we generally use iron or soft steel circuits, because the reluctance of these materials is low compared with the reluctance of other materials. However, an electric circuit is almost always made up entirely of copper or aluminum. The electricity is rarely made to flow through the air, except in certain radio appliances. But it is often necessary to cause magnetic flux to go a certain distance through the air. It is fortunate, therefore, that the reluctance of the air is only a few thousand times as great as the reluctance of most iron, while the resistance of the air is several million times the resistance of copper. Accordingly we can often use air as part of the magnetic circuit without having to overcome too great a reluctance in the circuit. If the reluctance of air were as much greater than the reluctance of iron as the resistance of air is greater than the resistance of copper, it would be practically impossible to make the powerful electromagnets demanded in the construction of large generators and motors.

68. Why Iron is Attracted to a Magnet. Because the reluctance of iron and steel is lower than the reluctance of air, iron and steel are attracted to a magnet. When a piece of soft iron, for instance, is placed in the air near a magnet, the magnetic lines will tend to pass through the piece of iron rather than the surrounding air, because the reluctance of the iron is lower than that of the air. Figure 95 shows this. A small piece of iron *A*, which was not magnetized, was placed in the magnetic field of the magnet *M*. The field is dis-

torted by the lines apparently trying to go through the iron, instead of through the air, because the iron offers an easier path. The iron now has a North and a South pole of its own, since lines of force enter one end and leave the other. We have, then, two magnets with their unlike poles near each other, and they therefore attract each other. This explains why any piece of soft iron placed near a magnet is always attracted to it. If the piece of iron is placed near the magnet, of course more lines flow through it than if it is placed farther away, and the iron becomes a stronger magnet and greater attraction exists between it and the magnet M.

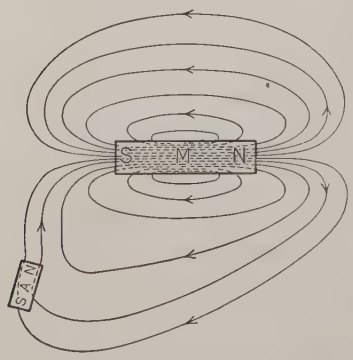


Fig. 95. The piece of iron A placed in a magnetic field becomes a magnet.

69. Magnetic Screens. This low reluctance of iron is used, also, whenever it is desired to reduce the magnetic field in a certain region. For instance, it is desired to free the space A, Fig. 96, of magnetic lines. It would do no good to surround it with copper, brass, wood or any other nonmagnetic material, because the reluctance of these is the same as that of air. A shield of soft iron is put in the field. The iron screen, since it has a much lower reluctance than air, takes up nearly all the lines in the field H and conducts them around the space A. The space is thus not **insulated** from the magnetic current, but the flux is **shunted** around it. Instruments are sometimes thus shielded from the effects of a magnetic field. Neither the air nor any other material can be said to be a magnetic insulator, because, as we have seen, the reluctance of air and all other nonmagnetic materials is not enough greater than that of iron and steel to prevent an appreciable magnetic flux from being set up in them. On the other hand, the **resistance** of air, rubber, enamel, etc., is so much greater than the **resistance** of metals, that these substances act as very efficient insulators to electricity and only a slight electric current leaks through them. Still, we have

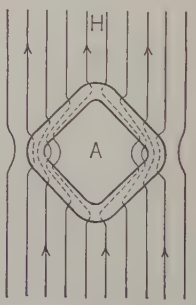


Fig. 96. The magnetic lines are nearly all shunted around the space A by the soft iron screen.

seen in Chapter IV that, good as these substances are, they are not absolutely perfect insulators and some minute part of the current always leaks out through the best insulated wires. This quantity of electricity is not to be compared, however, with the comparatively large magnetic flux that leaks out of magnetic paths even when they are composed almost entirely of iron and steel. It is rarely ever necessary, therefore, to resort to the method of shunting an electric current around a space we wish to be free of electricity, in the manner we have just shown magnetic flux to be shunted around a space we wish free from magnetism.

70. The Compass. A compass is merely a bar magnet suspended so that it can turn freely. We have seen that a magnetic line of force represents the direction in which a North pole is urged by the magnetic field. If a small magnet could be made with only one pole and that a North pole, and could be placed so that it was free to move near the North pole of a large magnet, it would be repelled from this North pole and travel around to the South pole, taking a path indicated by that line of force on which it happened to be placed. In fact a magnetic line of force may be defined as a line along which a North pole is repelled and a South pole attracted. But every magnet which has a North pole has also a South pole. Thus there is always a double action on a compass; the North pole tends to move in the direction of the lines of force and the South pole tends to move in the opposite direction. This causes a compass to swing around until it assumes a position in which it lies along a line of force, with its North pole pointing in the direction of the magnetic flux.

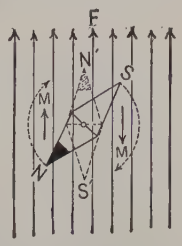


FIG. 97. The compass tends to turn to a position indicated by $S'N'$.

This is illustrated by Fig. 97. If we place the compass needle in the position S–N in a magnetic field F, the North pole N will be repelled along in the direction of the field, and the South pole S will be pulled along in the opposite direction. This will tend to swing the compass around on its axis until it takes the position S'–N', in which it is parallel to the lines of force in the field.

This also explains why iron filings scattered on a glass plate above a magnet arrange themselves along the lines of force. The filings all become small magnets, or compasses, and when the plate is tapped gently they are able to move, and naturally take positions along lines of force.

To understand why a compass at the surface of the earth always points approximately north, it is merely necessary to realize that the earth itself is a large magnet with the magnetic lines near its surface running from South to North. A compass thus takes a position along one of these lines.

The place to which the north end of a compass points is not the geographical North Pole of the earth, but a spot in Boothia, a peninsula in the northern part of Canada. All the earth's magnetic lines run in the direction of this spot although they do not all enter the earth at this one point. Most of them enter over an area of which this spot is the center, just as the lines in the bar magnet of Fig. 87 enter the magnet over an area near the pole. The earth's lines run through the earth and emerge over a similar area in the Antarctic region, thus giving on the earth's surface a practically uniform magnetic field flowing from south to north, called the Earth's Magnetic Field.

Note also that since the compass points towards Boothia, this spot is really the South Magnetic Pole. So the North pole of a compass really points to the South Magnetic Pole. But since this South Magnetic Pole lies near the North Geographical Pole, it causes no confusion to say that the compass points North. It must always be kept clearly in mind, however, that **a compass always points away from a North magnetic pole,** as this is the surest test of polarity in dealing with electric appliances.

71. Magnetic Field about a Straight Wire. If iron filings are sprinkled on a card through which a wire carrying a current is passing, as in Fig. 98, the filings will arrange themselves in circles around the wire, showing the existence of a circular field around the wire.

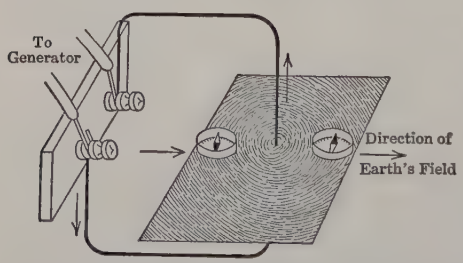


Fig. 98. Iron filings and compasses show that there is a circular magnetic field about a wire carrying a current.

In fact, wherever an electric current is flowing there exists a magnetic field. When the wire carrying the electric current is straight, the magnetic field about the wire is circular, and if it were possible to make a compass with a North pole only and place it near the wire, it would whirl around and around the wire in a circle, except as its path would be

modified by centrifugal force and gravity. If the current in the wire were reversed, the North pole would still whirl around the wire but in the opposite direction, showing that the field is the same shape but opposite in direction. Figure 99 shows this circular field about a straight wire.

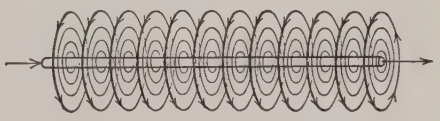


FIG. 99. The magnetic field about a wire carrying current.

If, now, we look along the wire in the direction in which the current is flowing, we note that the magnetic field forms whirls around the wire in the direction we would turn down a right-hand screw. Notice in particular that these whirls are **not spirals** but are **circles.** Figure 100 shows a cross-section of the wire and magnetic field, and represents the way the field would appear if we looked at the end of the wire with the current going away from us. In Fig. 101 the current is reversed. Notice that the field is also reversed in direction.

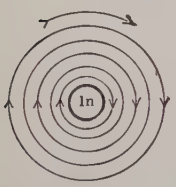


FIG. 100. The magnetic field about a wire carrying a current flowing into the paper.

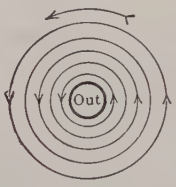


FIG. 101. The magnetic field about a wire carrying a current flowing up out of the paper.

72. The Thumb Rule for Wire. The best way to find the direction of the magnetic field about a wire carrying an electric current is by the thumb rule.

If we **grasp the wire with our right hand,** so that the **thumb** points in the direction of the **current,** the **fingers** will point in the direction of the magnetic **field.**

Similarly, if we know the direction of the magnetic field, we can find the direction of the current. For, if we place the fingers in the direction of the lines of force, the thumb will then point in the direction of the current. This is illustrated in Fig. 102.

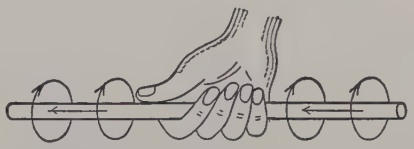


FIG. 102. The right-hand thumb rule for finding the relation between the directions of the magnetic field and the electric current in a conductor.

73. Field about a Coil Carrying a Current. If a wire is made into a loop as in Fig. 103, we find, by the **thumb** rule, that the lines

of force, which everywhere whirl around the wire, all enter the same face of the loop and all come out of the other face.

If we now place several loops together into a loose coil as in Fig. 104, most of the lines will thread the whole coil. If we make a

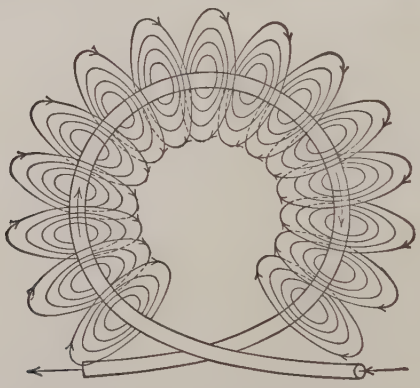


Fig. 103. The magnetic field about a single loop of wire carrying a current.

close coil, many more lines will thread the whole coil, and return outside the coil to the other end.

The reason why fewer magnetic lines encircle the separate loops of a closely wound coil, causing more lines to thread the entire coil, is shown in Fig. 105. This drawing represents an enlarged longitudinal section of the coil in Fig. 104. The current entering the ends

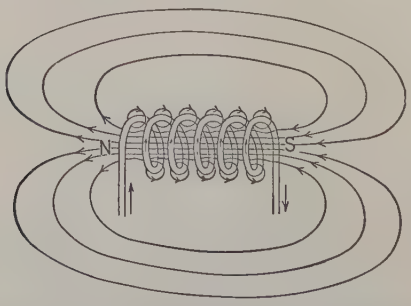


Fig. 104. The magnetic field about a loose coil of wire carrying a current.

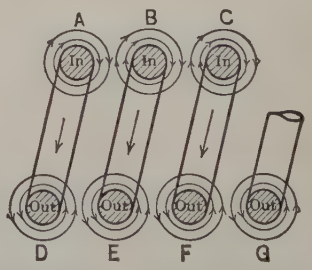


Fig. 105. A cross-section of the coil in Fig. 104.

of half loop at A, B and C, comes out again at D, E and F. If the wire ends, A and B, were pushed nearer one another, the field on the right side of A, being in the opposite direction, would partially neutralize the field on the left side of B. The space between the

wires A and B would thus be freer of lines of force. The magnetic lines now would be compelled to continue on through the coil, and would not slip into the first space between the loops and encircle only one coil wire with a separate field.

We thus have the same shaped field as in and about a bar magnet; one end is a North pole, since most of the lines come out of it, and the other end is a South pole, since most of the lines enter it.

74. Thumb Rule for Coil. In order to find the polarity of such a coil as described above, it is best to use a modified form of the Thumb Rule for a straight wire.

Grasp the coil with the right hand so that the **fingers** point in the direction of the **current** in the coil, and the **thumb** will point to the North pole. Note that the Thumb Rule for a coil has this in common with the Thumb Rule for a straight wire: The fingers are wrapped around the coil in the direction of the currents around the coil. In this case they are **electric** currents. Similarly the fingers are wrapped around the wire in the direction of the flux around the wire, which in that case is **magnetic** flux. Figure 106 illustrates this rule.

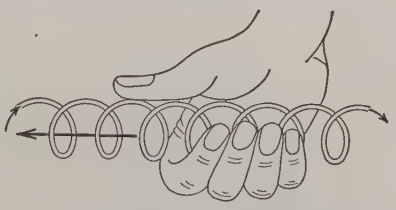


FIG. 106. The right-hand rule for finding the direction between the electric current in a coil and the magnetic field.

75. Coil Equivalent to a Bar Magnet. A coil like that of Fig. 104 with a current flowing through it is then the equivalent of a bar magnet with two poles. Its magnetic field has the same shape as the field of a bar magnet and it obeys the laws of a bar magnet. That is, unlike poles of two coils attract each other and like poles repel; if the coil is free to turn and is placed in a magnetic field, it will tend to take such a position that the lines inside the coil are parallel to the lines in the field.

76. Electromagnets. Magnetic Hoists. Very strong magnets are made by inserting a piece of iron or steel, called a **core**, into the coil. The reluctance of iron and steel is so much lower than that of air, that the same electric current in the coil sets up thousands of times as many magnetic lines in the iron core as it would in the air alone.

The powerful poles of magnetic hoists are built on this principle. Coils of wire are wound around a number of cores of soft iron. As long as there is no electric current going through the coils the iron

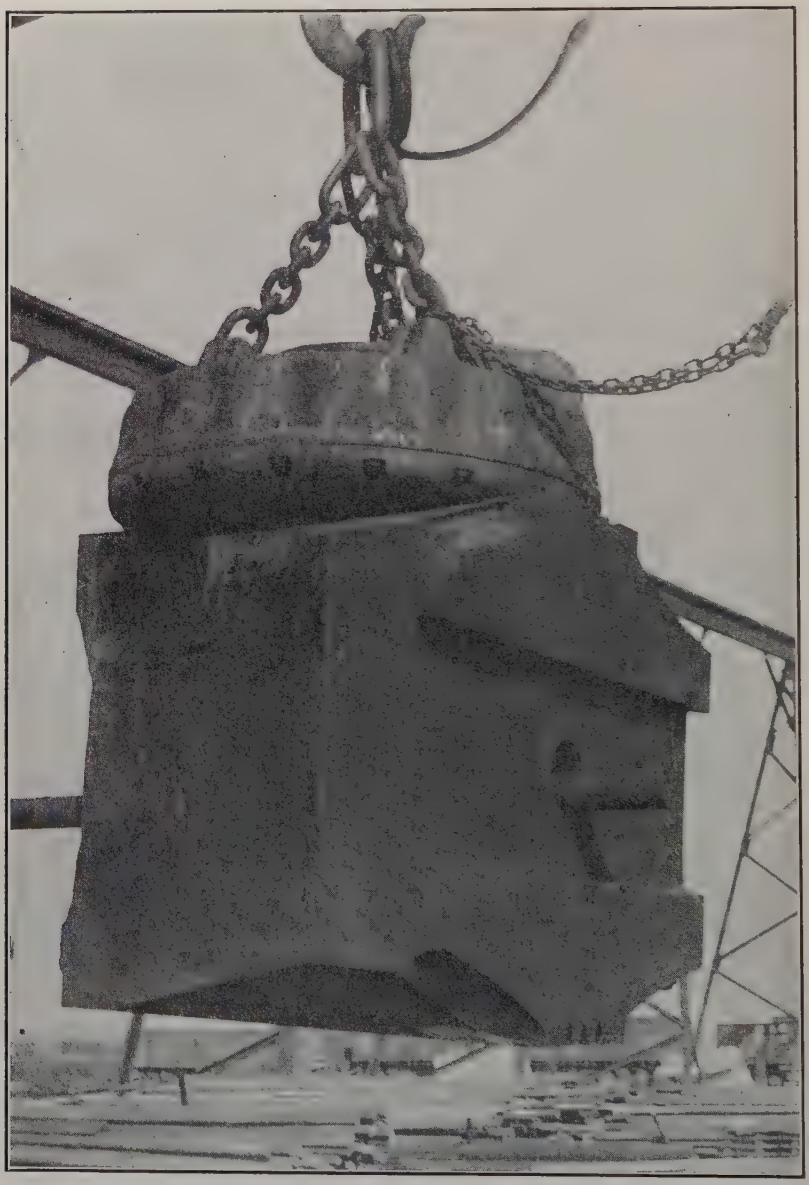


FIG. 107. An electromagnet lifting a load of 15,000 pounds.
Cutler-Hammer, Inc.

is not a magnet. The face of the iron cores is brought into contact with pieces of iron or steel castings, etc., and the current turned on. The iron cores now become such strong magnets that each square inch of their ends will lift from 100 to 200 pounds of iron.

To release the load of iron it is merely necessary to turn off the electric current, or to reverse the direction of it.

Figure 107 shows one of these magnetic hoists in action.

Prob. 6–5. Draw magnetic internal and external field for iron core with electric current flowing around it as indicated in Fig. 108.

FIG. 108. An electromagnet.

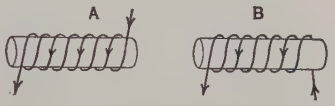


FIG. 109. Two electromagnets.

Prob. 7–5. Draw field between coils *A* and *B*, in Fig. 109. State whether attraction or repulsion exists between them.

77. Generator and Motor Fields. But the most important use of electromagnets is in generators and motors, where they are used to create the intense magnetic fields necessary in the economical generation of electric power. In Fig. 110, which represents a 2-pole motor, coils are wound on iron or soft-steel cores in such a way as to make one pole face a North pole and the other a South pole. The steel yoke *Y* joins the two cores together at one end, so that the magnetic circuit is composed entirely of iron except for the space *G* where the armature goes. A large part of the armature is also made of soft iron or soft steel. The magnetic circuit is then a nearly closed iron circuit in the direction indicated.

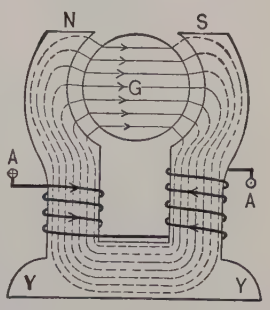


FIG. 110. The magnetic circuit in a bipolar motor.

In Fig. 111 is illustrated a more modern form of a 2-pole motor. Figure 112 shows how the current in the field coils is made to set up magnetic currents in the proper direction. Figure 113 shows the same thing for a four-pole motor.

Prob. 8–5. Show windings on cores *A* and *B* of an electric motor with direction of electric current indicated, and draw flux for magnetic circuit with direction of flux indicated in Fig. 114.

78. "Sucking" Coils. Just as a bar magnet will attract a piece of iron, so a coil, since it has a field like a bar magnet, will attract a piece of iron. When the plunger of soft iron in Fig. 115 is placed in the field of the coil, it becomes magnetized as marked, the North pole being nearest the South pole of the coil. Thus it is attracted into the coil, being drawn up until the centers of coil and plunger nearly coincide. The strongest attraction exists when the center of the iron plunger nearly coincides with the center of the coil.

FIG. 111. A two-pole motor frame.

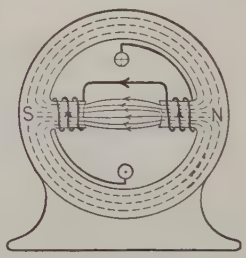


FIG. 112. The magnetic circuit in the two-pole motor frame of Fig. 111.

Extensive use is made commercially of this principle. It is the device used to operate a circuit breaker shown in Fig. 116. In this device the plunger is sucked up, only when the current in the coil becomes excessive. Then it automatically opens the switch, shutting off the power.

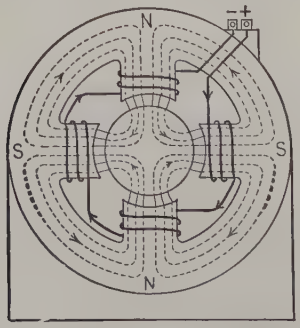


FIG. 113. The magnetic circuits in a four-pole motor.

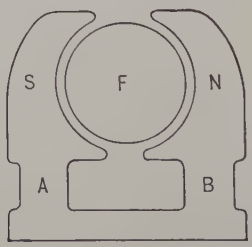


FIG. 114. The magnetic circuit of a two-pole motor.

79. Non-Inductive Coils. Since the direction of the magnetic field about a wire depends upon the direction of the electric current, it is seen that the fields of two currents flowing in opposite directions must oppose each other, and will neutralize each other if the wires are near enough together and the currents are equal. Use is made of this fact whenever it is desired to have a current with a very weak magnetic field. Two wires are wound into a single coil so

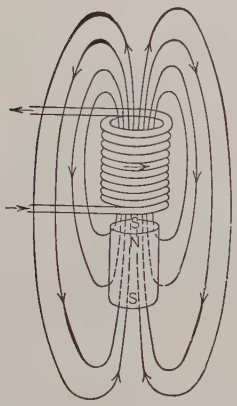


FIG. 115. A magnetic "sucking" coil.

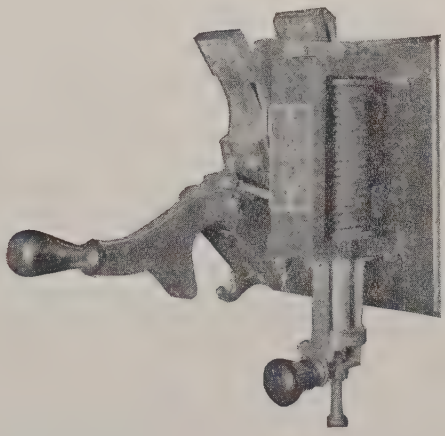


FIG. 116. A circuit breaker operated by a "sucking" coil.

that the current in one wire flows in one direction, and the current in the other wire in the other direction, around the coil. Such a coil has no perceptible magnetic field, and is said to be **non-inductively** wound. Resistance coils used in the measurement of resistance are wound in this way, as are almost all **resistance** coils in commercial ammeters and voltmeters.

SUMMARY OF CHAPTER V

A MAGNET is a body which possesses the property of attracting iron and steel.

THE POLES of a magnet are those regions where the attraction is greatest.

It is usual to represent the strength of these poles by lines called **MAGNETIC LINES OF FORCE.** These magnetic lines are represented as coming out of a **NORTH POLE**, going around the magnet, and into a **SOUTH POLE**, going through the magnet to a North pole.

Thus a **MAGNETIC LINE OF FORCE** forms a complete circuit.

A MAGNETIC FIELD is a space in which there are magnetic lines.

A RING may be magnetized so that all the lines lie inside the iron, thus producing no poles. Most electrical machines consist of magnets, the force lines of which lie partly in air and partly in iron or steel.

Magnetic lines act like stretched rubber bands and tend to SHORTEN. They also exert a LATERAL CROWDING effect upon one another. This explains the ATTRACTION OF UNLIKE POLES and the REPULSION OF LIKE POLES.

RELUCTANCE is the resistance which a substance offers to the magnetic flux.

The reluctance of iron and steel is low: thus a piece of iron in a magnetic field offers less reluctance than the air to the magnetic lines. The lines thus become denser in the iron, making it a magnet. This explains the attraction of iron and steel by a magnet.

NO MATERIAL is a good MAGNETIC INSULATOR; thus a space to be screened from magnetic lines must be surrounded by material of low reluctance, which will shunt most of the magnetic lines around the space.

A COMPASS is a magnet free to turn. It always takes a position lying along a line of force.

The EARTH is a large magnet with the magnetic South pole lying near the geographic North pole. The North pole of a compass therefore points to the magnetic South pole which is near the geographical North pole.

There is always a magnetic field about a wire carrying a current of electricity.

THUMB RULE FOR STRAIGHT WIRE. Grasp the wire with the right hand so that the thumb points in the direction of the current, the fingers then will point in the direction of the magnetic field around the wire.

An electric current flowing in a coil of wire makes an electromagnet of the coil, one end becoming a North pole and the other a South pole.

THUMB RULE FOR A COIL. Grasp the coil so that the fingers point in the direction of the current around coil, and the thumb points to the North pole. This is another form of the rule for a straight wire.

The powerful magnets composing the magnetic fields of generators and motors are made by coiling insulated wire around soft iron cores.

PROBLEMS ON CHAPTER V

Prob. 9–5. Draw the magnetic field for the motor in Fig. 117.

Prob. 10–5. Draw a 6-pole dynamo field, showing circuits and direction of flux, and of electric current.

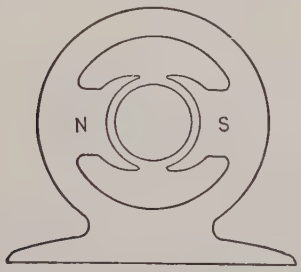


FIG. 117. The magnetic paths of a two-pole motor.

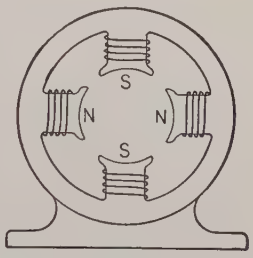


FIG. 118. The magnetic paths in a four-pole motor.

Prob. 11–5. Show current in windings on the 4-pole motor frame to produce North and South poles as marked in Fig. 118. Indicate magnetic flux as usual.

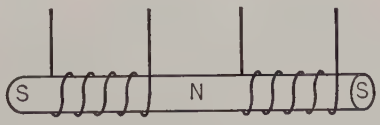


FIG. 119. An electromagnet with two coils.

Prob. 12–5. Draw magnetic field of Fig. 119, and put current arrows on windings to produce field as marked.

CHAPTER VI

THE MAGNETIC CIRCUIT

In order to make precise calculations of the efficiency and operating characteristics of motors, generators and other electrical devices, it is necessary to be able to calculate not only the electric quantities but also the magnetic quantities involved. Magnetism and electricity are so closely associated in all electrical appliances, and so interrelated and interdependent that we must be as familiar with the laws of the **magnetic circuit** as we are with those of the **electric circuit.** Fortunately, the same laws which are fundamental to the electric circuit are equally fundamental to the magnetic circuit. It is merely necessary, therefore, to become familiar with the laws of the electric circuit, as **restated** in the terms and units of the magnetic circuit.

80. Magnetic Flux (Lines of Force). In starting to study the laws of the magnetic circuit, we use the simplest circuit possible. Such a circuit can be represented by Fig. 120, which consists of a wooden ring, 6.25 centimeters mean diameter, made of 1.25-centimeter round stock. On this ring are wound 730 turns of insulated wire.

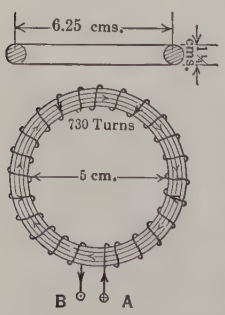


FIG. 120. A ring electromagnet.

If we send an electric current through the wire in the direction marked (in at A), a magnetic flux will be set up in the wooden ring in the direction indicated (clock-wise).

We have seen that this magnetic flux consists of lines of force, which in this case go around in the circuit composed of a wooden ring. The amount of this flux is measured by the number of lines which thread the circuit, just as the amount of an electric current is measured by the number of amperes which flow around an electric circuit. We represent the number of amperes in an electric current by the letter (I). Similarly we represent the number of lines in a magnetic circuit by the Greek letter (Φ), (pronounced, **phi**).

81. Magnetic Pressure. The Gilbert. But a current of electricity or a current of water cannot exist unless there is a pressure set up causing it to flow. And neither can a magnetic flux exist in a circuit unless there is a magnetic pressure set up which causes the magnetic flux to exist. An electric pressure may come from a battery cell or a generator, and is called the **electromotive force.** The magnetic pressure is usually set up by an electric current flowing in a coil, and is called the **magnetomotive force.** We measure the electric pressure in **volts** and represent it by the letter E. Similarly we represent the magnetic pressure, or magnetomotive force, by the letter F, and measure it in **gilberts.** The number of gilberts is equal to 0.4π, or 1.26, times the product of the amperes flowing in the coil by the number of turns of wire in the coil. Thus if an electric current of 2 amperes is flowing in the 730 turns of the coil on the ring of Fig. 120, a magnetic pressure of $1.26 \times 2 \times 730 = 1840$ gilberts is produced. A current of 3 amperes in this coil would set up a magnetic pressure of $1.26 \times 3 \times 730$ or 2760 gilberts.

The product of the number of turns in a coil by the amperes flowing through the turns is called the **ampere-turns.**

82. Magnetic Resistance (Reluctance). Whenever we wish to set up an electric current in a circuit, we always have to overcome the **resistance** of the electric circuit, which opposes the flow of the current. Similarly, we have seen that there is always a magnetic resistance, which we call the **reluctance,** and which always opposes the setting up of the magnetic flux in the circuit. We represent the electric resistance by the letter R, and measure it in **ohms.** We represent the magnetic resistance (reluctance) by the script letter $\mathcal{R}$, but no name is given the quantity.* Just as the resistance of an electric circuit depends upon the material and dimensions of the circuit, so the reluctance of a **magnetic circuit** depends upon the material and dimensions of the magnetic circuit.

83. Ohm's Law for the Magnetic Circuit. Thus in the magnetic circuit we have magnetic pressure setting up a magnetic flux against the resistance offered by the magnetic reluctance, just as in an electric circuit we have the electric pressure setting up an electric current against the opposition offered by the electrical resistance.

* In many references and texts, including earlier editions of this text, the name **oersted** is given to the unit of magnetic reluctance. In accordance with recent international agreement this name will now be used to designate the unit of **magnetizing force** (see Art. 87).

The amount of the electric current set up can be determined from Ohm's law:

$$\text{Electric Current} = \frac{\text{Electric Pressure}}{\text{Electrical Resistance}}$$

$$\text{Amperes} = \frac{\text{Volts}}{\text{Ohms}} = I = \frac{E}{R}.$$

Similarly the amount of the magnetic flux set up can be determined by Ohm's law of the Magnetic Circuit:

$$\text{Magnetic Flux} = \frac{\text{Magnetic Pressure}}{\text{Magnetic Reluctance}}$$

$$\text{Magnetic Lines} = \frac{\text{Gilberts}}{\text{Reluctance}}$$

$$\Phi = \frac{F}{\mathfrak{R}}.$$

Like Ohm's law for the electric circuit, this equation can have the three forms:

$$\Phi = \frac{F}{\mathfrak{R}} \qquad \text{Magnetic Lines} = \frac{\text{Gilberts}}{\text{Reluctance}}$$

$$\mathfrak{R} = \frac{F}{\Phi} \qquad \text{Reluctance} = \frac{\text{Gilberts}}{\text{Magnetic Lines}}$$

$$F = \Phi\mathfrak{R} \qquad \text{Gilberts} = \text{Magnetic Lines} \times \text{Reluctance}$$

Example 1. The reluctance of the wooden ring in Fig. 120 is 16 units. What magnetic flux would be set up in this ring if 2 amperes were sent through the 730 turns of the coil?

Solution.

$$\text{Magnetic Lines} = \frac{\text{Gilberts}}{\text{Reluctance}}$$

$$\Phi = \frac{F}{\mathfrak{R}}$$

$$= \frac{1.26 \times 2 \times 730}{16} = 115 \text{ Magnetic Lines.}$$

Example 2. The magnetic circuit of Fig. 121 is composed of iron. The coil has 280 turns. When 3.6 amperes are sent through the coil, a flux of 3000 lines is set up in the iron circuit. What is the reluctance of the circuit?

Solution.

$$\text{Reluctance} = \frac{\text{Gilberts}}{\text{Magnetic Lines}}$$

$$\mathcal{R} = \frac{F}{\Phi}$$

$$= \frac{1.26 \times 3.6 \times 280}{3000}$$

$$= 0.423 \text{ unit.}$$

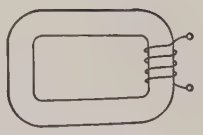


FIG. 121. An electromagnet composed of an iron ring.

Example 3. How many amperes must be sent through the coil on the wooden ring of Fig. 120, in order to set up a magnetic flux of 400 lines? The reluctance of the ring is 16 units.

Solution.

$$\text{Gilberts} = \text{Lines} \times \text{Reluctance}$$

$$F = \Phi\mathcal{R}$$

$$= 400 \times 16 = 6400 \text{ Gilberts.}$$

These 6400 gilberts must be made up of the product of the 730 turns times the amperes flowing through them times 1.26, because

$$\text{Gilberts} = 1.26 \times \text{Amperes} \times \text{Turns}$$

$$\text{Amperes} = \frac{\text{Gilberts}}{1.26 \times \text{Turns}}$$

$$= \frac{6400}{1.26 \times 730} = 6.96 \text{ Amperes.}$$

Prob. 1–6. What magnetic flux will be set up in a magnetic circuit of reluctance 2.5 if the magnetomotive force is supplied by a coil of 1236 turns carrying 0.42 ampere?

Prob. 2–6. The magnetic circuit of an electromagnet has a reluctance of 0.4. How great a magnetomotive force is necessary to set up a flux of 125,000 lines?

Prob. 3–6. A certain electromagnet has a flux of 200,000 lines. The coil has 325 turns carrying 2.4 amperes. What is the reluctance of the magnetic circuit?

Prob. 4–6. A coil consists of 1980 turns. The reluctance of the magnetic circuit is 0.008. How many amperes must flow through coil to set up a flux of 1,000,000 lines?

Prob. 5–6. How many turns must be wound on a brass ring, the reluctance of which is 1.73, if a current of 3.84 amperes is to set up 8600 lines?

Prob. 6–6. How many turns would be required to set up the same flux in Prob. 5–6 if the electric current were made twice as large?

84. Computation of Reluctance (Non-magnetic Materials). In computing the reluctance of a magnetic circuit, we use the same

method and the same laws employed in finding the resistance of an electric circuit.

When we wish to determine the resistance of wire, we make use of the resistivity, that is, the resistance of a unit wire.

Since the diameter of wire is measured in mils, and the length in feet, we use as a unit a mil-foot. And because most wire used for carrying electric currents is copper, we learned that the resistance of a mil-foot of copper is 10.4 ohms.

To find the resistance of a copper wire of any size or length, we have merely to multiply the resistivity, 10.4, by the length in feet, and divide by the cross-section area in circular mils.

Similarly, when we wish to determine the reluctance of a magnetic circuit, we make use of the reluctivity, which is the reluctance of a unit piece of the circuit. The unit piece is the centimeter cube.

While most magnetic circuits are composed of the magnetic substances, iron and steel, the reluctivity of these materials varies so widely that we do not attempt to learn the value of it. Rather, we learn the reluctivity of air, which is the same as that of practically all materials except iron and steel. The reluctance of a centimeter cube of air is unity.

To find the reluctance of a portion of any circuit composed of any material, except iron and steel, we merely have to multiply the reluctance of a centimeter cube, by the length of that part of the circuit in centimeters and divide by its cross-section area in square centimeters. This expressed in the form of an equation is:

$$\mathcal{R} = \frac{1 \times l}{A},$$

in which

l = length in **centimeters** of magnetic circuit,
A = area (cross-section) in **square centimeters** of magnetic circuit.

Note that this is similar in every detail to the equation for the resistance of a copper wire:

$$R = \frac{10.4\, l}{d^2},$$

in which

l = length of wire in feet,
d^2 = area of wire in circular-mils.

Example 4. Compute the reluctance of the wooden ring in Fig. 120, made of 1.25 cm. round stock, mean diameter of ring 6.25 cm.

Solution.

$$\begin{aligned}\text{Length of magnetic circuit} &= 3.14 \times 6.25 \\ &= 19.63 \text{ cm.} \\ \text{Cross-section area} &= 1.25^2 \times 0.785 \\ &= 1.227 \text{ sq. cm.} \\ \text{Reluctance} &= \frac{1 \times l}{A} \\ &= \frac{19.63}{1.227} \\ &= 16.0 \text{ units.}\end{aligned}$$

This equation is particularly useful in finding the reluctance of air gaps in magnetic circuits.

Example 5. In Fig. 122, the magnetic circuit is composed of an iron ring and an air gap. The length of the air gap is 1.6 cm. The cross-section of this gap is round in shape with a diameter of 2.13 cm. What is the reluctance of the air gap alone? (The method of finding

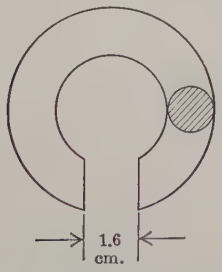


FIG. 122. A ring with an air gap.

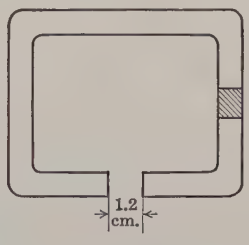


FIG. 123. A magnet made of rectangular stock.

the reluctance of the iron part of the circuit is explained in succeeding paragraphs.)

Solution.

$$\begin{aligned}\text{Reluctance of air gap} &= \frac{1 \times l}{A} \\ &= \frac{1 \times 1.6}{2.13^2 \times 0.785} \\ &= 0.450 \text{ unit.}\end{aligned}$$

Prob. 7–6. What is the reluctance of a brass bar, 40 cm. long, made of 2 cm. $\times$ 3 cm. rectangular stock?

Prob. 8–6. Compute the reluctance of the air gap in Fig. 123. The air gap is 1.0 cm. square and 1.2 cm. long.

Prob. 9–6. The length of an air gap is 0.875 cm. What cross-section must it have in order that the reluctance may not exceed 0.42?

Prob. 10–6. How many gilberts are necessary to send 1000 lines through brass plate 0.25 cm. thick having an area of 12 cm. × 4 cm.

Prob. 11–6. How many ampere-turns are necessary to send 1000 lines through an air gap 0.25 cm. long with a cross-section area of 12 cm. × 3 cm.?

85. Magnetic Field within a Long Coil or Solenoid. If we have a long coil, with air core, the length of which is at least ten times the diameter, we can easily find the **total** magnetic flux produced by a current in the coil. Figure 124 shows a long solenoid and the field

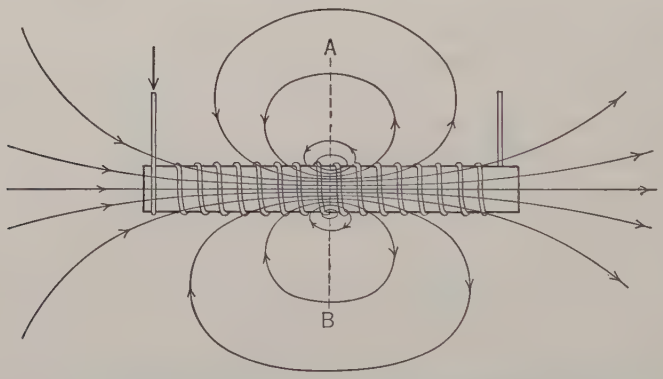


FIG. 124. A long solenoid, showing the magnetic field set up by current in the coil.

about it. Note that all the flux is collected inside the coil at the cross-section AB, mid-way between the ends.

Now it has been found that the **total** reluctance offered to the flow of magnetic flux is that of the air (or other non-magnetic material) **within** the coil. This is equivalent to saying that once the lines emerge from the coil they apparently take a path of zero reluctance through the air to the corresponding point on the other side of the coil.

Thus to find the flux through the center cross-section of the coil, we merely have to divide the **total** magnetomotive force by the reluctance of the air inside the coil. Note that as in the case of the electric circuit, we find the **total** flux by dividing the **total** magnetomotive force by the **total** reluctance. This result is true only at the center section of the coil where the total flux passes.

The reluctance of the air inside the coil $= \frac{l}{A}$ where

A = inside cross-section area of coil in square centimeters
l = length of coil in centimeters.

Therefore,

$$\Phi \text{ (total lines)} = \frac{\text{Total magnetomotive force}}{\text{Total reluctance}} = \frac{1.26\ NI}{\frac{l}{A}} = \frac{1.26\ NIA}{l}$$

where N = number of turns on coil
I = current in amperes.

This equation gives the flux at the center only of a long solenoid. Of this total flux, half leaves (or enters) the coil at the ends and the other half leaks through the sides.

Example 6. What magnetic flux is set up in an air-core coil 100 cm. long and 2 cm. in diameter, if it consists of 4000 turns in which 0.24 ampere is flowing?

Solution.

$$\Phi = \frac{1.26\ NIA}{l} = \frac{1.26 \times 4000 \times 0.24 \times 3.14}{100} = 38.0 \text{ lines (through the center section).}$$

Half of these, or 19.0 lines, pass out of the coil at one end and in at the other end.

Prob. 12–6. How many amperes must be sent through an air-core coil consisting of 400 turns in order to set up 2000 lines at the center of the coil? The coil is 30 cm. long and has a diameter of 2.5 cm.

Prob. 13–6. How much current must be sent through the coil of Prob. 12–6 in order to set up a flux of 2000 at the ends of the coil?

Prob. 14–6. What diameter must a long air-core coil have if a current of 8 amperes is to set up a flux of 3200 lines at the center of the coil? The length of the coil is 40 cm., and it is wound with 1500 turns.

86. Reluctance of Iron and Steel. The reluctivity of iron and steel used in the electrical industries is much lower than that of the non-magnetic materials, ranging from 0.016 to 0.00025 for the various grades. But it is impractical to state the reluctivity of any

particular grade of iron or steel, because even in a given sample of known composition and heat treatment, the reluctance will vary with the degree to which it is magnetized.

In this important respect, the reluctance of the magnetic materials differs from the resistance of electric conductors, and in fact from the reluctance of the non-magnetic materials. The **resistivity** of copper of a certain grade is always 10.4 ohms per mil-foot, if the temperature is standard, regardless of how many amperes are flowing through it, and the **reluctivity** of all non-magnetic materials is unity per centimeter cube, regardless of how many magnetic lines are set up in them.

But the reluctance of a piece of Standard Annealed sheet steel may be 0.0001 per centimeter cube when 20,000 lines are going through each square centimeter of it, and yet be only 0.00003 per centimeter cube when the magnetic flux is reduced to 12,500 lines per square centimeter.

Therefore, instead of being able to use a standard value for the reluctivity of each grade of iron and steel as we can for non-magnetic materials, we are compelled to use tables or curves showing the values of the reluctance of these materials when magnetized to different degrees.

87. Magnetization Curves. However, instead of plotting curves showing the values of reluctivity for different degrees of magnetization, it is just as easy to construct tables and plot curves showing the magnetizing force required to set up a given number of lines per square centimeter in the material. This saves a large amount of mathematical work, because instead of having to multiply the magnetic flux by the reluctance in each case, in order to find the number of gilberts necessary to set up this number of lines, we can find the necessary number of gilberts per centimeter directly from the tables or curves.

If we want to know the reluctivity we have only to divide the gilberts per centimeter by the flux density as given in the table or taken from curves. So in using these tables and curves, we are really making use of Ohm's law for the magnetic circuit, although much of the mathematics has been done in making the curves and tables, and therefore does not appear in our work.

In order to measure the magnetic quantities needed for tables and curves, a sample of the desired iron or steel is made up into a ring, as in Fig. 120. It is closely wound with wire of a known number of turns and a known current is passed through the coil. This sets up

magnetic flux in the sample, and the amount of flux can be measured by a special galvanometer circuit. We thus know the number of gilberts required to set up a measured number of lines in a sample of known length and cross-section.

For the purpose of making tables which can be used with as little mathematics as possible, it is customary to divide this number of gilberts by the length of the magnetic path and thus determine the number of gilberts necessary to set up this number of lines in a piece of the material one centimeter long.

Thus we send 2 amperes through the 730 turns on the ring in Fig. 120. We would have a magnetomotive force, F, of $1.26 \times 2 \times 730$ or 1840 gilberts magnetizing a magnetic circuit 6.25×3.14 or 19.6 centimeters long. To magnetize one centimeter of this circuit would require 1840/19.6 or 93.7 gilberts. From this value we may determine the number of gilberts necessary to magnetize any length of this iron to the same degree. For instance, to set up this number of lines in a piece of this iron 40 centimeters long and of the same cross-section would require 40×93.7 or 3750 gilberts.

This value of gilberts per centimeter is called the **magnetizing force** for this degree of magnetization, and is represented by the letter H. Although the magnetizing force is always in gilberts per centimeter, this unit is given the name **oersted.** Thus in the example above, the magnetizing force is 93.7 oersteds.

The student should get the distinction between F and H clearly in mind. The magnetizing force H is really the magnetic pressure required to send a given number of lines through one centimeter of the magnetic circuit. It is similar to the voltage required to send a given electric current through a mile of wire of given dimensions. Just as we would speak of the voltage drop per mile of wire in that case, we can speak of H as the magnetic pressure drop per centimeter of material.

H is the number of gilberts required to send a given number of lines through **one** centimeter of a given circuit, and is expressed in **oersteds.**

F is the number of gilberts necessary to send a given number of lines through **any given** length of circuit and therefore represents **gilberts.** To find F, the gilberts for a given length of circuit, we merely have to multiply H, the oersteds, by l, the length of the material in the circuit. Expressed as an equation:

$$F = Hl.$$

Again, in order to make mathematical work easy, we do not put in the table the total number of lines set up in the test ring, but only the lines set up per square centimeter of cross-section.

Thus if the 1840 gilberts (93.7 oersteds) set up 21,400 lines in a ring of 1.23 square centimeters cross-section, we divide this number of lines by the cross-section area and state that 21,400/1.23 or 17,400 lines per square centimeter were set up. Knowing that 93.7 oersteds can set up 17,400 lines per square centimeter, we can find the number of lines set up by this magnetic pressure in a circuit of any cross-section. For instance, the number of lines set up by this magnetizing force in a piece of this iron with a 4-square-centimeter cross-section area, would be $4 \times 17{,}400$ or 69,600 lines.

This value of lines per square centimeter is called the **flux density** or **degree of magnetization,** and is measured in **gausses,** one gauss being merely one line per square centimeter. It is represented by the letter B. It corresponds to the amperes per square inch cross-section of an electric circuit. For instance, it is often stated that the current density in the copper parts of certain electrical devices, such as switchblades, bus bars, etc., should not exceed 1000 amperes per square inch. To carry 2000 amperes, therefore, it is necessary to use blades of 2 square inches cross-section. Similarly, if 10,000 gausses, or lines per square centimeter, is the flux density at which a certain grade of steel should be magnetized for the most efficient design, then to carry 20,000 lines it would be necessary to use a steel piece of 2 square centimeters.

Again we should get this distinction between Φ and B clearly in mind.

B is the flux density or number of **lines per square centimeter** or the **gausses** in the material.

Φ is the number of lines flowing through **any** given area of the material. To find Φ, we merely have to multiply B, the number of lines per square centimeter, by A, the area. Expressed as an equation:

$$\Phi = BA.$$

88. Typical B-H Curves. In the manner described above, specimens of iron and steel used in magnetic circuits of electrical machinery have been tested for the number of gausses set up by different magnetizing forces, ranging from a few oersteds up to values well beyond those in commercial use. Curves have been plotted between the values of H and the corresponding values of B. These curves are called magnetization, saturation, or B–H curves.

Figure 125 is a set of standard curves for soft steel castings, cast iron, wrought iron and for Standard Annealed sheet steel. These curves represent the average for materials used by large manufacturing companies and are values commonly met with in commercial materials. The facility which the use of these curves lends to magnetic computations is illustrated by the following examples.

Example 7. What electric current must be sent through the coil in Fig. 120 in order to produce a magnetic flux of 19,350 lines in the ring, if a wrought iron ring is used?

Solution.

$$\text{Area of ring} = 1.25 \times 1.25 \times 0.785$$
$$= 1.23 \text{ square centimeters.}$$
$$B = \frac{19{,}350}{1.23} = 15{,}750 \text{ gausses.}$$

From *B–H* curve for wrought iron, to magnetize wrought iron to flux density of 15,750 gausses requires 13 oersteds.

$$\text{Length} = 6.25 \times 3.14 = 19.6 \text{ centimeters.}$$

To magnetize 19.6 centimeters of wrought iron to a flux density of 15,750 gausses requires $19.6 \times 13 = 255$ gilberts or $\frac{255}{1.26} = 202$ ampere-turns. The current necessary $= \frac{202}{730} = .277$ amp.

Example 8. If the ring in the above example were soft cast steel, how many ampere-turns would be necessary to magnetize it to the same flux density as in above example?

Solution.

From *B–H* curve for cast steel, to magnetize cast steel to a flux density of 15,750 gausses requires 49 oersteds.

To magnetize 19.6 centimeters of cast steel to this flux density requires 19.6×49 gilberts $= \frac{960}{1.26}$ or 762 ampere-turns.

In order to show the iron and steel characteristics more completely, the curves of Fig. 125 are plotted to a much smaller scale than would be satisfactory for use in practical design. Actual flux densities rarely exceed 15,000 gausses because the required magnetizing current increases very rapidly beyond the bend, or "knee" of the magnetization curve. Hence, it is usually sufficient to measure and plot only a small part of the magnetization curve which includes the desired range of flux densities. This portion can be plotted to a large scale so that more precise readings can be taken from it.

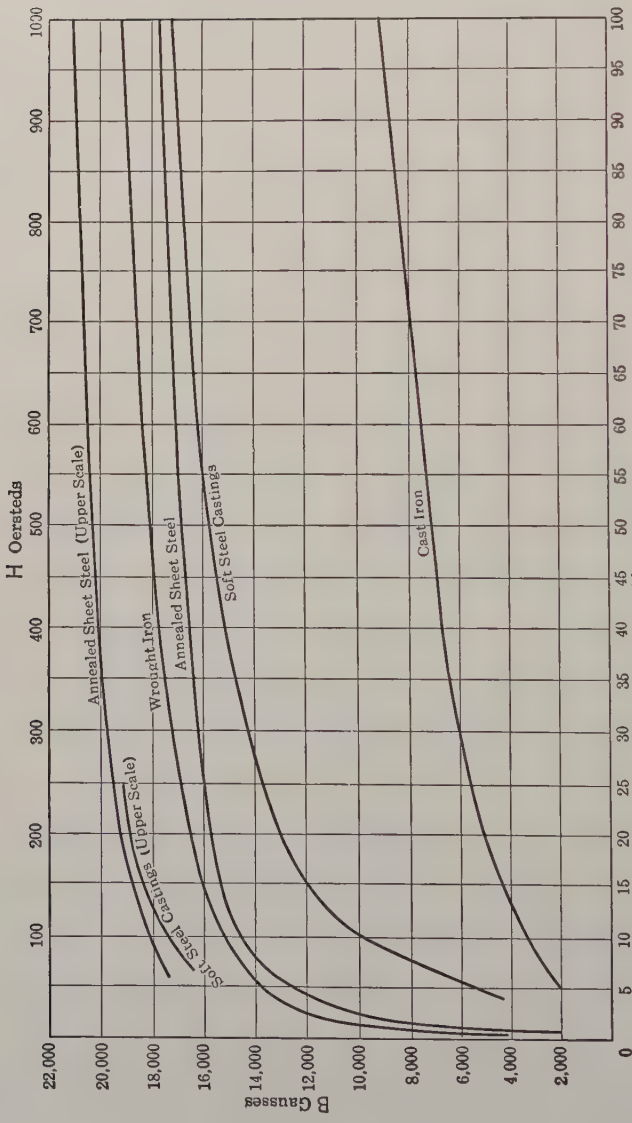


Fig. 125. Typical magnetization curves of irons and steels.

Prob. 15–6. The magnetic circuit of Fig. 126 was made up of annealed sheet steel pieces, having an average length of long sides of 9 centimeters and of short sides of 7 centimeters. Each side is 2.5 centimeters wide. The pile of sheets is 2.75 centimeters deep. If 90,000 lines are to be set up in this core, how many ampere-turns are needed in the coil? Subtract 12% from the depth to allow for the insulation between the sheets.

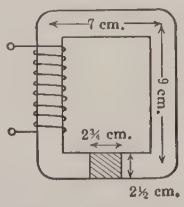


FIG. 126. An electromagnet of square stock.

Prob. 16–6. If the core of Fig. 126 were a solid soft steel casting, how many ampere-turns would be needed to set up 90,000 lines?

Prob. 17–6. How many ampere-turns would be needed to set up 90,000 lines in a core of brass sheets of same size as that of Fig. 126? No insulation between sheets.

Prob. 18–6. If a core is made up of Standard Annealed steel sheets only 75% as deep as that of Prob. 15–6 but otherwise of the same dimensions, how many ampere-turns are required to set up a flux of 90,000 lines? Sheets insulated as in Prob. 15–6.

Prob. 19–6. If the core in Prob. 18–6 were to have the same flux density as the core in Prob. 15–6, how many lines would be set up?

Prob. 20–6. How many ampere-turns would be needed to set up the flux of Prob. 19–6?

Prob. 21–6. Each of two electromagnets is made of wrought iron. Each has a length of 2 feet. The first has a circular cross-section area of 4.8 square inches and is to be magnetized to carry 380,000 lines. The second has a circular cross-section area of 2.4 square inches and is to carry 190,000 lines.

(*a*) How many ampere-turns must be applied to the first?
(*b*) How many ampere-turns must be applied to the second?
(*c*) If the same size wire were used in both coils, about how would the lengths of the amounts wound on the two compare?

Prob. 22–6. How many ampere-turns would be needed for the first magnet of Prob. 21–6 if it were to carry only the same number of lines as the second magnet?

89. Saturation Point. In Fig. 127, which is a copy of the *B–H* curve for annealed sheet steel, we have a typical shape of the magnetization curve of magnetic materials. The peculiar shape of this curve should be noted in detail as it is typical of all such magnetization curves.

In general there are two stages of magnetization. In order to set up a few lines of force, a relatively small number of oersteds is needed. For instance, to set up the first 8500 gausses, we need only

1.5 oersteds, but it becomes increasingly difficult to set up further lines. This is shown by the fact that about 60 oersteds are required to set up 17,000 gausses. In other words, to double the number of gausses we have to use about forty times the magnetizing force. If we wish to increase the number of lines even slightly beyond this point, the number of oersteds required assumes a tremendous proportion. For instance, if we try to set up 17,500 gausses, which is an increase of 500 gausses, we should have to use 90 oersteds.

The two stages of magnetization may be defined as follows. The first stage is that during which it is comparatively easy to add lines of force to the steel. This would take in the part of the curve up to that part marked " *X*," Fig. 127.

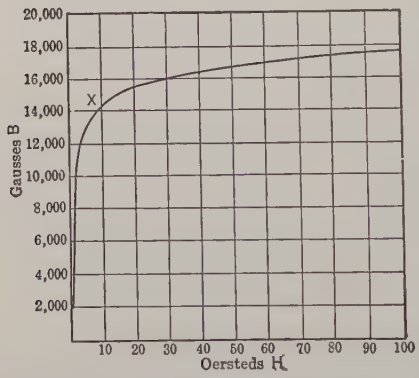


FIG. 127. The magnetization curve of annealed sheet steel.

The second stage comprises all that part of the curve beyond the point, " *X*," where it is much more difficult to add lines of force. The point " *X* " is commonly called the Saturation Point. Note particularly that the saturation point is not that point at which the steel is so full of lines that no more can be added, but is that point beyond which it is extremely difficult to add more lines. In fact, there is practically no point beyond which it is impossible to set up more lines. It is probably true that if we continued to add ampere-turns indefinitely to a cast steel magnetic circuit, we should continue to increase slightly the number of lines in the circuit.

The saturation point then really means that point beyond which it is usually unprofitable to magnetize the steel because of the greatly increased difficulty in setting up lines of force. The point lies at the " knee " of the curve, where it bends into a nearly straight line running almost horizontally.

If we carry the magnetization far enough the steel will probably behave like a non-magnetic material and the same number of ampere-turns would be needed to set up additional lines that would be required to set up the same number of additional lines in so much air.

In actual practice, the magnetization is rarely carried much beyond the saturation point.

90. Permeability and Its Relation to Flux Density. Another way of stating the facts in the above section would be to say that in the first stage of magnetization, that is, as far as the saturation point, the reluctivity of the steel is comparatively low. Beyond this point the reluctivity of the steel increases rapidly until finally it would probably reach that of air. However, in this connection, instead of using the term reluctivity, it is customary to use the term permeability, which is merely the reciprocal of the reluctivity, just as in any direct-current electric circuit, conductivity is the reciprocal of resistivity. Since the reluctivity is the ratio of the oersteds to the lines set up in a centimeter cube of the material, so the permeability would be the inverse ratio, or the ratio of the lines set up to the oersteds for a centimeter cube of the material. In other words, it represents the number of lines set up in a centimeter cube by one oersted.

The permeability of non-magnetic materials is a constant and equals $\frac{1}{1}$ or 1. Or we may state it as follows: one oersted will set up one line of force in a centimeter cube of a non-magnetic material. Thus one is the permeability of a non-magnetic material.

The permeability of a magnetic material is not a constant but, as we have seen, depends upon the number of gausses which the material contains. Thus, in cast steel the permeability when there are 7000 gausses equals $\frac{7000}{6.5}$ or 1075. When there are 15,500 gausses, the permeability equals $\frac{15,500}{47.5}$ or 326. When there are 18,600 gausses, the permeability of cast steel equals $\frac{18,600}{160}$ or 116. Note that the permeability decreases very rapidly. In other words, when the steel contains only 7000 gausses, each oersted sets up 1075 gausses. When the material contains 15,500 gausses, one oersted sets up only 326 gausses. When there are 18,600 gausses, one oersted sets up only 116 gausses. Thus a statement of the permeability of the material really shows how many gausses are set up by each oersted. The permeability, therefore, may be defined as the ratio of B to H. This is usually expressed in the equation:

$$\mu = \frac{B}{H},$$

where μ stands for the permeability.

The curves drawn between the corresponding values of B and μ for the different materials are called permeability curves. Figure 128 is the permeability curve corresponding to the B–H curve of annealed steel. Note that the permeability in this material is comparatively high up to a moderate degree of magnetization. Beyond that point the permeability drops off very quickly as the flux density gets higher.

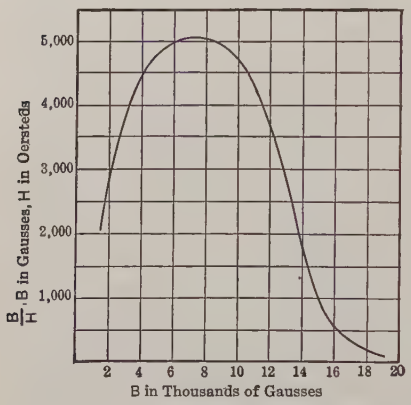


Fig. 128. The relation between flux density and permeability of annealed sheet steel.

The equation given above that $\mu = \dfrac{B}{H}$ provides a very simple way of dealing with magnetic circuits containing air or other non-magnetic materials. Since for these materials, $\mu = 1$, we can write the equation

$$B = H.$$

Hence, **in the non-magnetic materials,** the number of gausses set up is the same as the number of oersteds of magnetizing force.

Example 9. In Example 7 (Art. 88) what is the permeability of the wrought iron ring?

Solution.

$$\mu = \frac{B}{H} = \frac{15{,}750}{13} = 1212.$$

Example 10. If the ring in Example 7 were made of brass, how many ampere-turns would be needed to magnetize it to the same flux density?

Solution. Since brass is non-magnetic, the magnetizing force H (in oersteds) must equal the flux density B (in gausses). Hence

$$H = 15{,}750 \text{ oersteds.}$$

Since the length of the path is 19.6 centimeters, a total of $15{,}750 \times 19.6$ gilberts is required. Thus

$$F = 15{,}750 \times 19.6 = 308{,}700 \text{ gilberts}$$
$$= \frac{308{,}700}{1.26} = 245{,}000 \text{ ampere-turns.}$$

Note that the value of F for brass is just 1212 times as great as for the wrought iron ring, and in Example 10 we found that the permeability of the wrought iron was 1212.

91. Series Magnetic Circuits. The same laws apply to series magnetic circuits as to series electric circuits.

(*a*) The total magnetic flux through all parts of a series circuit is the same. The flux density of different parts, however, may be different. Thus if 20,000 lines are present in a circuit which in some places has an area of 2 square centimeters, the flux density in those places will be 10,000 gausses. If in the same circuit there are places where the area is 4 square centimeters, the flux density in those places will be only 5000 gausses. In all parts of the circuit, however, there are 20,000 lines total flux.

(*b*) The reluctance of a series circuit is the sum of the reluctances of the several parts.

(*c*) The total magnetic pressure (in gilberts) required to set up a certain magnetic flux in a series circuit is the sum of the pressures required to set up the same flux in each of the parts.

This does not mean that a separate coil is used to supply magnetic pressure for each part; a single coil can supply the total pressure required by the series magnetic circuit, just as a single battery supplies all the voltage required in a series electric circuit.

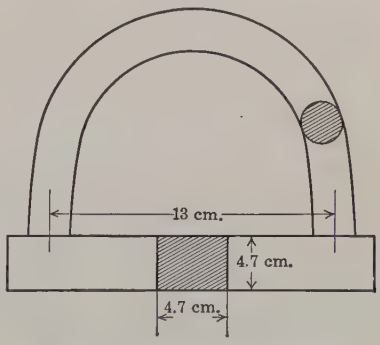


FIG. 129. A series magnetic circuit.

Example 11. In Fig. 129, the U-shaped portion is made of round wrought iron 3.5 centimeters in diameter. Its average length is 35 centimeters. The ends are flat and make a close fit on the surface of the bottom piece of cast iron, which has a cross-section 4.7 centimeters square. The average length of the path through the cast iron is 18 centimeters. If no air space is assumed to exist between the wrought iron and the cast iron, how many ampere-turns must be used to set up a magnetic flux of 130,000 lines through the magnetic circuit?

Solution.

$$\text{Cross-section area of wrought iron} = 0.785 \times 3.5 \times 3.5$$
$$= 9.62 \text{ square centimeters}$$

$$\text{Flux density } (B) \text{ in wrought iron} = \frac{130{,}000}{9.62} = 13{,}500 \text{ gausses.}$$

Magnetizing force (H) required to set up 13,500 gausses (from curve for wrought iron) = 4 oersteds

Magnetomotive force (F) required to set up 13,500 gausses in 35 centimeters of wrought iron $= 35 \times 4$
$= 140$ gilberts

Cross-section area of cast iron $= 4.7 \times 4.7$
$= 22.1$ square centimeters

Flux density (B) in cast iron $= \dfrac{130{,}000}{22.1}$
$= 5880$ gausses.

Magnetizing force (H) required to set up 5880 gausses (from curve for cast iron) = 28 oersteds

Magnetomotive force (F) required to set up 5880 gausses in 18 centimeters of cast iron $= 18 \times 28$
$= 504$ gilberts

Pressure required for entire magnetic circuit
$= 140 + 504$
$= 644$ gilberts
$= \dfrac{644}{1.26} = 511$ ampere-turns

The coil can be wound entirely on the U-shaped piece of wrought iron, or on the cast iron bar, or part on each — in whatever manner is most convenient.

Example 12. If there is an air gap 0.25 millimeter long where each end of the U-shaped piece joins the flat bar, Fig. 129, how many ampere-turns are required to set up the 130,000 lines?

Solution. The flux density in the air is the same as in the wrought iron, or 13,500 gausses. Hence, since for air $B = H$

Magnetomotive force for air gaps $= Hl$
$= 13{,}500 \times 0.05$
$= 675$ gilberts
$= \dfrac{675}{1.26} = 535$ ampere-turns

Total ampere-turns $= 511$ (iron) $+ 535$ (air)
$= 1046$ ampere-turns.

Prob. 23–6. Compute the number of ampere-turns necessary to magnetize the circuit of Fig. 129 with a flux of 140,000 lines. Air gap 0.002 inch at each end.

Prob. 24–6. If the U-shaped piece in Fig. 129 were cast steel, what number of ampere-turns would be necessary? Other data as in Prob. 23–6.

Prob. 25–6. The magnetic circuit of a certain 2-pole generator is made up of the following parts in series. Two cast-steel cores each 13.2 square inches in cross-section and 6 inches long; one cast-steel yoke 8 inches long, 12 square inches cross-section; two standard annealed sheet-steel pole pieces average length $2\frac{1}{4}$ inches, average cross-section 12.5 square inches; one armature core of standard annealed sheet steel average cross-section 13 square inches, average length of magnetic path $2\frac{3}{4}$ inches; two air gaps each with cross-section area 18 square inches and $\frac{3}{32}$ inch long. How many ampere-turns are necessary if the flux is to be 1,080,000 lines? Assume all the flux flows through the whole of the above circuit.

Prob. 26–6. If the maximum current allowed in the coils of Prob. 25–6 is 0.8 ampere, how many turns are necessary?

92. Parallel Magnetic Circuits. In all multipolar and in many bipolar dynamos the magnetic paths through the machines are in parallel. In dealing with these parallel magnetic paths, it is only necessary to keep in mind that the same general laws for the flow of electric currents through parallel electric circuits apply equally well to the magnetic flux through parallel magnetic paths. It is particularly important to keep in mind that the same voltage which forces an electric current through one path in a parallel circuit also forces electric currents through all the other parallel branches of the circuit.

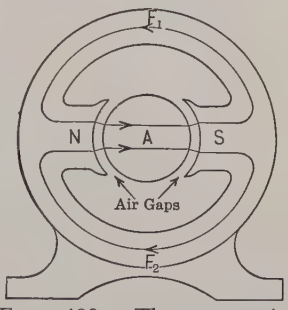


FIG. 130. The magnetic paths through a two-pole generator.

Note how this same law applies to the ampere-turns needed to force magnetic flux through parallel circuits. In Fig. 130 representing the magnetic circuit of a bipolar machine, those parts of the circuit composed of the North pole N, the armature A, the South pole S, and the two air gaps, are in series and carry the whole magnetic flux (disregarding any leakage). As this magnetic flux enters the frame it divides into two paths, F_1 and F_2, which we will assume have the same reluctance, as they will (approximately) in a well-designed machine. Such a circuit is similar to the electric circuit in Fig. 131. Here the lamp, L, carries the full electric current, and corresponds to the series part of the magnetic circuit made up of

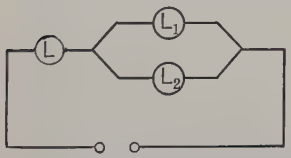


FIG. 131. The electric analogy of the magnetic circuits of Fig. 130.

the two poles, armature and air gaps. Each of the two lamps, L_1 and L_2 (assumed to have the same resistance), carries one-half the current of the lamp L. The voltage necessary to force an electric current through this series-parallel combination equals the sum of the voltage necessary to force it through the lamp L plus the voltage to force half this current through either lamp L_1 or L_2, inasmuch as the same voltage which will force half the current through either L_1 or L_2 will force the other half through the other parallel lamp.

Similarly, the gilberts necessary to force a magnetic flux through the combination in Fig. 130 equal the sum of the gilberts necessary to force the full flux through the two poles, armature and air gaps, plus the gilberts necessary to force half this flux through either F_1 or F_2, inasmuch as the same gilberts which force this half flux through either F_1 or F_2, force the other half of the flux through the other parallel path.

Example 13. In Fig. 131 the resistance of lamp L is 40 ohms, and the resistance of L_1 and L_2 is 100 ohms each. What voltage must be applied to the combination to cause lamp L to carry 0.6 ampere?

Solution.

$$\begin{aligned}
\text{Voltage necessary for } L &= 0.6 \times 40 \\
&= 24.0 \text{ volts.} \\
\text{Voltage necessary for either } L_1 \text{ or } L_2 &= 0.3 \times 100 \\
&= 30 \text{ volts.} \\
\text{Voltage for combination} &= 24 + 30 \\
&= 54 \text{ volts.}
\end{aligned}$$

Example 14. In Fig. 130, the length of cast-steel pole pieces N and S is 10 centimeters each, average cross-section is 100 square centimeters. Average length of magnetic path through annealed sheet steel armature is 7 centimeters, average cross-section is 130 square centimeters. Length of air gaps is 0.30 centimeter each, cross-section is 200 square centimeters. Average length of each path through cast-steel frame is 45 centimeters, area is 52 square centimeters. How many ampere-turns must be put in the field coils to set up a flux of 1,500,000 lines in the armature?

Solution.

Pole pieces, cast steel:

$$\begin{aligned}
B \text{ (gausses)} &= \frac{1{,}500{,}000}{100} \\
&= 15{,}000 \text{ gausses.} \\
H \text{ (from curve)} &= 38 \text{ oersteds.} \\
F \text{ (for two-pole pieces)} &= 38 \times 20 \\
&= 760 \text{ gilberts.}
\end{aligned}$$

Armature, annealed sheet steel:

$$B \text{ (flux density)} = \frac{1{,}500{,}000}{130}$$
$$= 11{,}500 \text{ gausses.}$$
$$H \text{ (from curve)} = 3.5 \text{ oersteds.}$$
$$F = 7 \times 3.5$$
$$= 24.5 \text{ gilberts.}$$

Air gaps:

$$B = \frac{1{,}500{,}000}{200}$$
$$= 7500 \text{ gausses.}$$
$$H = 7500 \text{ oersteds.}$$
$$F \text{ (for two gaps)} = 7500 \times 0.60$$
$$= 4500 \text{ gilberts.}$$

Total for pole pieces, armature and air gaps $= 760 + 24.5 + 4500$
$= 5285$ gilberts
$$= \frac{5285}{1.26} = 4200 \text{ ampere-turns.}$$

Frame, cast steel (one path only need be considered).

$$\text{Flux through one path} = \frac{1{,}500{,}000}{2}$$
$$= 750{,}000 \text{ lines.}$$
$$B \text{ (flux density)} = \frac{750{,}000}{52}$$
$$= 14{,}400 \text{ gausses.}$$
$$H \text{ (from curve)} = 32 \text{ oersteds.}$$
$$F \text{ (for either path)} = 45 \times 32$$
$$= 1440 \text{ gilberts}$$
$$= \frac{1440}{1.26} = 1140 \text{ ampere-turns.}$$
$$\text{Ampere-turns for total circuit} = 4200 + 1140$$
$$= 5340 \text{ ampere-turns.}$$

In some devices, parallel magnetic circuits are employed in which the branches carry different amounts of flux. This would be the same as using two different sizes of lamp L_1 and L_2 in Fig. 131. Exactly the same laws apply to this case as to the one shown above, but it has been found desirable to compute such problems in a slightly different manner. Example 16 in the next section illustrates a case of this type.

Prob. 27–6. The hoisting magnet of Fig. 132 is made of cast steel and has the following dimensions: Length of A, 30 centimeters, cross-section 150 square centimeters; average length of B_1 and B_2, 45 cm. each, cross-section 80 square centimeters; piece C is of wrought iron, total average length of C, 35 centimeters, cross-section, 120 square centimeters; air gaps each 0.09 centimeter. How many ampere-turns must be applied to A to set up a flux of 2,200,000 lines?

Prob. 28–6. If the air gaps in Prob. 27–6 are increased to 0.14 centimeter each, how many ampere-turns will be needed?

Prob. 29–6. If the piece *C*, Fig. 132, is cast iron, with an average length of 35 centimeters, and cross-section of 300 square centimeters, and air gaps are 0.25 centimeter each, how many ampere-turns must be used? Other data in Prob. 27–6.

Prob. 30–6. If the air gaps in Prob. 27–6 are increased to 0.30 centimeter each, and the flux changed to 1,250,000 lines, how many ampere-turns are necessary?

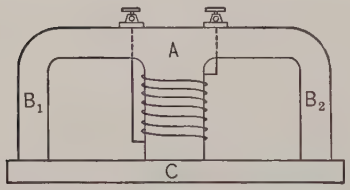


Fig. 132. A three-pole lifting magnet.

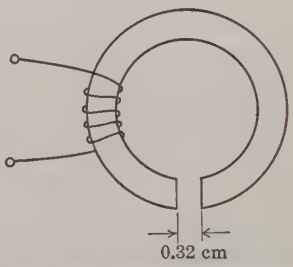


Fig. 133. A ring magnet with an air gap.

93. To compute Total Flux, for Given Conditions. Thus far in problems on magnetic circuits, parts of which contain iron and steel, we have known the flux density to which the materials were magnetized, and have been able to determine the reluctance and the ampere-turns necessary to set up the flux. If, however, we wish to find out how many lines are set up in a certain path of iron or steel by a given number of ampere-turns, we cannot divide the gilberts by the reluctance in order to find the number of lines, because the reluctance itself, as we have seen, depends upon the number of lines per square centimeter. Since we do not know the number of lines per square centimeter, we do not know the reluctance.

It is usual, therefore, in this kind of problem, to make an estimate of the approximate number of gausses, and note from the curves how many gilberts are necessary. From the information obtained by this estimate we can make closer estimates, and obtain more definite information, until we finally estimate very closely the number of lines set up by a given number of gilberts. This method is illustrated in the following examples.

Example 15. How many lines are set up in the wrought-iron ring of Fig. 133, when 3060 ampere-turns are applied to it? The ring has a cross-section of 20 square centimeters and an average length of 50

centimeters. The air gap is 0.32 centimeter long with an average cross-section of 25 square centimeters.

Solution.

Since air has so much greater reluctance than iron, we may suspect that the greater part of the gilberts will be used in sending the magnetic flux across the air gap.

We have a total of $3060 \times 1.26 = 3857$ gilberts available to overcome the total reluctance. Suppose we assume that a certain percentage, say 80%, of the total reluctance is caused by the air gap. Then 80% of the magnetomotive force will be used in the air gap, or

$$F \text{ (air gap)} = 0.8 \times 3857 = 3085 \text{ gilberts}$$
$$H = \frac{3085}{0.32} = 9640 \text{ oersteds}$$
$$B = 9640 \text{ gausses.}$$

The total flux in the air gap is hence

$$\phi = BA = 9640 \times 25 = 241{,}000 \text{ lines}$$
$$\text{For the iron, } B = \frac{241{,}000}{20} = 12{,}050 \text{ gausses.}$$
$$H \text{ (from curve)} = 2.5 \text{ oersteds}$$
$$F \text{ (in iron)} = 2.5 \times 50 = 125 \text{ gilberts.}$$

By this assumption we have used $3085 + 125 = 3210$ gilberts instead of 3857. Evidently we assumed too small a flux since the total F is only $\frac{3210}{3857}$ or 83% of the available number of gilberts. If we increase the total flux by this 17% shortage, the magnetomotive force required will probably be too high because the iron will begin to take a proportionately larger share of the total number of gilberts. Let us try a 15% increase for a second trial.

$$\phi = 241{,}000 \times 1.15 = 277{,}000 \text{ lines}$$
$$B \text{ (air)} = \frac{277{,}000}{25} = 11{,}080 \text{ gausses.}$$
$$B \text{ (iron)} = \frac{277{,}000}{20} = 13{,}850 \text{ gausses.}$$
$$H \text{ (air)} = 11{,}080 \text{ oersteds}$$
$$H \text{ (iron, from curve)} = 5 \text{ oersteds}$$
$$F \text{ (air)} = 11{,}080 \times 0.32 = 3545 \text{ gilberts}$$
$$F \text{ (iron)} = 5 \times 50 = 250 \text{ gilberts}$$
$$\text{Total } F = 3795 \text{ gilberts}$$

This value is now only about 1.6% too low, and it will be sufficiently precise to increase the total flux by this amount as the final result, or

$$\phi = 101.6 \times 277{,}000 = 282{,}000 \text{ lines.}$$

Example 16. The parallel magnetic circuit of Fig. 134 is a soft steel casting. A coil of 450 turns on B carries a current of 1.8 amperes. Compute the flux in each path, given the following dimensions.

	Length of path	*Cross-section area*
A	35 cm.	4 sq. cm.
B	10 cm.	20 sq. cm.
C	30 cm.	12 sq. cm.

Solution. The magnetomotive force supplied by the coil on B is

$$F = 1.26 \times 450 \times 1.8 = 1021 \text{ gilberts}$$

With the direction of current as shown in Fig. 134, the flux passes up through B, divides in the neighborhood of point x (not necessarily an equal division), part passes through path A and the remainder through path C. The two parts of the flux come together again near y and re-enter path B.

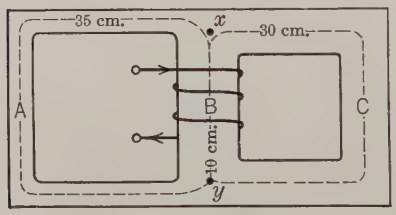


FIG. 134. A parallel magnetic circuit with branches of unequal size.

A certain amount of the magnetomotive force of 1021 gilberts set up by the coil is required to force flux through B, just as a part of the electromotive force of a dry cell is required to send current through its own internal resistance. Therefore, the magnetomotive force between x and y will not be the full 1021 gilberts but will be somewhat less because of the drop in B.

Let us assume as a first approximation that 10% of the F is used in leg B. Then the F across x–y will be

$$F_{xy} = 0.9 \times 1021 = 918.9 \text{ gilberts}$$

This value of F sends flux through both A and C, as was shown in Art. 92, so we can find the fluxes set up in these paths.

$$H_A = \frac{F_{xy}}{\text{length of path } A} = \frac{919}{35} = 26 \text{ oersteds}$$

$$H_C = \frac{F_{xy}}{\text{length of path } C} = \frac{919}{30} = 31 \text{ oersteds}$$

From the curve for soft steel castings,

$$B_A = 13{,}700 \text{ gausses}$$
$$B_C = 14{,}300 \text{ gausses}$$

$$\phi_A = B_A A_A = 13{,}700 \times 4 = 54{,}800 \text{ lines}$$
$$\phi_C = B_C A_C = 14{,}300 \times 12 = 171{,}600 \text{ lines}$$

Total flux $= \phi_B = \phi_A + \phi_C = 226{,}400$ lines

$$B_B = \frac{226{,}400}{20} = 11{,}320 \text{ gausses}$$

From curve, $H_B = 13$ oersteds

$$F_B = 13 \times 10 = 130 \text{ gilberts}$$

We assumed that 10% of the total F, or 102 gilberts, would be required in leg B, but we now find that the remaining 919 gilberts set up enough flux in legs A and C to require 130 gilberts in leg B. The difference between the assumed value of F_B and the computed value is only 28 gilberts. This difference amounts to only $\frac{28}{35}$ oersted in path A and $\frac{28}{30}$ oersted in path C and will produce only a small change in the fluxes through those paths and hence only a small change in the total flux. Hence the drop in path B must be much nearer 130 gilberts than 102 gilberts. Assuming a 130-gilbert drop in path B,

$$F_{xy} = 1021 - 130 = 891 \text{ gilberts}$$

$$H_A = \frac{891}{35} = 25 \text{ oersteds} \qquad B_A = 13{,}600 \text{ gausses.}$$

$$H_C = \frac{891}{30} = 30 \text{ oersteds} \qquad B_C = 14{,}200 \text{ gausses.}$$

$$\begin{aligned} \phi_A &= 13{,}600 \times 4 = 54{,}400 \text{ lines.} \\ \phi_C &= 14{,}200 \times 12 = 170{,}400 \text{ lines.} \\ \phi_B &= \phi_A + \phi_C = 224{,}800 \text{ lines.} \end{aligned}$$

$$B_B = \frac{224{,}800}{20} = 11{,}240 \text{ gausses.}$$

$H_B = 13$ oersteds (as closely as the curve can be read).
$F_B = 10 \times 13 = 130$ gilberts.

Since this calculated value of F_B agrees with the assumed value, the fluxes are approximately

$$\begin{aligned} \phi_A &= 54{,}000 \text{ lines.} \\ \phi_B &= 224{,}000 \text{ lines.} \\ \phi_C &= 170{,}000 \text{ lines.} \end{aligned}$$

We are not justified in giving the results to any greater precision because the B–H curve is not plotted to a large scale. Note that the values of H in all cases are given only to the nearest oersted because fractions of an oersted cannot be estimated accurately in the curves.

Prob. 31–6. An 800-turn coil on the magnet of Fig. 129 carries a current of 2.6 amperes. The U-shaped part is cast steel, 33 centimeters long, and the flat part is cast iron. The air gaps are each 0.05 centimeter long and have 10.5 square centimeters area, and all

other dimensions are as in Example 11. How many lines of flux will be set up?

Prob. 32–6. If a flat piece of wrought iron replaced the cast iron in Prob. 31–6, how many lines would be set up?

Prob. 33–6. An electromagnet is made up of three sections in series as follows: cast iron 26 inches long, 22 square inches cross-section; two air gaps each $\frac{1}{4}$ inch long, 25 square inches cross-section; a section 6 inches long, area 5.5 square inches, made up of fine particles of magnetite iron, the reluctivity of which is $\frac{1}{10}$ that of air. What will be the total flux if the magnet is wound with 4000 turns carrying 1.8 amperes?

Prob. 34–6. The dynamo of Fig. 130 is wound with 2 coils of 2000 turns each, and carrying 1.2 amperes. If the dimensions are as in Example 14 what flux will be produced in the pole pieces?

Prob. 35–6. Branch C of the magnetic circuit of Fig. 134, has an air gap 0.1 centimeter long cut in it. If the effective area of the air path is 13.5 square centimeters and all other data is as in Example 16, find the flux set up in each branch.

94. The Tractive Force of Electromagnets.* When it is desired to compute the attraction or pull which a magnet exerts upon a piece of iron or steel, if substantially all of the magnetomotive force is in the air gap, use is made of the formula

$$f = \frac{8.94\ B^2 A}{10^8},$$

where f = pull in pounds,
B = flux density in air gap in gausses,
A = area of air gap in square centimeters.

Example 17. Compute the pull of the horseshoe magnet on the flat piece in Fig. 135, from the following data. Sufficient current is sent through the coils to produce a flux of 300,000 lines.

Length of U-shaped wrought iron magnet = 48 centimeters.
Length of magnetic path through wrought iron bar = 21 centimeters.
Length of each air gap = 0.15 centimeter.
Number of turns in each coil = 1500.

* For practical data on the construction of electromagnets, see C. R. Underhill's "Solenoids, Electromagnets and Electromotive Windings," D. Van Nostrand Co.; Standard Handbook for Electrical Engineers; American Handbook for Electrical Engineers; Proceedings of American Institute of Electrical Engineers, etc.

Solution.

$$f = \frac{8.94\ B^2 A}{10^8}.$$

$$A = 5 \times 5 \times 0.7854 = 19.6 \text{ square centimeters.}$$

$$B = \frac{300{,}000}{19.6} = 15{,}300 \text{ gausses.}$$

$$f = \frac{8.94 \times 15{,}300^2 \times 19.6}{10^8}$$

$$= 410 \text{ pounds.}$$

Total force for two ends:

$$2 \times 410 = 820 \text{ pounds.}$$

If we can decrease the area of the ends of the magnet without materially decreasing the flux, we can increase the pull of the magnet. This seems unlikely, but it is true, because while we decrease A in the formula we increase B proportionally, and B is squared in the formula.

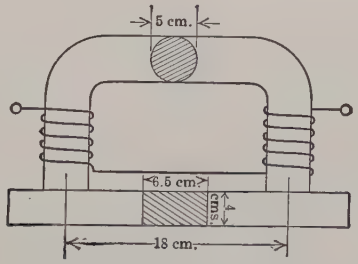


FIG. 135. A lifting magnet.

For instance, let us chamfer 0.30 centimeter off the ends of the magnet in Fig. 135, and adjust the current to keep the total flux constant.

$$\text{The diameter of air gap} = 5 - 0.60 = 4.4 \text{ centimeters.}$$

$$\text{Area of air gap} = 4.4 \times 4.4 \times 0.7854$$

$$= 15.2 \text{ square centimeters.}$$

$$B \text{ in air gap} = \frac{300{,}000}{15.2} = 19{,}750 \text{ gausses.}$$

$$f = \frac{8.94\ B^2 A}{10^8}$$

$$= \frac{8.94 \times 19{,}750^2 \times 15.2}{10^8}$$

$$= 530 \text{ pounds.}$$

$$\text{Total force} = 2 \times 530$$

$$= 1060 \text{ pounds.}$$

Note that decreasing the area of the air gap produced the surprising result of increasing the pull from 820 to 1060 pounds.

A striking example of a useful application of this principle exists in the design of the powerful electromagnets used in hospitals for extracting steel and iron particles from the eye. These magnets are constructed with conical shaped ends in order to provide the maximum flux density and therefore the maximum tractive force.

An undesirable example of this principle is the strong pull of the field on the armature toward one pole if the clearance between the pole and the armature gets decreased through bearing wear or other means. This decreases the fringing and lessens the area of the air gap which has been shortened.

Prob. 36–6. Calculate the tractive force in the magnet of Fig. 135 if 1.25 amperes are sent through the coils.

Prob. 37–6. If the ends of the magnet in Fig. 135 are chamfered 0.2 centimeter all around, what current will be required in the coils in order to produce the same tractive force as in Prob. 36–6?

Prob. 38–6. If the magnet of Fig. 135 is to be used to lift a cast iron piece weighing 300 pounds, and the air gap at each pole is 0.25 centimeter, find the current required in the windings. Assume that the area of the air path is 10% greater than that of the U-shaped steel, and that the ends of the magnet are not chamfered.

95. Hysteresis. In Fig. 127, we saw how the flux density B increases when the magnetizing force H increases, even when the curve was carried to a point considerably beyond saturation. If now we gradually take away the magnetizing force, let us see what happens.

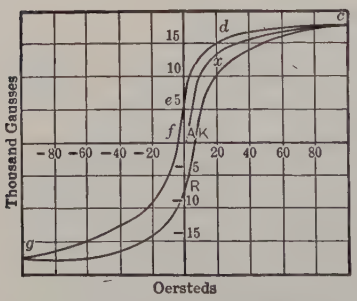


Fig. 136. A hysteresis loop.

In Fig. 136, the line Axc is the magnetization curve of soft cast steel. It is a reproduction of the curve of cast steel in Fig. 125. At point c the flux density is 17,400 gausses, and the magnetizing force 100 oersteds.

When, however, the magnetizing force H is decreased, the flux density B does not decrease along the same line by which it had increased, but follows the curve $cdef$. At the point e, although the magnetizing force H has become zero, there still remains a flux density, B, of 7000 gausses. The flux density, B, therefore, is said to lag behind the force H. This lagging is called **hysteresis,** and is the cause of a certain loss in every alternating-current machine and in

the armatures of direct-current machines. The reason for this is seen if we consider the rest of the curve.

At the point e, as has been said, although H has become zero, the value of B has only decreased to 7000 gausses, represented by the line Ae. This value is called the **remanence.** To get this magnetic remanence out of the iron, it is necessary to set up a magnetizing force, H, in the opposite direction, of 6 oersteds. This force is represented by the line Af and is called the **coercive** force.

If now this magnetizing force in the opposite direction, $-H$, is increased, a flux density, $-B$, will be set up in the opposite direction and a curve of magnetization in this direction can be drawn as fg, until B has as large a negative value at g as it had a positive value at c.

The magnetizing force is again gradually decreased, and again the flux " lags " behind the force, until when H has again become zero, the value of B has decreased to R only. There is thus a remanence of about 7000 gausses, AR, left in this direction, to remove which again requires a coercive force AK.

To bring the flux density up to the value c again, we have to increase this force to its original value for the density at this point. The cycle thus completed is called the " Hysteresis Loop."

It can be shown that the area of this loop represents the work done in overcoming the hysteresis effect on a cubic centimeter of this specimen of iron, just as the area of a steam indicator card represents the work done in overcoming the resistance to motion offered by the piston. Only, in this case, all work done, as represented by the area of the loop, is wasted.

This can be illustrated as follows:

Since the value of B, *i.e.*, the magnetic density of the iron, always lags a little behind the magnetizing force, some extra force must be exerted continually to urge on the change in the magnetization. When the flux density is being increased, the value of H, or magnetizing force, has to be a little greater than would be necessary, were there no hysteresis effect. Similarly, as the iron is being demagnetized, the magnetizing force must be larger in the opposite direction in order to overcome the drag of the magnetic lines which seem to persist in the iron. Thus there is a constant action taking place between the magnetized iron and the magnetic field of the coil.

This is similar to the case of an automobile being towed on a level road with the throttle completely closed, the ignition shut off and the clutch and gears both in, so that the engine is being driven by the

rear wheels. If the pistons and valves allowed no air to leak in or out and there were no friction, it would require no work to turn the engine over. The air would be compressed in the cylinders during the compression stroke and would expand during the working stroke, giving back just as much energy as had been expended in compressing it. But, there is always a slight air leak, and the pressure on the working or expansion stroke would be a little less than at a corresponding point on the compression stroke. As a result, even if there were no friction, not all the energy of compression would be returned on the expansion stroke and the engine would use up energy as the car was being towed, with the clutch and gears in.

The amount of energy lost during each cycle of operation would be the area of a loop, a curve drawn between pressure and piston displacement.

Similarly, the area of the hysteresis loop represents the energy lost during one cycle of magnetization as described above. This is expressed by an equation as follows:

$$W_h = 0.796 \frac{(BH)}{10^8} \text{ watt-seconds per cubic centimeter,}$$

where

(BH) = area of loop plotted with
B = flux density in gausses,
H = magnetizing force in oersteds.

With every cycle (double alternation) of an electric current in a machine, there is always this hysteresis loss in the iron cores. The greater the number of alternations a second, the greater the total loss. For this reason, the iron to be used in Alternating-Current machines, and in armatures in general, has to be selected with proper care as to its hysteresis qualities.

If a piece of annealed steel or annealed sheet-steel is put to this test, the loops plotted will be very narrow and contain little area (Fig. 137), showing the loss to be small.

In the case of steel which has not been so well annealed, the loop will widen out (Fig. 138) and contain a larger area. For this reason, either standard annealed sheet-steel or annealed silicon-steel is used in transformer cores, armatures, etc.

Example 18. Find the hysteresis loss per cubic centimeter, per cycle, in the steel used to obtain loop in Fig. 137. Area of loop equals 10 square inches, and the curve is plotted to a scale of 25 oersteds per inch horizontally and 25,000 gausses per inch vertically.

Solution. On horizontal axis:

1 inch = 25 oersteds; magnetizing force H.

On vertical axis:

1 inch = 25,000 gausses; flux density B.

1 square inch = $25 \times 25{,}000 = 625{,}000$ (BH units).

Area of curve:

10 square inches = $10 \times 625{,}000 = 6{,}250{,}000$ (BH units).

Hysteresis loss:

$$W_h = 0.796 \frac{6{,}250{,}000 \ (BH \text{ units})}{10^8}$$

$$= 0.0498 \text{ watt-second per cc.}$$

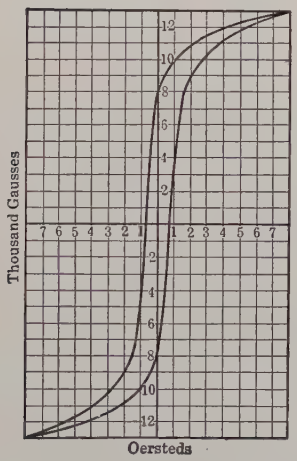


FIG. 137. A hysteresis loop for annealed steel.

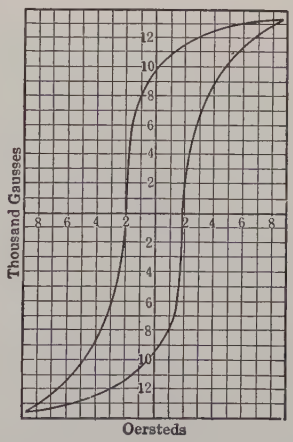


FIG. 138. A hysteresis loop for steel which has not been as well annealed as that of Fig. 137.

Example 19. If 20 pounds of the steel in Example 18 go through the cycle of magnetization 60 times per second, what power will be lost on account of hysteresis?

Solution.

$$20 \text{ pounds} = \frac{20 \times 1728}{460} = 75 \text{ cubic inches.}$$

$$= 75 \times 16.4 = 1230 \text{ cubic centimeters.}$$

From above example:

1 cubic centimeter loses 0.0498 watt-second per cycle.

Then 1230 cubic centimeters would lose:

$$0.0498 \times 60 \times 1230 \text{ watt-seconds in 60 cycles,}$$
$$= 3670 \text{ watt-seconds.}$$

Since this loss occurred in one second, the power lost = 3670 watts.

96. Computation of Hysteresis Loss. Dr. Steinmetz found by a series of tests that, when the variation of flux is sinusoidal, the hysteresis loss can be expressed approximately as a formula, which may be written as:

$$P = \frac{KB_{\text{max.}}^{1.6}}{10^7}$$

in which: P = loss in watts per cubic centimeter of magnetic material for 1 cycle per second,

$B_{\text{max.}}$ = greatest flux density in gausses during cycle,

K = Steinmetz's hysteresis constant, depending upon material used.

Note that this formula can *not* be applied to rotating iron masses where the flux variation is *not* sinusoidal.

For ordinary so-called "annealed electrical sheet-steel," such as is commonly used in the cores of armatures, etc., the constant (K) may be taken as 0.0011. For silicon steel such as is used in transformers, a fair value K is .00065. For its value in connection with other materials, see any electrical handbook.

Example 20. A transformer core of standard annealed sheet-steel weighs 42 pounds and is supplied by a 30-cycle circuit. Maximum flux density = 10,100 gausses. What is the hysteresis loss in the core? Sheet steel weighs 0.283 pound per cubic inch.

Solution.

Loss per cubic centimeter per cycle per second

$$= \frac{0.0011 \times 10{,}100^{1.6}}{10^7}$$
$$= 0.000281 \text{ watt.}$$

Loss per cubic cm. for 30 cycles per second

$$= 0.000281 \text{ watt} \times 30$$
$$= 0.00843 \text{ watt.}$$

$$\text{Core contains } \frac{42}{0.283} = 148.3 \text{ cubic inches.}$$
$$= 148.3 \times 16.4 = 2430 \text{ cubic centimeters.}$$
$$\text{Loss in core} = 2430 \times 0.00843$$
$$= 20.5 \text{ watts.}$$

Prob. 39–6. A transformer core contains 60 pounds of sheet steel (specific gravity 7.8). The magnetic flux, which has a maximum value of 12,000 gausses, goes through 60 cycles per second. What is the loss in kilowatts?

97. Magnetic Qualities of Nickel, Cobalt and Alloys. While iron and steel are usually spoken of as the magnetic materials, it is true that both nickel and cobalt are magnetic to some extent, though far less than iron and steel. Nickel reaches its highest permeability, about 225, at a flux density of about 1600 gausses, and cobalt has its highest permeability, about 195, at approximately 4000 gausses. In order to compare nickel and cobalt with other materials, it is only necessary to remember that non-magnetic materials have a permeability of 1 at all flux densities and that annealed silicon steel has a permeability of about 4000 at 10,000 gausses. Thus, nickel and cobalt at best are about 200 times as permeable as non-magnetic materials, and only about $\frac{1}{20}$ as permeable as commercial silicon steel.

However, when these elements are combined with iron, very interesting and useful alloys are produced. Certain alloys of nickel and iron, known by various trade names such as permalloy,* hypernik, and mumetal, have very high permeabilities. For example, in permalloy a magnetizing force of 0.4 oersted produces a flux density of 3540 gausses, corresponding to a permeability of $\frac{3540}{0.4}$, or 8850. This alloy is composed of about 78.5 per cent nickel and 21.5 per cent iron.

Another group of alloys contain iron, nickel, and cobalt, and are called " perminvar."† These alloys, after proper heat treatment, have unusually constant permeability and comparatively small hysteresis loss.

In contrast with these alloys is the cast iron known as " nomag " and developed by Ferranti, Ltd. of England. This material has a permeability of 1.04 which is practically that of air and other non-magnetic materials.

98. Rational System of Magnetic Units. The system of magnetic units used in this chapter is called the C.G.S. (Centimeter,

* Arnold & Elmen, " Permalloy, a New Magnetic Material of Very High Permeability," Bell System Technical Journal, July, 1923. A. E. Kennelly, " The Reluctivity of the Recently Discovered Magnetic Metal Permalloy," Franklin Institute Journal, February, 1924.

† " Magnetic Properties of Perminvar," G. W. Elmen, Bell System Technical Journal, Jan., 1929.

Gram, Second) system, because these three units are taken as the base units.

Frequently dimensions are given in inches, and it is more convenient then to use what is known as the English Rational System of units.

In this system, the unit of flux is 1 line just as in the C.G.S. system, but the unit of magnetomotive force is 1 ampere-turn, which is 1.26 gilberts. Since we wish to have

$$F = \phi \mathcal{R}$$

in this system, the rational unit of reluctance (no name for this unit) must also be 1.26 times as large as the C.G.S. unit.

In terms of rational units, the reluctivity of air (reluctance of an inch cube) can be found as follows:

C.G.S. reluctance of an inch cube

$$= \frac{\text{length in cm.}}{\text{area in sq. cm.}} = \frac{2.54}{2.54 \times 2.54} = 0.3937$$

$$\text{Rational reluctance} = \frac{\text{C.G.S. reluctance}}{1.26} = \frac{0.3937}{1.26}$$

$$= 0.3125$$

Thus the reluctance of a piece of non-magnetic material in rational units is

$$R = \frac{0.3125\, l}{A}$$

where

l = length in inches

A = area in square inches.

The permeability of air in this system is $\frac{1}{0.3125}$ or 3.20. Similarly, the permeability of all materials is 3.20 times the C.G.S. permeability.

Flux density B is in lines per square inch and magnetizing force H in ampere-turns per inch.

The B–H curves and permeability curves have exactly the same shape and can be supplied with both sets of units on the axes, so that the values can be used directly for computation in both systems.

It should be remembered, however, that any particular calculation must be carried out in one set of units. Thus, if reluctance is computed in rational units, it must be used with magnetomotive force in ampere-turns, never in gilberts.

Example 21. How many ampere-turns are required to send 10,000 lines through a circuit of wood, 50 inches long and 12 square inches in area?

Solution.

The same equation applies as in the C.G.S. system.

$$R = \frac{0.3125\, l}{A}$$
$$= \frac{0.3125 \times 50}{12}$$
$$= 1.302$$
$$F = \Phi\mathfrak{R}$$
$$= 10{,}000 \times 1.302$$
$$= 13{,}020 \text{ ampere-turns.}$$

Prob. 40–6. A cast-iron ring 12 in. in mean diameter and having a cross-section area of 3 sq. in. has an air gap 0.2 in. long. Determine the number of ampere-turns needed to produce a flux of 2500 lines, if the iron is working at a permeability, in rational units, of 600.

Prob. 41–6. How many lines would be produced by the same number of ampere-turns, if the cast-iron ring of Prob. 40–6, were replaced by one of copper?

SUMMARY OF SYSTEMS

		C.G.S. System	Rational System
Magnetomotive force	F	1 gilbert $= \frac{1 \text{ ampere-turn}}{1.257}$	ampere-turn
Flux	Φ	1 maxwell = 1 line of force	line of force
Reluctance	$\mathfrak{R}$	C.G.S. unit = 0.796 rational unit	rational unit
Flux density	B	1 gauss = 1 line per sq. cm. = 6.45 lines per sq. in.	lines per sq. in.
Magnetizing force	H	1 oersted $= \frac{2.54 \text{ amp.-turns per in.}}{1.257}$	ampere-turns per in.
Permeability	μ	lines per sq. cm. set up by 1 oersted = 3.20 × rational value for μ	lines per sq. in. set up by 1 ampere-turn per inch.

To reduce:	gilberts to ampere-turns,	multiply by 0.796;
	C.G.S. $\mathfrak{R}$ to rational $\mathfrak{R}$,	" 0.796;
	gausses to lines per square inch,	" 6.45;
	oersteds to ampere-turns per in.,	" 2.02;
	C.G.S. value of μ to rational value of μ,	" 3.20.
To reduce:	ampere-turns to gilberts,	multiply by 1.257;
	rational $\mathfrak{R}$ to C.G.S. $\mathfrak{R}$,	" 1.257;
	lines per sq. in. to gausses,	" 0.155;
	ampere-turns per in. to oersteds,	" 0.495;
	rational value of μ to C.G.S. value of μ,	" 0.312.

SUMMARY OF CHAPTER VI

THE MAGNETIC CIRCUIT.

1. The magnetic flux follows a complete magnetic circuit just as the electric current follows a complete electric circuit.

2. The magnetic flux is measured in lines of force while the electric current is measured in amperes.

3. The magnetic flux is set up by the magnetic pressure, called the magnetomotive force, which is measured in gilberts. The electric current is caused to flow by the electric pressure, called the electromotive force, which is measured in volts.

4. The magnetic flux is limited by the reluctance of the magnetic circuit. The flow of the electric current is limited by the resistance of the electric circuit.

5. The law of the magnetic circuit is analogous to Ohm's law for the electric circuit.

OHM'S LAW.

Magnetic Flux	Electric Current
Flux equals pressure divided by reluctance.	Current equals pressure divided by resistance.
Lines of force equal gilberts divided by reluctance.	Amperes equal volts divided by ohms.
Gilberts = 1.26 × ampere-turns.	
$\Phi = \frac{F}{\mathcal{R}}$.	$I = \frac{E}{R}$.

The RELUCTANCE of non-magnetic materials is computed by an equation analogous to the equation for the electrical resistance.

Reluctance	Resistance
$\mathcal{R} = \frac{K \times l}{A}$	$R = \frac{K \times l}{A}$
K for non-magnetic materials = 1.	K for copper = 10.4.
l = length in centimeters.	l = length in feet.
A = area in square centimeters.	A = area in circular mils.

Reluctances in series are added just as resistances in series are added.

Reluctances in parallel are treated just as resistances in parallel are treated.

Flux at centre of LONG COIL (air core) $= \frac{1.26\ NIA}{l}$.

The RELUCTANCE OF A PIECE OF IRON OR STEEL is not constant but depends upon the degree of magnetization.

B-H CURVES.

In order to facilitate the computation of magnetic circuits containing iron and steel, curves are usually drawn between a series of values of the magnetizing force (H) in oersteds and the corresponding values of the flux density (B) in gausses (lines per square centimeter). These are called magnetization curves.

TWO STAGES OF MAGNETIZATION. SATURATION POINT

The magnetic curves have the same general shape for all magnetic materials. The magnetization may be divided into two stages. The first stage consists of that part of the curve where the flux density reaches only moderate values and the permeability is comparatively high, relatively only few oersteds being required. This continues until the saturation point is reached, beyond which it is increasingly difficult to add to the flux density. Each small increase of flux density requires a larger number of oersteds. The magnetization

curve, therefore, makes a more or less sharp bend or "knee" at the saturation point.

It is not generally economical to operate machines with a flux density much beyond the saturation point.

SERIES AND PARALLEL MAGNETIC CIRCUITS.

The rules for applying Ohm's law to series and parallel magnetic circuits are the same as for applying Ohm's law to series and parallel electric circuits.

The value of the fraction $\frac{B}{H}$, which is the ratio of the flux density to the magnetizing force, is called the PERMEABILITY. It may be thought of as the number of gausses which one oersted will set up in the material at that degree of magnetization.

COMPUTATION OF FLUX FOR COMPOSITE CIRCUITS.

The flux which a given number of ampere-turns will set up in a composite circuit of magnetic material cannot easily be computed directly. A cut and try method, however, may be used satisfactorily and an approximation made to any desired degree of precision. This may require several trial computations.

TRACTIVE FORCE OF ELECTRIC MAGNETS.

The general equation for the tractive force of magnets may be expressed as:

$$f = \frac{8.94\ B^2 A}{10^8},$$

where f = tractive force in pounds, B = gausses in air gap.
A = area in sq. cm.

HYSTERESIS.

When we remove the magnetizing force from a magnetic material, the flux does not become zero in the material, because there is a certain "lag" in the magnetic flux with regard to the magnetic force. This lagging is called hysteresis.

Since the flux in armature cores and alternating-current appliances must be reversed many times a second, a certain amount of power must always be expended to overcome the effect of this lagging. Soft iron and annealed steel exhibit this property of hysteresis to a very limited degree. It is, therefore, customary to use these materials whenever frequent and large changes are required in magnetic flux. The hysteresis loss in watts in one cubic centimeter of iron or steel in which the flux changes at the rate of one cycle per second can be computed from Steinmetz's formula when the maximum flux does not greatly exceed 10,000 gausses.

Steinmetz's formula for hysteresis loss:

$$P = \frac{K B_{max.}^{1.6}}{10^7}$$

in which P = loss in watts per cubic centimeter of magnetic materials for one cycle per second,

$B_{max.}$ = greatest flux density in gausses during cycle,
K = Steinmetz's hysteresis constant depending upon material used.

MAGNETIC QUALITY OF NICKEL AND COBALT.

Nickel and cobalt are magnetic to a slight extent, having a maximum permeability about two hundred times that of air and about 1/20 that of annealed steel. When alloyed with iron, they produce materials of unusual magnetic properties.

OTHER SYSTEMS OF MAGNETIC UNITS.

There are several systems of magnetic units none of which has been standardized as yet. The system used in this book is called the C.G.S. system. The relative values of units of the English Rational system can be found by means of conversion factors which are found on page 156.

PROBLEMS ON CHAPTER VI

Prob. 42–6. A solenoid 40 inches long and 2.5 inches in diameter is made of 385 turns of No. 14 B. & S. copper wire. If this coil is energized from a 6-volt storage battery what is the total flux through the central cross-section?

Prob. 43–6. Assuming that the flux in Prob. 42 is uniformly distributed over the coil area, what is the magnetizing force in oersteds at the center of the coil, mid-way between the two ends?

Prob. 44–6. The average density of the earth's magnetic field is about 0.6 gauss. What current would be required in the solenoid of Prob. 42 in order to produce an average flux density at the center equal to this value?

Prob. 45–6. In order to raise a certain weight the flux density in a horseshoe-shaped wrought-iron hoisting magnet must be 105,000 lines per square inch; length of iron circuit 20 inches. If there are 1428 turns carrying two amperes, how near to the weight must the magnet be brought to lift it? Neglect the reluctance of the weight.
From Caldwell's " Engineering Problems."

Prob. 46–6. It is desired to design a wrought-iron horseshoe magnet to raise 1000 pounds. The length of magnetic path in the steel is 50 centimeters and there would be 2 air gaps of 0.18 centimeter each. Density in the iron is to be 16,000 gausses; in the air gap 15,000 gausses. On account of its great cross-section, neglect the reluctance of the weight. Find the diameter of the iron core of the magnet and the number of ampere-turns needed.
From Caldwell's " Engineering Problems."

Prob. 47–6. A wrought-iron ring 10 inches in diameter and 4.2 square inches in cross-section is opened by a saw cut 0.5 inch wide. A coil is wound on the ring to magnetize it, and a flux density of 18,000 gausses is set up. What is the force acting to draw the ring together?

Prob. 48–6. How many ampere-turns are required to set up the flux in Prob. 47?

Prob. 49–6. In order to magnetize an iron rod, a magnetomotive force of 400 ampere-turns is necessary. How many volts must be applied to a coil of 196 turns and 48 ohms resistance?

Prob. 50–6. It is desired to magnetize a cast-steel ring 15 inches in diameter to a density of 85,000 lines per square inch. 780 turns of No. 23 B. & S. copper wire are used. Average length of each turn is 11.5 inches. How many volts must be applied to coil in order to set up desired flux density?

Prob. 51–6. How many ampere-turns are necessary to magnetize a 35-centimeter cast-steel bar of 10 square centimeters cross-section so that it has a flux of 125,000 lines? Bar is bent into a ring.

Prob. 52–6. A magnetic circuit is made up of 300 centimeters of wrought iron 50 square centimeters in cross-section; 1.1 centimeters of air, 56 square centimeters in cross-section. Find number of ampere-turns necessary to set up 700,000 lines of force in above circuit.

Prob. 53–6. The magnetic circuit of a generator is made up of: cast-iron yoke, length 12 inches, flux density 40,000 lines per sq. inch; two wrought iron cores each 4.5 inches long, flux density 80,000; two air gaps each 0.08 inch long, flux density 55,000; path through armature of standard annealed sheet-steel 6 inches long, flux density 85,000. Find the number of ampere-turns required in the field coils.

Prob. 54–6. A wrought-iron hoisting magnet of the form shown in Fig. 139 has the following dimensions:

Length of path through U-shaped part	= 36 inches
" " " flat piece	= 15 "
" each air gap	= 0.25 inch
Cross-section of U-shaped part	= 14 square inches
" " flat piece	= 30 " "
" " each air gap	= 16 " "
Number of turns in coils (each)	= 3200 turns
Current in coils = 1.24 amperes	

Find total flux.

Prob. 55–6. What is the force of attraction between the U-shaped part and the flat piece in Prob. 54?

Prob. 56–6. What current must flow in the magnet coils of Prob. 54, if the total force of attraction is to be 1 ton?

Prob. 57–6. A wrought-iron ring 240 centimeters long is to carry a flux of 1,000,000 lines. Flux density is to equal 15,000 gausses. (*a*) What is the cross-section area? (*b*) What is the permeability at this density? (*c*) How many ampere-turns are required? (*d*) What is the reluctance of circuit?

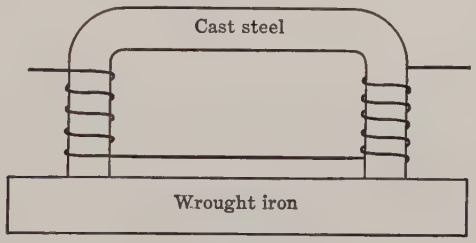


Fig. 139. A lifting magnet.

Prob. 58–6. The magnetic paths in the cast-steel piece of Fig. 140 have the following dimensions:

Path	*Length*	*Cross-section.*
A	30 cm.	50 sq. cm.
B	50 cm.	25 sq. cm.
C	100 cm.	25 sq. cm.

The coil on *A* has 225 turns and carries 6 amperes. What is the total flux in each branch?

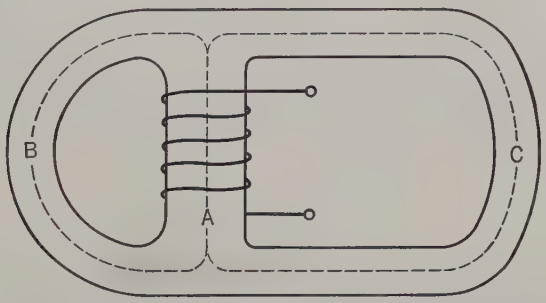


Fig. 140. A parallel magnetic circuit of cast-steel.

Prob. 59–6. Find the total flux in each branch of the circuit of Prob. 58 if branch *B* has an air gap 0.02 cm. long in it. Assume the cross-section of the air gap is 60 sq. cm.

Prob. 60–6. In Fig. 222, the field cores are of wrought iron each 5 inches long and an average cross-section area of 20 square inches. Yoke is of cast steel 12 inches long, 15 square inches cross-section; armature core is of sheet steel, magnetic path 5.5 inches long, with an average cross-section of 11.5 square inches. Two air gaps each 0.25

inch long, of 38 square inches cross-section area. If 1,600,000 lines are desired in each air gap, how many ampere-turns must the field coils produce?

Prob. 61–6. The sheet-steel core of a transformer weighs 50 pounds and has a cross-section area of 11.5 square inches. When excited by a 40-cycle supply the maximum flux in the core is 800,000 lines. What is the hysteresis loss? Steel weighs 0.284 pound per cubic inch.

Prob. 62–6. The silicon-steel core of a transformer weighs 280 pounds. If it is used on an alternating-current circuit of 133 cycles per second, what will be the hysteresis loss? $B_{max.} = 10{,}000$ gausses. Silicon steel weighs 0.284 lb. per cu. in.

Prob. 63–6. What would the hysteresis loss be in Prob. 62–6 if the transformer were used on a 60-cycle circuit?

CHAPTER VII

THE GENERATOR

Thus far we have been studying the laws controlling the action of electric and magnetic pressures and currents. We are now to consider how electric pressures and currents are generated and maintained.

99. The Direct-Current Generator. Practically all modern electric power is produced by generators driven by steam engines or water turbines. A typical direct-current generator is shown in Fig. 141. It is driven by a steam turbine and delivers 1000

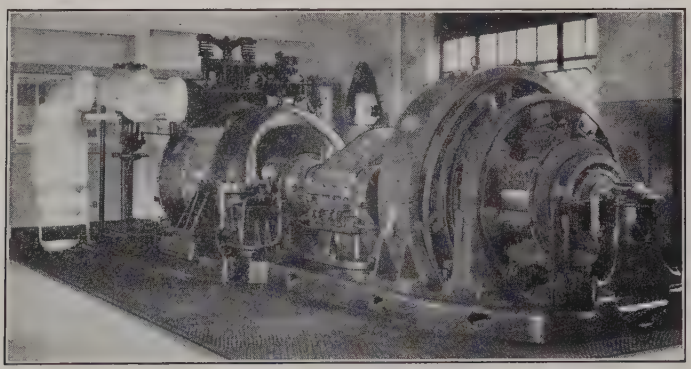


Fig. 141. A 1000-kw. direct-current generator driven by a steam turbine. *General Electric Co.*

kilowatts. In studying the principles of construction and operation of such a machine, we shall need to use all the laws which we have learned concerning both the electric and magnetic circuits. In addition to these we must become familiar with the "Law of Electromagnetic Induction."

100. Electromagnetic Induction. The generator consists fundamentally of a number of loops of insulated wires revolving in a strong magnetic field in such a way that these wires cut across the lines of magnetic force. This cutting of the lines of force sets up an electromotive force along the wires.

We have shown that whenever there is an electric current present there is also present a magnetic field. It is not true that wherever a magnetic field exists, there also exists an electric current, in the ordinary sense; but we can say that **wherever a conductor moves in a magnetic field in such a way as to cut lines of force, an electromotive force is set up.** It is on this principle that the electric generator works.

101. The Direction of Induced Electromotive Force. In constructing an electric generator we must know the laws concerning the direction and the amount of the e.m.f. induced in a wire cutting magnetic lines. The direction can best be determined from the study of a diagram like Fig. 142.

If the wire AB is moved **down** across the lines of force as marked, an electric pressure will be set up in the wire from A to B. If the

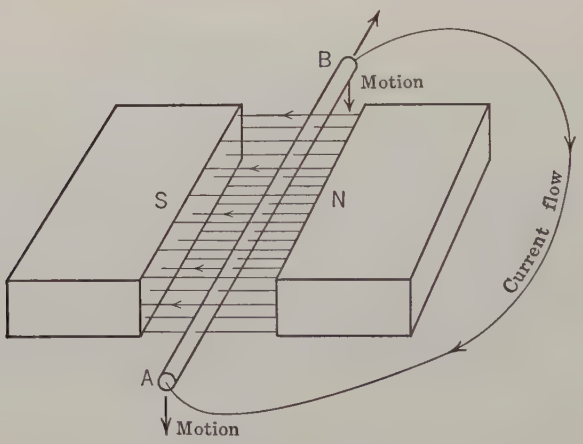


FIG. 142. An e.m.f. is induced in the wire from A to B when the wire cuts down across the magnetic flux.

wire were moved **up** across the lines, the pressure would be set up in the opposite direction, from B to A. If the direction of the magnetic field were reversed, the pressure would then be set up in the reverse directions for the same motions of the conductor.

Thus the direction of the induced voltage is seen to depend upon the direction of the flux and the direction of the motion of the conductor.

The rule for finding the direction of the induced electromotive force is as follows.

Extend the THUMB, FOREFINGER and MIDDLE FINGER of the RIGHT hand at right angles to one another. Let the THUMB point in the direction of the motion, the FOREFINGER in the direction of the magnetic flux, then the MIDDLE FINGER will be pointing in the direction of the induced electromotive force.

As an aid to the memory, note that the initial letters of Forefinger and Flux are the same, and those of Middle Finger and (electro) Motive Force are alike.

Figure 143 brings out this relation clearly.

102. Amount of Induced E.m.f. The amount of the induced electromotive force depends upon the number of lines cut per second. It has been found that if a conductor cuts 100,000,000 lines per second, a pressure of 1 volt is produced between the terminals. This number 100,000,000 is usually written 10^8, and we read it "ten-to-the-eighth."

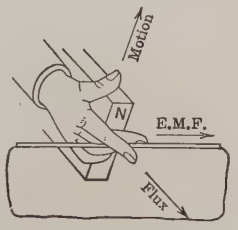


FIG. 143. The right-hand rule.

Example 1. A wire passes 40 times a second across the pole face of a field magnet the flux density of which is 15,000 lines per square inch. The dimensions of the face are 30 × 20 inches. What average e.m.f. is induced in the wire?

Solution.

$$\text{Total flux} = 30 \times 20 \times 15{,}000 = 9{,}000{,}000 = 9 \times 10^6 \text{ lines}$$

$$\text{Lines cut per second} = 9 \times 10^6 \times 40 = 360 \times 10^6 = 3.6 \times 10^8$$

$$\text{Volts} = \frac{3.6 \times 10^8}{10^8} = 3.6 \text{ volts.}$$

Prob. 1–7. How many volts are induced in a wire which cuts 6×10^7 lines in 0.24 second?

Prob. 2–7. A wire is cutting through a field of 4,500,000 lines at an average rate of 3600 times per minute. What voltage (average) is induced in the wire?

Prob. 3–7. A wire 1 meter long is moving at a velocity of 10 meters per second at right angles to the earth's magnetic field. If the flux density is 0.6 gauss, what voltage is induced in the wire?

Prob. 4–7. If a wire which is cutting through a magnetic field at the rate of 60 miles per hour has an induced voltage of 1.8 volts across its terminals, how fast must it move to set up 4.2 volts?

103. Current in Revolving Loop. The e.m.f. of a generator is the voltage induced in loops of wires revolving in such a way as to

cut the lines of force of a powerful magnetic field. Let us consider the case of a single loop of wire so revolved.

In Fig. 144 and 145, the loop of wire is revolving as shown by the arrow.

In Fig. 144, the side of the loop BD is moving **down** across the lines. By applying the right-hand rule, the induced e.m.f. is found to be in the direction of D to B as marked. The side AC is moving

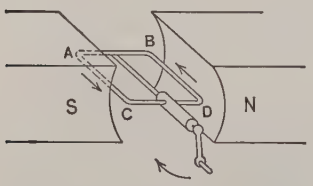


Fig. 144. With the revolving loop in this position a maximum e.m.f. is induced in the direction $DBAC$.

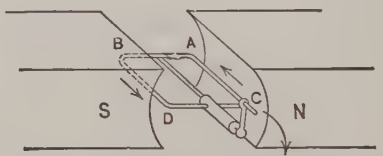


Fig. 145. With the revolving loop in this position a maximum e.m.f. is induced in the direction $CABD$.

up and the induced e.m.f. is from A to C as marked. Thus a current will flow around the loop in the direction $ACDB$, as marked.

In Fig. 145, the wire BD has now turned to the opposite side and is moving **up** instead of **down.** The induced e.m.f. has now reversed and is from B to D. For the same reason, the e.m.f. in AC is reversed and is from C to A. This now causes a current to flow around the coil in the direction $BDCA$, which is the reverse of the direction of the current when the coil was in the other position.

Thus the current in a revolving coil flows one way during half the revolution and in the reverse direction during the other half.

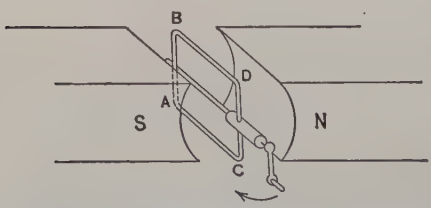


Fig. 146. No voltage is induced when the loop is in this position.

There is then an alternating current in a closed loop of wire revolved in a magnetic field.

The electromotive force will be at its maximum value in **one direction** when the coil is in the position of Fig. 144, and at its maximum value in the **opposite direction** when in the position of Fig. 145, because at these positions the sides of the coils are cutting the greatest number of lines per second. When the loop is vertical, as in Fig. 146, the sides of the

loop are moving **parallel** to the magnetic lines and are **not cutting them.** There is then no e.m.f. induced in the loop. It is then said to be in the " zero position."

104. Curve of Electromotive Force. In Fig. 146, the coil is at the " zero position." The horizontal position of Fig. 144 would be

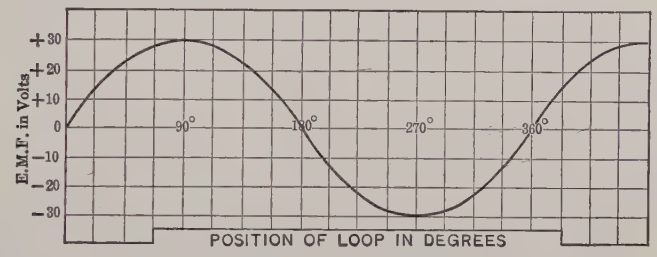


FIG. 147. A sine curve of induced e.m.f.

90° from the zero position. When the coil has become vertical again, it would be again in a zero position, at 180° from the first zero position. When again horizontal, as in Fig. 145, it would be 270° from the first zero position.

We may plot a curve using the **position** (in degrees from the zero position) as the abscissae and the **induced e.m.f.** as the ordinates.

When the e.m.f. is in one direction, we call it positive (+), and in the other, negative (−).

We shall obtain a curve which may have a shape something like that of Fig. 147, if the field is uniform. Or the curve may have more the shape of Fig. 148 if the armature has an iron core and the pole faces are so shaped that the magnetic lines " fringe out " very little. A curve like Fig. 149 would result if the poles were narrow and there was a considerable " fringe " to the magnetic field.

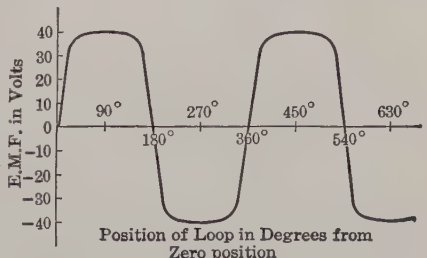


FIG. 148. A flat top wave of induced e.m.f.

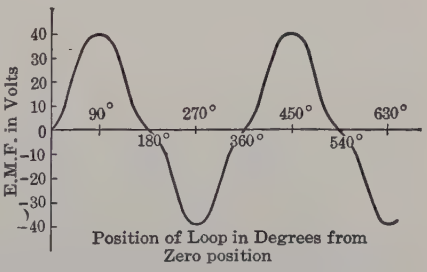


FIG. 149. A peaked wave of induced e.m.f.

These curves all show that the voltage (induced e.m.f.) rises rapidly to a maximum at the 90° position (horizontal) where the lines are being cut at the greatest rate. It falls again to zero at the 180° position (vertical) when the sides of the loop are moving parallel to the lines and are thus not cutting them. Then the sides begin cutting in the opposite direction and a voltage is induced in the opposite direction (which we have agreed to call " negative "). The voltage in this direction increases rapidly to a maximum at the 270° position (horizontal) where the lines are again being cut at the greatest rate (but in the opposite direction). Again it decreases as the coil begins to approach the zero (vertical) position where the sides again move parallel to the lines, and thus again at this point no voltage is induced. This " cycle " of events, as it is called, takes place every complete revolution in a two-pole machine, the maximum voltage reached in each direction depending upon the number of lines cut per second, at the instant the sides are cutting at the greatest rate.

In order to get a high maximum voltage in a generator:

First. The inside of the coil is filled with an iron core to increase and concentrate the magnetic flux.

Second. A great many turns of wire are wound in series on this core, so that the voltage across the ends at any time is the sum of the voltages in each turn.

The turns of wire and the core, together with the device for taking out current from the turns, form the **armature** of most direct-current and many alternating-current generators.

The current **within** the armature of a direct as well as of an alternating-current generator is always an alternating current, and goes through much the same " cycle " of values described by the curve in Fig. 147.*

Whether a machine delivers an alternating or a direct current, then, must depend upon the device used for taking off the current from the armature. This device may consist of either collecting rings, which deliver an alternating current, or a commutator, which delivers a direct current.

105. Collecting Rings. If we have the outside circuit **continually connected** to the two ends of the revolving coils, we must get the same kind of voltage across the outside circuit that we have across the inside. That is, we would get an alternating current

* The one exception to this statement is the unipolar generator, which as yet has come into very little use.

from the machine, since, as we have seen, the **voltage inside** of the coil **is always alternating.** This voltage is brought out by connecting the ends of the wire of the coils to rings, called **collecting rings.** Brushes bearing on these rings will lead the alternating voltage and current to any desired outside circuit.

Figure 150 shows the principle of collecting rings. C_1, C_2 represent the sides of a coil of wire in the armature; R_1 and R_2, the collecting rings joined one to each end of the coil; B_1 and B_2, the brushes; and L, a bank of lamps as outside circuit.

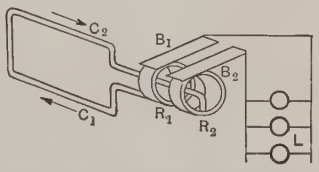


FIG. 150. The collecting rings deliver an alternating current to the brushes.

Since the lamps always form a continuous circuit with the coil, they will always have the same voltage drop that the coil has. The curves of Fig. 147, 148 or 149 will then be a fair representation of the e.m.f. across the lamps as well as across the armature.

106. The Commutator. If we can connect each brush first to one end and then to the other end of the revolving coil, making the change just at the instant the voltage and current in each side are reversing, we shall then have the voltage and current on the outside always in the same direction. This is accomplished by means of a **commutator.** Figure 151 shows the principle of the commutator. The segments S_1 and S_2 are attached each to an end of the revolving coil C_1, C_2. The brushes B_1 and B_2 collect the voltage and current and deliver them to the lamp bank L.

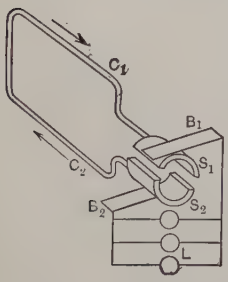


FIG. 151. The commutator delivers a direct current to the brushes.

Assume that, at the instant chosen, the current in C_1 is coming to the brush B_1. The current in the side C_2 must be going in the opposite direction or away from the brush B_2. B_1 is then $+$ and B_2 is $-$. The brush B_1 bears on the segment S_1 as long as the current in C_1 is in this direction. As soon as the current in C_1 changes to the opposite direction, the segment S_1 has left the brush B_1 and the segment S_2 has come into contact with B_1. But just as the current in C_1 has reversed, so the current in C_2 has also reversed, and the current now flows to segment S_2. S_2 now delivers current to brush B_1 which keeps it still the positive brush. Thus B_1 is always positive. In the same way it is seen that B_2 will

always be the negative brush. The current delivered to the lamps then will always flow in the same direction.

A single turn of wire, of course, does not deliver a steady current, even with the commutator, although the current is always in the same direction. The brushes are so placed that the change from one

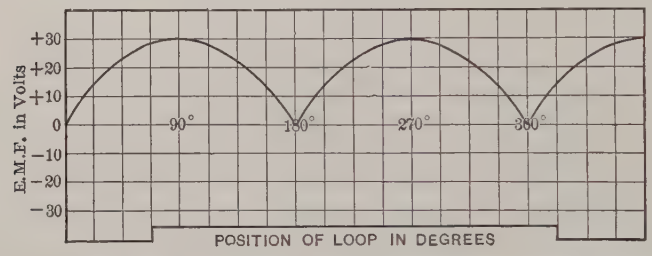


Fig. 152. The e.m.f. wave form of a single-coil direct-current generator.

segment to another is made when there is no current flowing in the coil. At this moment, also, there would be no current flowing in the lamps. So we would have a Pulsating Current in the lamps of practically the same shape as the voltage curve of Fig. 152. The current on the inside of the coil is represented by a curve similar to that of Fig. 147. The values of the voltages on curve in Fig. 152

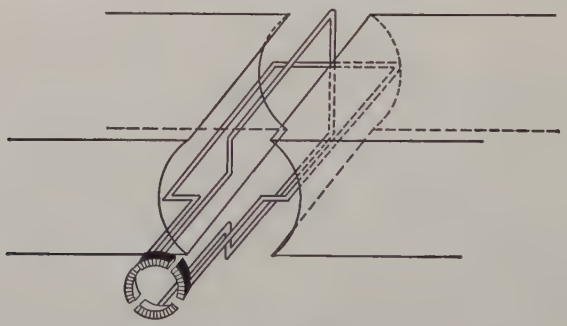


Fig. 153. A two-coil direct-current armature and poles.

are the same as those on curve in Fig. 147, the voltage across the lamps being zero when the voltage across the coil is zero, and greatest when greatest across the coil. The sign, however, for the voltage across the lamps is always positive (+), since the current is always in the same direction. Curve 152 then is Curve 147, with the negative loops turned into positive ones.

To obtain a practically steady direct current, a large number of

coils is placed around the core, and the commutator divided up into a correspondingly large number of segments. The turns of wire of the different coils are distributed around the circumference so that at as many instants as possible some of them are cutting lines at the maximum rate. For instance, if we have just two coils, as in Fig. 153, placed at an angle of 90° to each other with the ends joined to a commutator with four segments, the voltage across one coil

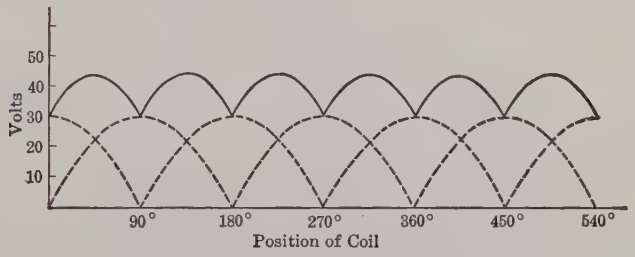


FIG. 154. The solid line is the wave form of a two-coil direct-current armature.

would always be at a maximum when the voltage across the other coil was zero. The curves of voltage in these coils would then be like those of Fig. 154. Note that there is no instant at which the resultant pressure is less than 30 volts. If we now place the brushes on the commutator correctly, we can obtain an e.m.f. between the brushes which will always be between 30 and 42 volts. The brush voltage would then be indicated by the heavy wavy line in Fig. 154.

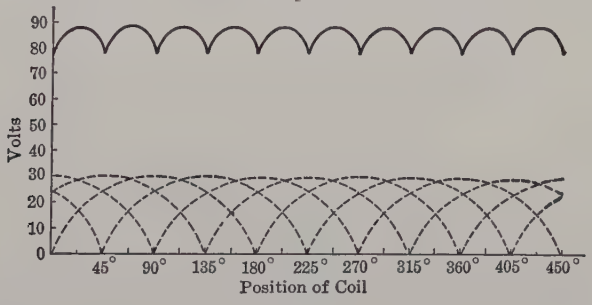


FIG. 155. The wave form of a four-coil direct-current armature.

This voltage, it is seen, is much more steady than that of a single coil. By adding two more coils so that all were equally spaced around the core, we could obtain a voltage still more nearly constant as shown by the heavy line in Fig. 155. The generation of a practically constant voltage is merely a matter of adding coils enough

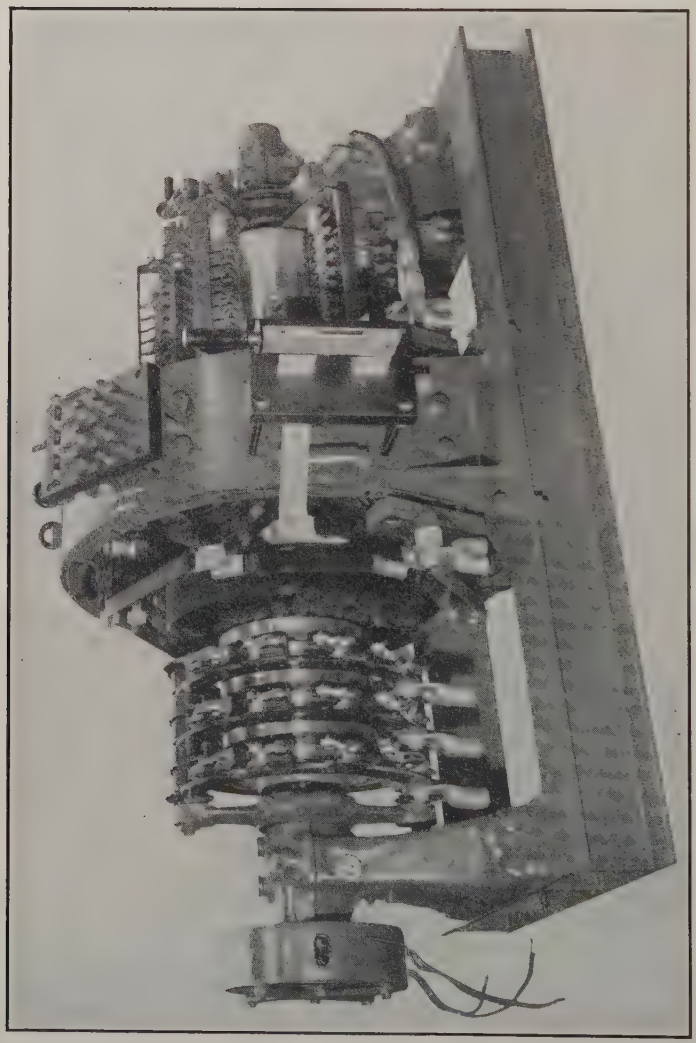


FIG. 156. A double-current generator. *General Electric Co.*

to the armature and making the corresponding additions to the number of commutator segments.*

107. Armature Same for Direct as for Alternating Current. It is seen from the above that an alternating electromotive force is set up in the coils of every armature. Whether the machine delivers direct current or alternating current depends entirely upon whether the armature is fitted with a commutator or with collecting rings. In fact the same armature may be fitted with both collecting rings and a commutator, and thus deliver both alternating and direct currents. Figure 156 shows such an arrangement. On the left-hand side of the machine are seen the collecting rings. Brushes placed on these rings will have an alternating e.m.f. across them when the armature is revolved in the field. On the right is a commutator with a large number of segments. Brushes placed on the commutator at proper positions will have a direct e.m.f. across them, and since there is a large number of coils in the armature, the e.m.f. will be steady.

108. Armature Cores. We have seen that in a modern generator the electromotive force is produced in coils revolving in the space between the poles of electromagnets. In order that there may be as many magnetic lines as possible in this space, the armature coils are wound on laminated annealed steel cores. These cores fill up practically all the space between the poles of the machine, leaving only small air gaps between the poles and the armature. Thus the reluctance of the armature path is made small and a high flux density is maintained in the air gaps.

Note that in Fig. 157 the magnetic lines have paths which are made up entirely of steel except for the small air gaps.

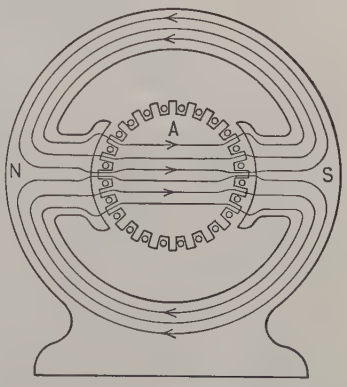


FIG. 157. The magnetic paths in a slotted-core armature.

Many types of armature cores have been developed but at present practically the only one in commercial use is the drum type such as is illustrated in Fig. 157. This core is made up of thin annealed steel punchings about 0.014 inch thick. These punchings are stacked to form a cylinder and keyed to a shaft. The coils are

* For the practical shaping and connection of these coils see "Industrial Electricity" by W. H. Timbie.

formed by winding insulated wire in the slots on the outside of the core. Figure 158 represents one of these annealed steel discs or punchings. Figure 159 shows the armature core formed by such discs with some of the armature coils in place.

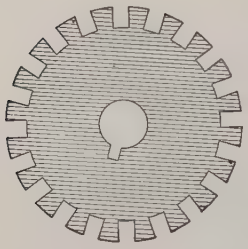


Fig. 158. A soft steel punching for a laminated core.

While, as we have said, the ring-type of armature is not in general use, the actions within the core and winding of an armature can be illustrated best by diagrams of a ring-wound armature. We will, therefore, use the ring armature with its clearer diagrams to explain the fundamental principles underlying the generation of e.m.f. in armature coils. It must be remembered, however, that the same principles apply equally well to drum armatures, and that the ring armatures are used only for the sake of clearness, not because they are in commercial use.

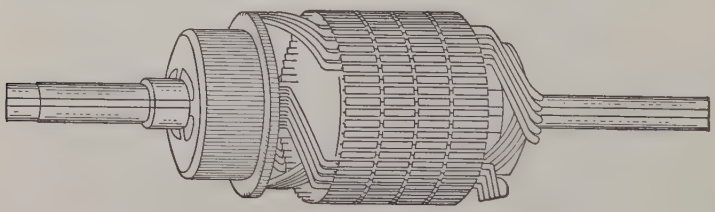


Fig. 159. A partly-wound drum armature.

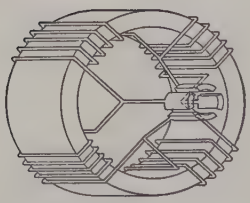


Fig. 160. A ring-wound armature.

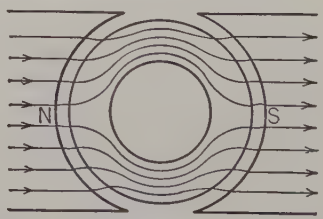


Fig. 161. The magnetic field is parallel to the axis of the poles when no current is being delivered by the armature.

The ring core is similar to the drum except that it is hollow and the wires do not lie entirely on the outside of the cylinder but are wound in and out. Figure 160 shows the method of winding a ring armature.

109. Armature Reaction. Cross Magnetization. When a ring armature is placed in position between the poles of a generator, the

magnetic lines flow in paths illustrated by Fig. 161. Suppose now we had six coils wound on the core of Fig. 161, and the core is rotating in a clockwise direction as marked in Fig. 162. The conductors on the outside of the core would be moving **down** across the south pole and a voltage would be induced which, according to the right-hand rule, would tend to set up current in the coils as marked. The three coils on the right-hand side of the armature would, therefore, deliver current to the lower brush.

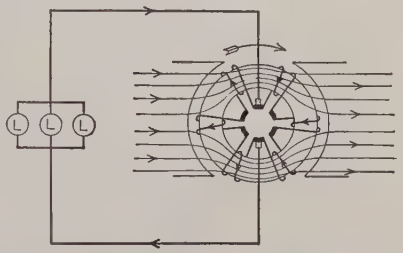


FIG. 162. A six-coil ring-wound armature delivering no current.

The conductors on the other side of the armature would be moving **up** across the north pole and would set up an e.m.f. as marked on the coils, according to the right-hand rule, and would tend likewise to deliver current to the lower brush. If the brushes are connected as in Fig. 162 to an outside circuit, the current will flow as marked. We would, therefore, have a wound steel ring with currents flowing in the windings. These currents would set up a magnetic field of their own within the steel ring in accordance with the principles which we have learned in the previous chapter. This magnetic flux would enter the top of the ring and come out at the bottom as in Fig. 163. We thus have a steel ring with magnetic lines flowing from top to bottom placed in a field in which the lines flow from left to right as in Fig. 164. This magnetic field set up in the armature core is called the **cross magnetization** of the armature, because it is at right angles to the magnetic field between the poles. The effect of these two fields at right angles is to distort the field between the poles, causing the lines to crowd toward the upper tip of the north pole and down toward the lower tip of the south pole. Of course at no load there would be no current flowing in the armature coils, and there would be no cross magnetization and the magnetic field would

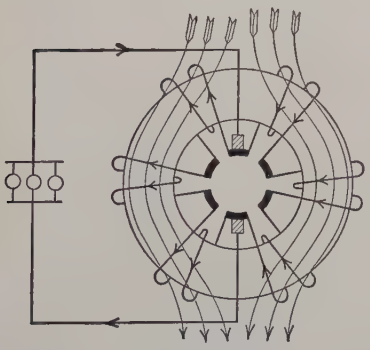


FIG. 163. The magnetic field set up by the current in a six-coil armature.

not be distorted. The field is distorted only when there is a current flowing in the armature coils.

Figures 165 and 166 show the result of this cross magnetization when a current is flowing in the armature coils. Line *ab* is perpendicular to the field when no distortion takes place. Line *cd* is perpendicular to the resulting field when the cross magnetization of the current has distorted the field. The angle *l* represents the amount of distortion.

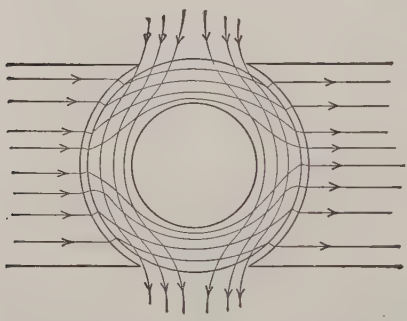


FIG. 164. The combination of the field set up by field coils and that set up by the armature windings.

110. Brush Setting for Sparkless Commutation. The result of this field distortion when a current flows in the armature coils is apparent when it comes to setting the brushes on the commutator. The brushes must be placed in such a position on the commutator that the least possible sparking exists between the commutator segments and the brushes. Sparking at brushes pits the copper of the commutator segments and roughens it so that the brushes jump and make very uneven contact,

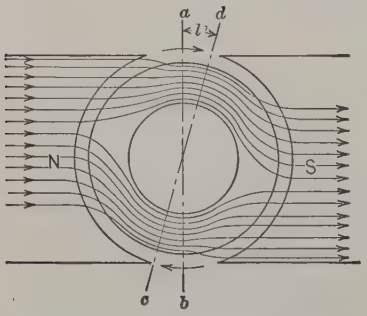


FIG. 165. The magnetic field is shifted *l*° from the zero axis.

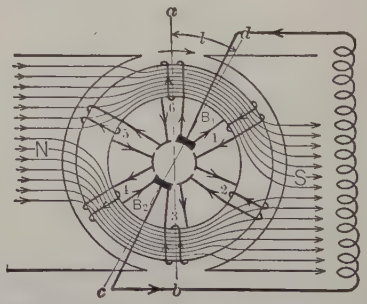


FIG. 166. The brushes are shifted to the axis *cd*, *l*° ahead of the zero axis.

causing still further sparking and more rapid deterioration of the commutator.

As a general principle it may be said that the brushes are set on a line practically at right angles with the magnetic lines. Thus with no current flowing in the armature coils the proper position of the

brushes would be on the line *ab*, Fig. 165 and 166. But when a current is flowing in the armature and the magnetic field has shifted, the brushes have to be placed on the line *cd* in order to have them on a line perpendicular to the direction of the field.

Formerly it was necessary actually to shift the brushes more and more forward in the direction of the rotation of the armature as more and more current was taken from the brushes. In modern machines, however, provision is made, principally by supplying commutating poles, by which this shifting is unnecessary, the same result being produced in other ways. The action of these commutating poles is explained on page 256.

In order to comprehend this action, it is necessary to have a clear understanding of the magnetic and electrical action which takes place in the armature, when no commutating poles are used, and then note how the commutating poles correct any bad effects of these actions.

111. Generated Voltage of a D-C Generator. Since the average voltage generated in an armature conductor is always one-hundred-millionth of the magnetic lines cut per second, we have merely to know the number of conductors in one path between the brushes, the number of magnetic flux lines per pole, and the speed of rotation of the armature in order to determine the voltage.

In Fig. 166, each of the six armature coils has a voltage wave induced similar to that of Fig. 147, but spaced like the waves in Fig. 155. Since each coil cuts the same number of magnetic lines in one revolution, however, the **average** voltage in each coil is identical. Note in Fig. 166 that coils 1, 2 and 3 are in series between the brushes but that this series combination is in **parallel** with the series combination of coils 4, 5 and 6. Thus the armature winding is divided into two parallel paths, each path containing three coils in series. Since the voltage across a parallel circuit is the same as the voltage across any branch, the voltage between brushes B_1 and B_2 is the voltage either of coils 1, 2 and 3 or of coils 4, 5 and 6.

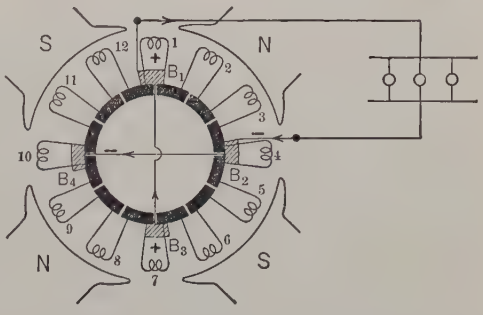


FIG. 167. A twelve-coil armature.

The number of conductors per path depends upon the total num-

ber of active conductors and the number of paths into which they are divided. Thus in Fig. 167, the conductors are divided into 4 paths by the 4 brushes.*

Example 2. In Fig. 166 there are 10 turns in each coil, and 5,000,000 magnetic lines per pole. The armature makes 2400 revolutions per minute. What will be the average e.m.f. generated between brushes?

Solution.

Total conductors in one path between brushes

$$= 10 \times 3 = 30.$$

Total flux cut by each conductor each revolution

$$= 5{,}000{,}000 \times 2 = 10{,}000{,}000 \text{ lines.}$$

$$\text{Number of revolutions per second} = \frac{2400}{60} = 40 \text{ r.p.s.}$$

$$\text{Total flux cut per second} = 30 \times 10{,}000{,}000 \times 40.$$

$$\text{Voltage generated between brushes} = \frac{30 \times 10{,}000{,}000 \times 40}{10^8}$$

$$= 120 \text{ volts.}$$

Prob. 5–7. The armature of a certain 6-pole d-c generator has 240 active conductors arranged in 6 parallel paths. If the flux per pole is 1,800,000 lines, what e.m.f. is generated when the speed is 1200 r.p.m.?

Prob. 6–7. What e.m.f. is induced in an 8-pole, 8-path generator having 1200 active conductors? Speed = 940 r.p.m.; flux = 4,200,000 lines per pole.

Prob. 7–7. If the armature of the generator in Prob. 5 were replaced by one having 1600 active conductors, at what speed would the machine have to run in order to induce twice the e.m.f.?

112. Multipolar Generators. Armature Voltage, Current and Resistance Relations. Large generators are usually of the multipolar type, because they can be made of lighter weight for the same output as a bipolar, and can be operated at a lower speed for the same voltage.

Voltage. The voltage of a multipolar generator is the voltage across one armature path from brush to brush. This was explained in Art. 111.

Current. Since all the armature paths are in parallel and the currents in all paths are equal, the current delivered by a multipolar generator must be the current in each path multiplied by the number of paths.

* For methods for determining the number of armature paths see Chapter X on Armature Windings in "Industrial Electricity" by W. H. Timbie.

Resistance. The armature resistance of a multipolar generator is the resistance of a number of parallel paths all of equal resistance, and must therefore be equal to the resistance of one path divided by the number of paths.

It is essential that all coils in the armature have exactly the same resistance so that the resistance of each path will be the same and thus allow each path to carry the same armature current. This avoids unequal heating in the armature and unequal armature reaction and mechanical stress. It is also essential that the coils be placed symmetrically around the armature core in order that the voltage induced in all paths may be the same.

Example 3. If the current in each path of the armature in Fig. 167 is 16 amperes, what current does the armature deliver?

Solution.

There are four armature paths in Fig. 167.

Armature current delivered $= 4 \times 16 = 64$ amperes.

Example 4. If the resistance of one armature path of Fig. 167 is 0.22 ohm, what is the armature resistance?

Solution.

Armature resistance = resistance of 4 paths in parallel

$$= \frac{0.22}{4} = 0.055 \text{ ohm.}$$

Prob. 8–7. In Fig. 167, each pole carries a flux of 3,800,000 lines. There are 5 turns per coil and the armature rotates at 1800 r.p.m. What is the voltage between brushes?

Prob. 9–7. If the armature of Prob. 8 is to deliver 60 amperes, what current must flow in each armature path?

Prob. 10–7. If the resistance of the armature in Prob. 9 is 0.028 ohm, what is the resistance of each path?

Prob. 11–7. The armature of a 4-pole machine is wound with 52 coils, each composed of 6.5 feet of No. 12 copper wire. There are 4 brushes and 4 parallel paths. Assuming that each brush short circuits one coil, what is the armature resistance?

Prob. 12–7. An 8-pole generator with 8 paths through the armature has 1,750,000 lines per pole and 464 active conductors. What e.m.f. is produced when the speed is 1080 r.p.m.?

SUMMARY OF CHAPTER VII

ELECTROMAGNETIC INDUCTION. When an electric conductor cuts magnetic lines of force, an E.M.F. is set up in the conductor, proportional to the rate of cutting.

DIRECTION OF INDUCED E.M.F. (RULE.) If the Thumb, Forefinger, and Middle Finger of the Right Hand are held at right angles to one another, with the Thumb extended in the direction of the motion of the Conductor, the Forefinger in direction of the Flux, then the Middle Finger will indicate the direction of the Induced E.M.F.

AMOUNT OF E.M.F. When 10^8 magnetic lines are cut per second, one volt of E.M.F. is induced.

MODERN GENERATORS are, fundamentally, loops of wire revolved so as to cut through a strong magnetic field. An Alternating E.M.F. is induced in each loop, the curve of which approximates a Sine Curve.

If COLLECTING RINGS are attached to ends of these coils, an alternating current is delivered by the Generator.

If a COMMUTATOR is used a direct current is delivered. Since the armature coils are each connected across a gap between two commutator segments, each brush must continually short circuit one coil.

ANGLE OF LEAD. AXIS OF COMMUTATION. The brushes must be so placed that the coils short circuited by them are not cutting lines of force at the instant of the short circuit. This position, owing to the cross magnetization due to the current in the armature coils, is on a line ahead of the neutral axis by the angle of lead. This position is called the AXIS OF COMMUTATION.

THE NEUTRAL AXIS may be defined as a line which bisects the angle between the axes of the poles.

THE E.M.F. INDUCED IN A D-C ARMATURE equals $\frac{1}{10^8}$ of the number of lines cut per second by the conductors in series between any two adjacent brushes; that is, in any single armature path.

THE ARMATURE RESISTANCE, from plus terminal to negative terminal, equals the resistance of each path divided by the number of paths.

PROBLEMS ON CHAPTER VII

Prob. 13–7. A 4-pole generator has 1,500,000 flux lines per pole and rotates at 1800 r.p.m. The armature is to be wound with 4 paths and is to generate 216 volts. How many conductors are required?

Prob. 14–7. If the armature of Prob. 13 were used in a 6-pole, 6-path machine having the same flux per pole and operated at the same speed, what voltage would be generated?

Prob. 15–7. A 4-pole, 4-path generator has 3,500,000 lines per pole. The armature is drum wound with 256 active conductors, and runs at a speed of 1400 r.p.m. What is the no-load voltage?

Prob. 16–7. The armature of Prob. 15–7 is wound with 2300 feet of No. 10 B. & S. copper wire. What is the armature resistance?

Prob. 17–7. The armature of a bipolar generator has 400 active conductors. There are 6,000,000 lines of flux per pole. At what speed must the armature revolve to induce 115 volts?

Prob. 18–7. The armature of a 6-pole generator has 1140 active conductors. The speed is 860 r.p.m. Area of each pole face is 80 square inches. Flux density in air gap = 60,000 lines per square inch. Number of armature paths, 6. What voltage is induced?

Prob. 19–7. The field of a 2-pole generator has 6,000,000 lines. In each armature path there are 50 active conductors in series. If these conductors cut the field 2400 times per minute, how much voltage is induced in each armature path?

Prob. 20–7. A 12-pole, 12-path generator has an e.m.f. of 620 volts when running at 120 r.p.m. If the armature is wound with a total of 3600 active conductors, what is the flux per pole?

Prob. 21–7. The drum armature of a bipolar generator is wound with 500 active turns of No. 8 B. & S. copper wire. Each turn is 3.5 feet long. What is the armature resistance?

Prob. 22–7. Allowing 1000 amperes per square inch of cross-section of conductor, how many amperes should generator of Prob. 21–7 deliver?

Prob. 23–7. Assuming that the 500 turns on the armature of Prob. 21–7 are active, what e.m.f. must be generated to maintain a brush potential of 110 volts at full load? Full-load current as found in Prob. 22–7.

Prob. 24–7. What must be the number of lines per pole in Prob. 23–7? R.p.m. = 1170.

Prob. 25–7. The armature of an 8-pole, 8-path generator has 944 active conductors and rotates at 240 r.p.m. The no-load voltage is 115 volts. If the area of each pole face is 136 square inches, what is the flux density?

Prob. 26–7. When generator in Prob. 25–7 is delivering 2000 amperes, the brush potential is 105 volts. If the armature resistance is 0.004 ohm, what must the flux density be at this load? The speed is unchanged.

Prob. 27–7. The resistance of the armature of a 10-pole, 10-path generator is 0.005 ohm. Speed = 232 r.p.m. When the generator delivers a current of 1200 amperes at 115 volts the flux per pole is 5,800,000 lines. How many active conductors are there in the armature?

Prob. 28–7. In designing a generator to have a no-load voltage of 240 volts, a drum armature is wound with 800 conductors. Two brushes are used which reduce the number of active conductors to 780 and there are two parallel paths. The flux per pole is 3,500,000 lines. How many poles must generator have in order to run as nearly as possible at a speed of 480 r.p.m.?

Prob. 29–7. What would the flux per pole in Prob. 28–7 have to be to produce exactly the 240 volts desired?

CHAPTER VIII

THE OPERATION OF DIRECT-CURRENT GENERATORS

Direct-current generators are of several types which differ widely as to the class of service which they are designed to deliver. In order to be able to select the generator best suited to meet particular requirements it is necessary to be familiar with the operating characteristics of each type.

113. Field Excitation of a D-C Generator. Direct-current generators and motors can be classified by means of the method used to excite the field magnets. There are two general types, named according to the method of excitation: (*a*) separately-excited; (*b*) self-excited.

In a separately excited machine, the current for the field coils is taken from some source outside the machine. This outside source may be a storage battery or another generator.

In a self-excited machine, the current for the field coils is drawn from the armature of the machine itself. Most direct-current generators are self-excited. But since the behavior of the separately excited machine is more easily understood, this type will be studied first.

114. Separately Excited Generators. Low-voltage generators (50 volts or under) are usually separately excited. They are in

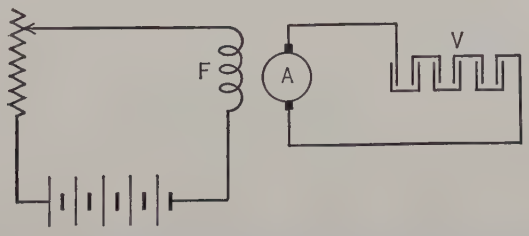


Fig. 168. A separately excited generator feeding a series of electroplating vats.

general use to operate electroplating vats and to charge storage batteries. They have the advantage of easy voltage control, both as to amount and polarity, *i.e.*, as to which brush is maintained

positive and which negative. Self-excited machines sometimes reverse their polarity on stopping and starting. Figure 168 shows a generator with the fields separately excited from a storage battery B. The current from the armature of the generator is supplying electroplating vats (V).

115. External Characteristics of Separately Excited Generator. That which we most desire to know about a generator is what change takes place in the terminal voltage of the machine when the load or armature current is changed. The relation between the terminal voltage and the load current is called the **external characteristic** of a generator.

In Fig. 169, the curve A is the external characteristic of a certain low-voltage separately excited generator. Note that when no cur-

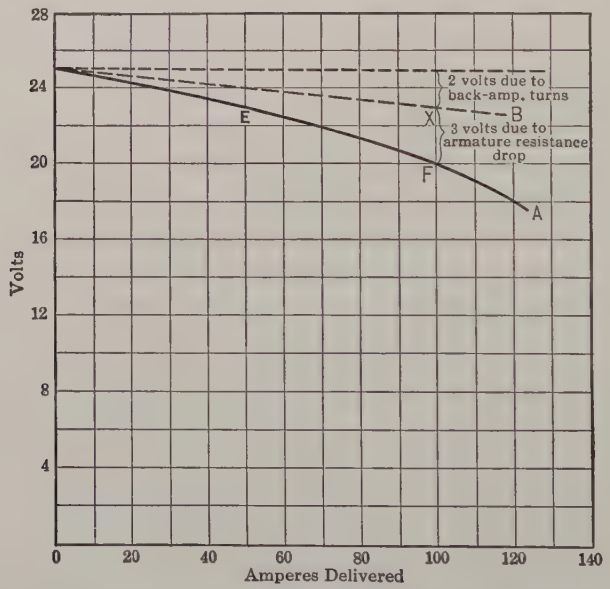


FIG. 169. The curve A shows the relation between the load and the terminal voltage of a separately excited generator.

rent is being taken from the generator, the terminal voltage is 25 volts, which in this case represents the e.m.f. of the generator. As more and more current is delivered, the voltage drops until it becomes 20 volts when delivering 100 amperes.

A generator whose terminal voltage drops as the armature current is increased is said to have a **drooping characteristic.**

The droop in the curve as the current delivered increases is

caused by two things: first, the increase in the voltage used up in overcoming the internal resistance of the armature; second, the back magnetization, or demagnetization of the armature, caused by shifting of the brushes from the zero position.

If the only cause of the droop were the internal resistance of the armature, the full-load voltage would be 22 volts instead of 20 volts.

Assume the resistance of the armature to be 0.030 ohm. When no current is being delivered, no voltage is used up in overcoming the internal resistance. The terminal voltage is then the e.m.f. of the generator, or 25 volts. When full load (100 amperes) is being delivered, 100×0.030 or 3.0 volts of the 25 volts will have to be used up in forcing this current through the armature. There will be left for the terminal voltage at full load only $25 - 3.0$ or 22.0 volts.

But the terminal voltage at 100-ampere load has actually dropped to 20 volts on curve of Fig. 169 instead of to 22 volts only, showing that some other action is also causing the terminal voltage to fall off as the current is increased. This action, we have said, is the demagnetization by the current in the armature windings, and is explained as follows.

116. Demagnetization of Armature. Back Ampere-Turns. If Fig. 170 represents the ring-wound armature of a generator, it

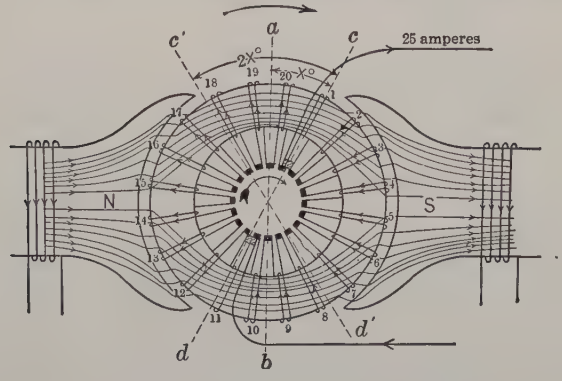


FIG. 170. The neutral axis has shifted $X°$ forward.

will be noted that each armature path contains 10 armature coils, one of which is always short-circuited by a brush. The zero position of the brushes, that is, the position when no current is being delivered, is on the axis *ab*. When a full-load current of 100 amperes is being delivered, the brushes are shifted forward to the neutral po-

sition on the axis *cd* in order that the coil which the brush must short-circuit shall be one which is not cutting lines of force. We remember that this brush shift is necessary on account of the current in the armature coils setting up a cross magnetic field which shifted the main field into the position shown in Fig. 166.

But this shifting of the brushes produces a secondary effect in the armature magnetization. Note in Fig. 170 that the current in coils 18, 19 and 20 and coils 10, 9 and 8 now tends to set up a field directly opposite to the main field. This causes, not a shift, but an actual weakening of the field. And since the voltage of the generator is caused by the armature winding cutting the lines of force, a decrease in the voltage must necessarily take place if the number of these lines is diminished.

In order to find the effect of these " back-armature turns " we merely multiply the number of turns in the double angle of brush shift (2 *X* in Fig. 170) by the current in these turns. This gives the **ampere-turns** of demagnetizing force of the armature. Subtract this value from the ampere-turns in the field windings and the result is the resultant ampere-turns tending to send flux through the fields. The effect on the voltage may be determined from the magnetization curve commonly called the " **saturation curve** " of the machine. This curve is obtained by running the machine unloaded at constant rated speed and measuring the terminal voltage for different values of the ampere-turns in the field coils.

We have assumed that the machine we are considering is a low-voltage generator with a full-load current of 100 amperes. The magnetization, or saturation, curve is shown in Fig. 171.

When the field current is made great enough to produce 1500 ampere-turns in the field coils, the no-load terminal voltage becomes 25 volts.

When the machine is delivering 100 amperes, 50 amperes must flow through each conductor in the armature (since there are two paths in a 2-brush machine). There are 3 coils of 2 turns each, or 6 back turns on the armature with the brushes in the position indicated in Fig. 170, so there must be 6×50 or 300 demagnetizing ampere-turns in the armature at full load.* This would reduce the magnetizing force on the field from 1500 to 1200 ampere-turns, which in turn would decrease the e.m.f. of the machine to 23 volts as seen

* Note that the demagnetizing pressure of coils 8, 9 and 10 is not subtracted also, because the magnetic pressure of these coils is in parallel with the pressure of coils 18, 19 and 20.

from Fig. 171. This generator therefore generates 23 volts only at full load instead of 25 volts as at no load.

At half load of 50 amperes the demagnetizing action of the "back" turns would be $25 \times 6 = 150$ and the total magnetizing effect on the field $1500 - 150 = 1350$ ampere-turns. This would produce 24 volts according to Fig. 171. Thus we might draw a curve between the actual generated voltage and the load on the generator. This curve would be like curve B in Fig. 169.

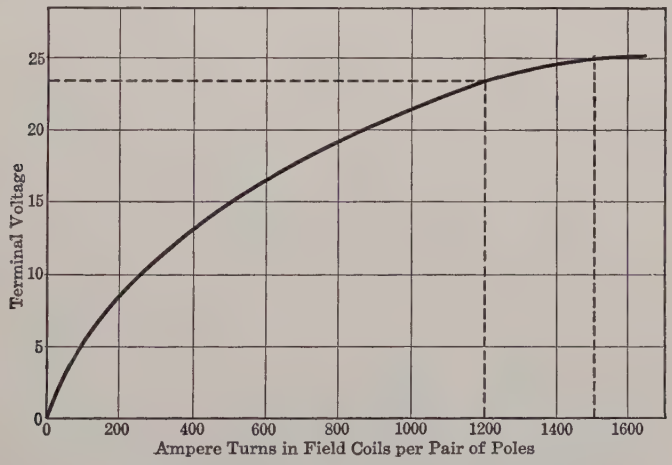


FIG. 171. The magnetization curve of a separately excited generator.

The curve A of terminal voltage is now obtained by subtracting the armature IR drop from this generated voltage curve. Thus the point F of 20 volts is obtained by subtracting $100 \times 0.030 = 3.0$ volts from the value of point X, or 23 volts.

We have seen that for half load the generated voltage is 24 volts. The armature IR drop would be $50 \times 0.03 = 1.50$ volts. The terminal voltage for half load would be $24 - 1.5 = 22.5$ volts, as seen from point E on curve A. Curve A is the **external characteristic** curve of the generator.

Thus to find the external characteristic curve of a separately excited generator without commutating poles

First: From saturation curve and back ampere-turns for various loads, construct curve between generated voltages and load currents.

Second: Subtract the respective values of IR drop from the generated voltage curve for the different load currents.

Note that the demagnetizing ampere-turns were caused entirely by the shifting of the brushes. Most modern machines of 110 volts or more are supplied with commutating poles, as explained in the next chapter, which make it unnecessary to shift the brushes from the zero axis. The direct-current generators and motors of 110 volts or over, described in the rest of this text, will be assumed to have commutating poles, and the brushes accordingly will be placed on the zero axis.

Figure 223 shows the appearance of a dynamo equipped with commutating poles. Figure 224 gives the wiring diagram of a machine equipped with commutating poles.

The external characteristic of a separately excited generator equipped with commutating poles is practically a straight line.

117. Regulation and Control. Another way of stating the characteristic of a generator is to give its "**voltage regulation.**"

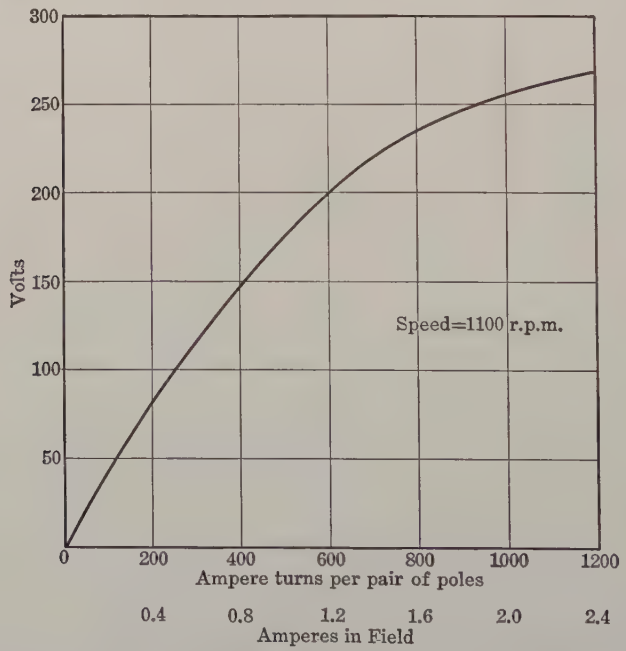


FIG. 172. The magnetization curve of a separately excited generator.

By **regulation** is always meant some change that automatically takes place when the load is changed. Unless it is otherwise stated, the change in the load is assumed to be between no load and full load. Thus the voltage regulation of the above generator would be

stated as 5 volts, since the voltage changed from 25 volts to 20 volts as the load increased from zero to 100 amperes (full load).

It is usual to express the voltage regulation as the percentage which this voltage change is of the voltage at full load. We would therefore say that the above generator has $\frac{5}{20}$ or 25 per cent regulation. The smaller the regulation, the better the separately excited generator is for most purposes.

By **control** is meant some change that is made to take place in the characteristic by the manipulation of auxiliary apparatus. For instance, as will be seen in the next section, an attendant may increase the field current of a generator by cutting some external resistance out of the field, and thus raise the terminal voltage.

Regulation refers to some change that is inherent in the design of the machine and takes place when the load is changed. **Control** refers to some change that can be produced by manipulating auxiliary apparatus whether the load is changed or not.

Example 1. The saturation curve of a 6-pole, 6-path generator running at 1100 r.p.m. is shown in Fig. 172. With no commutating poles on the machine, the brushes are given a 5° lead (forward shift). The armature has 360 active conductors and a resistance of 0.18 ohm.* The full-load current is 60 amperes. With a field excitation of 700 ampere-turns per pair of poles, what will be the regulation of the machine when operated as a separately excited generator?

Solution.

Voltage drop through armature $= 0.18 \times 60 = 10.80$ volts.

Back turns in armature $= \frac{5 \times 2}{360} \times 360 = 10$ turns per pair of poles.

Back ampere-turns per pair of poles in armature at full load $= 10 \times \frac{60}{6} = 100$ ampere-turns.

* In all problems in this text the term armature resistance includes the resistance of the armature windings and of the brush contact. The resistance of the windings is constant but the resistance of the brush contact decreases in almost direct proportion to the increase in current. This causes the voltage drop between each brush and the commutator to remain practically constant from $\frac{1}{2}$ load up to heavy overloads. For carbon brushes of medium hardness this drop is 1 volt per brush, or 2 volts per machine.

The American Institute of Electrical Engineers advises that an allowance be made for 2 volts drop for the brush contact resistance of direct-current machines with ordinary carbon brushes, and that the resistance of the armature be considered that of the windings only. However it usually causes very slight error to consider the resistance of windings and contact as constant, as will be done in this text.

No load voltage (700 ampere-turns in field) shown by curve in Fig. 172, 220 volts.
At full load resultant field ampere-turns = 700 − 100 = 600 ampere-turns.
Volts induced by 600 ampere-turns (Fig. 172) = 200 volts.
Volts used in armature = 10.8.
Terminal voltage at full load = 200 − 10.8 = 189.2 volts.

$$\text{Regulation} = \frac{220 - 189.2}{189.2} = 16.3\%.$$

This regulation is not excessive, as the external characteristic of a separately excited generator has a decided droop.

Prob. 1–8. What will be the terminal voltage of the generator of above example when delivering 40 amperes?

Prob. 2–8. (*a*) What will be the voltage regulation of the generator in above example if operated at a field excitation of 980 ampere-turns per pair of poles?

(*b*) At 490 ampere-turns per pair of poles?

(*c*) What conclusions can you draw concerning the effect upon regulation of operating a separately excited generator with low field excitation?

Prob. 3–8. Redraw the saturation curve of the generator in Example 1 with the generator running at $\frac{2}{3}$ of the speed of the example. (*a*) Calculate the full-load regulation at this speed. Field excitation, 700 ampere-turns per pair of poles.

(*b*) What conclusions do you draw about the effect of reduced speed upon the regulation of a separately excited generator?

Prob. 4–8. Compute the voltage regulation of the generator in Example 1, supplied with commutating poles, and with brushes on the zero axis.

Prob. 5–8. The following data are given for a generator.

No. of poles = 4.
No. of brushes = 4.
Brush lead = 3.8°.
No. of active conductors = 320.
Turns in field (per pair of poles) = 520
Full-load current = 22 amperes.
Armature resistance (including brush contact) = 0.63 ohm.
Field resistance = 110 ohms.
Speed = 1150 r.p.m.

Ampere-turns per pair of poles	No-load voltage	Ampere-turns per pair of poles	No-load voltage
0	0	500	253
100	62	600	280
200	118	700	297
300	172	800	309
400	217	900	317

Construct the external characteristic of this generator when operated separately excited. Voltage across the field coils = 110 volts.

Prob. 6–8. Construct the external characteristic of the generator in Prob. 5, when supplied with commutating poles. Brushes are set on the zero axis.

118. Voltage Control of Separately Excited Generators. The voltage of a separately excited generator can be controlled by inserting a rheostat in series with the field coils as shown in Fig. 168. When it is desired to lower the voltage, more resistance is cut into the field circuit which decreases the field current. This causes a decrease in the magnetic lines in the air gap and a corresponding decrease in voltage. By cutting resistance out from the field circuit more current enters the field coils, the magnetic lines are increased in number and the voltage raised.

Example 2. How much resistance must be introduced into the field circuit of Example 1 to produce a no-load voltage of 200 volts? The voltage applied to the field is to be 110 volts.

Solution.

In the previous example the field had a current of 1.4 amperes which induced a no-load voltage of 220 volts.

In order to induce a no-load voltage of 200 volts there is required, according to Fig. 172, a field current of 1.2 amperes.

To produce 1.2 amperes the total field resistance must be

$$\frac{110}{1.2} = 91.6 \text{ ohms.}$$

$$\text{Resistance of field in example} = \frac{110}{1.4} = 78.5 \text{ ohms.}$$

$$\text{Added resistance required} = 91.6 - 78.5 = 13.1 \text{ ohms.}$$

Prob. 7–8. How much resistance of a field rheostat must be introduced into the field of the generator in Prob. 5–8 to produce a terminal voltage of 120 volts when the machine is delivering 20 amperes? The voltage applied to the field to be the same as in Prob. 5–8.

Prob. 8–8. What voltage must be applied to the fields of Prob. 6–8 to produce a terminal voltage of 239 volts at a load of 20 amperes and a speed of 1000 r.p.m.?

Prob. 9–8. At what speed must the separately excited generator of Prob. 6–8 run in order to produce the same full-load voltage as in Prob. 6–8, when 27 ohms resistance have been inserted in the field?

119. Self-Excited Generators. When the current for the field coils is drawn from the armature of a generator itself, the generator is said to be self-excited. There are three distinct methods of taking the current from the armature.

First. Series wound. Figure 173 shows the method of exciting the field coils by leading all the current from the armature through the field coils. Note that the field coils are thus placed in series with the line supplied by the generator. Thus the same current that goes through the main line also goes through the field coils.

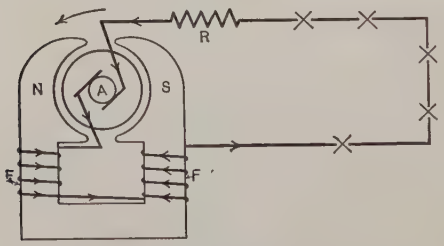


FIG. 173. A series generator feeding five series arc lamps.

Second. Shunt wound. Figure 174 shows the method of connecting the field coils in shunt with the main circuit. Note that when the current leaves the positive brush it divides into two parts, one part of the current going through the load supplied by the machine, the other part going through the field coils. Thus the field coils are in parallel with the main circuit, and are said to be shunted across the line.

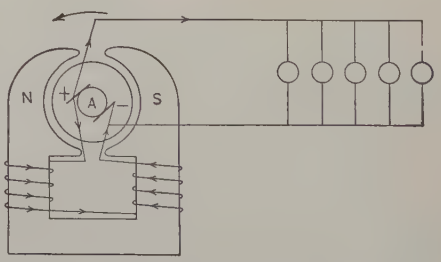


FIG. 174. A shunt generator feeding five lamps in parallel.

Third. Compound wound. The compound method of exciting the field coils is a combination of the shunt and series methods. Notice in Fig. 175 that there are two sets of field coils. One set, the shunt coils labeled B, is shunted across the line; the other set, the series coils labeled C, is connected in series with the line. These three types of self-excited generators have distinct characteristics and therefore have particular

uses. In all of these figures, the commutating poles are omitted for the sake of simplicity.

120. The Characteristic of a Series Generator. When a series generator is not delivering current, there is no current in the field coils. It might be supposed that, if the field coils were producing no magnetic flux, there would be no magnetic flux in the air gap for the armature to cut. But the magnetic poles and frame of the generator are made of steel or some other magnetic material. It will be remembered that even after the magnetizing force has been removed from a steel electromagnet, some lines of force, called the residual magnet-

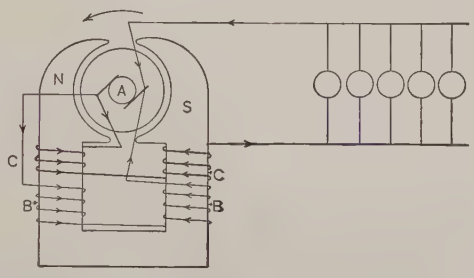


FIG. 175. A compound generator feeding five parallel lamps.

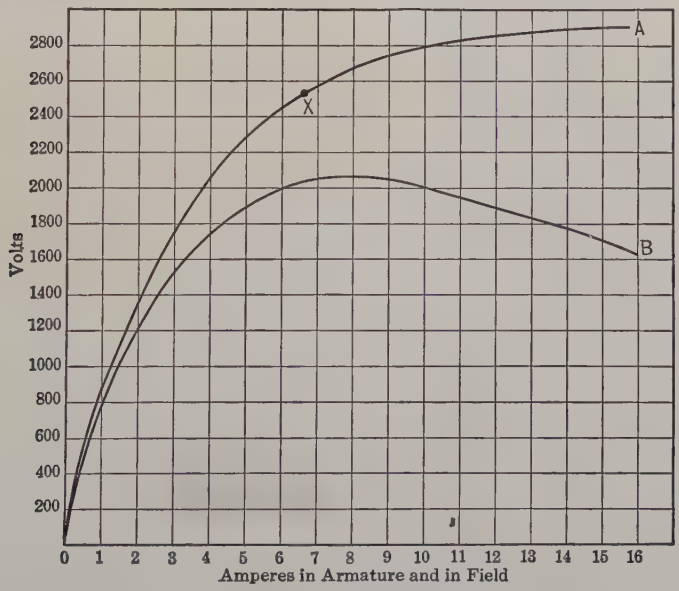


FIG. 176. The curve *B* is the relation between load and terminal voltage of a series generator.

ism, remain in the magnetic circuit. Thus when the armature is revolved, there are a few magnetic lines to be cut, and a small voltage is set up. In Fig. 176, curve *A* is the generated voltage, and

curve B, the characteristic curve of a series generator of 80 ohms armature resistance and equipped with commutating poles. Note that when no current is being supplied by the machine, there is still a generated and a terminal voltage of nearly 50 volts, due to the armature cutting the lines of force of the residual magnetism.

If, now, the terminals are connected to an external circuit containing an adjustable resistance and the resistance is set so that one ampere flows through the external circuit, the generated voltage rises to 880 volts, and the terminal voltage to about 800 volts. Since one ampere is flowing in the external circuit, there is also one ampere flowing in the field coils. This current in the field coils greatly increases the number of magnetic lines in the air gap and thus causes the large increase in generated voltage. The terminal voltage equals the generated voltage, 880, minus the voltage drop, 80, required to send the one ampere through 80 ohms of the armature resistance.

If the resistance of the external circuit is lowered enough to allow 2 amperes to flow through the external circuit and consequently through the field coils, the generated voltage rises to 1360, due to the increase in the number of magnetic lines, and the terminal voltage rises to 1200 volts. As more and more resistance is cut out of the external circuit and thus more and more current is allowed to flow through the external circuit and through the field coils, the generated voltage continues to rise as shown by curve A. The terminal voltage also rises until it reaches about 2060 volts with 8 amperes flowing through the circuit. This process is called the " building up " of the generator. The building-up process cannot go on indefinitely because as the saturation point is reached, it becomes more and more difficult to add lines of force to the magnetic field. Note that after the point X on the curve, Fig. 176, has been reached, a large increase in current is required for a small increase in terminal voltage. In fact, if more than 8 amperes are taken from the machine, the terminal voltage drops, although the generated voltage continues to rise slightly.

The general shape of the external characteristic of a series generator is therefore a curve similar to curve B in Fig. 176. If the generator is not supplied with commutating poles the demagnetizing action of the armature current will cause the terminal voltage curve to drop still farther from the generated voltage curve.

Prob. 10–8. Figure 177 is the magnetization curve of a series generator. Resistance of armature = 14 ohms; of series field =

8 ohms. Turns in series field per pair of poles = 300. Speed = 1600 r.p.m. Construct the external characteristic of this generator, carrying the value of the armature current up to 7 amperes. The generator is supplied with commutating poles.

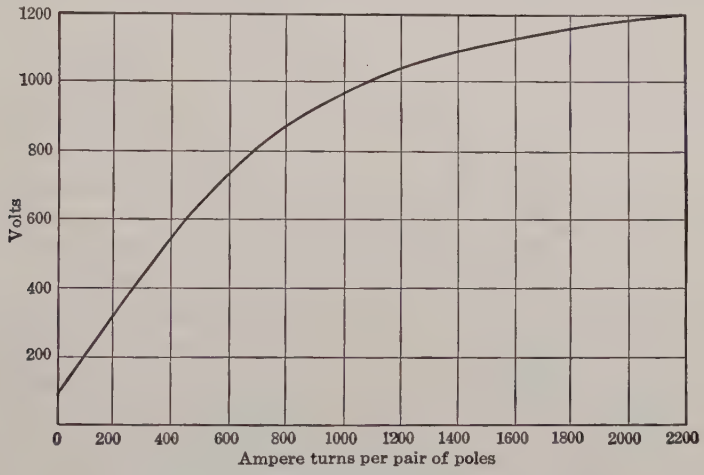


FIG. 177. The magnetization curve of a generator.

Prob. 11–8. Determine the terminal voltage of series generator of Prob. 10 for a load current of 5 amperes if the speed is decreased to 1200 r.p.m.

Prob. 12–8. The commutating pole windings of the generator in Prob. 10 are disconnected and it is found necessary to advance the brushes 9° ahead of the zero axis. There are two brushes and 2000 effective armature conductors. Plot the external characteristic of the generator under these conditions.

Prob. 13–8. If it is desired to raise the terminal voltage of the generator in Prob. 12 to a value of 940 volts when delivering 4 amperes, how much must the angle of brush lead be decreased?

121. Use of Series Generators. Series generators at present have but a limited use in the industries. They were formerly used in connection with street lighting by arc lamps connected in series. The arc lamp has been displaced to a large extent by series tungsten lamps, and series generators are still used in some places to furnish the current for these systems.

In order to operate properly, arc lamps and series tungsten lamps require a constant current; that is, the current must not appreciably rise or fall in any lamp, and since the lamps are all in series, the current through the line must be constant. For this reason, some controlling device must be connected to the line which will

correct any change which may occur in the line and maintain a constant current.

Because these series generators are used in connection with a line which demands a constant current, they are commonly called "**constant-current generators.**" It is very important to keep in mind that these machines are called constant-current generators not because they automatically tend to deliver a constant current, but because they are used in connection with lines in which the current must be kept constant. They do not of themselves tend to deliver a constant current. This characteristic must be controlled by some external device.

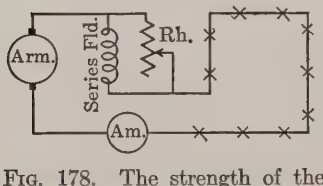


FIG. 178. The strength of the series field is controlled by the amount of resistance in the shunt *Rh*.

The most common device for this purpose is illustrated in Fig. 178. The current through the series field is controlled by means of a shunt (*Rh*). Whenever anything happens to a lamp it automatically short-circuits itself. This decreases the external resistance and allows more current to flow through the circuit. But by cutting out a little of the rheostat resistance more current is shunted around the series field, which is thus weakened. This results in a smaller generated voltage. By properly adjusting (*Rh*) the current can thus be maintained constant.

122. Caution Concerning Series Lighting Circuits. Although the voltage across any single series lamp will be small, nevertheless the voltage across the generator may be several thousand volts and the voltage from line to ground will on the average be one half the terminal voltage of the generator. There is, therefore, great danger to life in handling a series lamp when it is in service. For instance, if any part of the frame becomes grounded to one side of the line, anyone touching the lamp while standing on the ground might be subjected to a pressure upward of a thousand volts. Too much care cannot be exercised in handling any part of a series lighting system.

Prob. 14–8. If a shunt of 80 ohms resistance is placed around the series field of the generator in Prob. 10–8, what will be the terminal voltage when the machine is delivering 7 amperes?

Prob. 15–8. If 15 ohms are suddenly cut out of the external circuit in Prob. 14–8, what must the value of the series-field shunt be made in order to keep the current at 7 amperes?

123. Self-Excited Generators. Shunt Wound. The second type of self-excited generators is the shunt wound. In this machine, part of the current delivered by the generator is shunted from the line and through the field coils. Thus the armature current consists of two parts: first, that part which is delivered to the line, and second, that part which is delivered to the field coils.

Since none of the current shunted through the field coils ever gets into the line, and therefore cuts down by just so much the current which the generator might deliver to the line, it is desirable to make the field current as small as possible. Therefore the field coils consist of many turns of comparatively small wire. This causes the field coils to have a high resistance and allows but a small current to be forced through them.

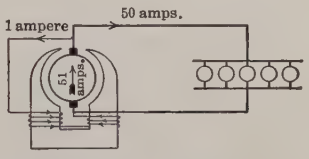


FIG. 179. The current in the armature of a shunt generator is the sum of the currents in the line and in field coils.

In Fig. 179, which represents a shunt generator, note that the armature delivers a current of 51 amperes to the brushes. Of this current, 50 amperes go through the line to the lamps and return to the armature, and 1 ampere goes through the field coils and returns to the armature.

124. Building-up of Voltage in a Shunt Generator. The voltage of a shunt generator builds up in the same manner as that of the series generator. The small amount of residual magnetism in the field is cut by the armature conductors when the armature is revolved, and sets up a small induced voltage between the brushes. Since the field coils are connected directly across the brushes, this small induced voltage sends a small current through the coils, which increases the magnetic lines in the field. The increased field strength causes an increase in the voltage induced in the armature. This higher voltage in turn increases the current through the field coils, which still further increases the magnetization and so on until the normal voltage of the machine at a given speed and field resistance is reached.

This building-up action is the same for any self-excited machine and often requires from 20 to 30 seconds before the normal voltage is attained. It does not continue indefinitely because the resistance of the field coils always sets a definite limit to the current in the coils, and when the magnetization gets beyond the saturation point, the reluctance of the magnetic path increases all out of proportion to

the increase in current. So only a few magnetic lines are added for even a considerable increase in the field current.

This is seen by a consideration of Fig. 180. The curve A is the magnetization curve of a shunt-wound generator having, say, 2000 field turns per pair of poles. The resistance of the field coils is 178.6 ohms. The current through these field turns must be produced by the terminal voltage. On the other hand, the magnetic field, by the cutting of which the voltage is produced, must be set

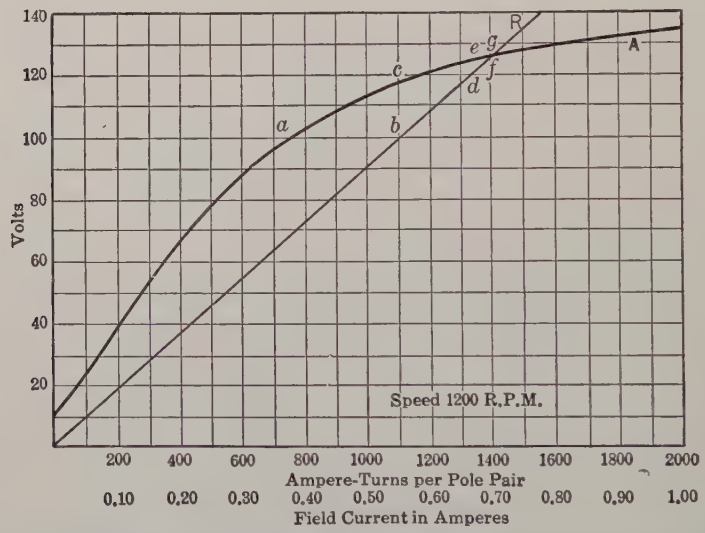


FIG. 180. The open-circuit voltage of a shunt generator is determined by the point where the field-resistance line and the magnetization curve intersect.

up by the current in the field coils. The generated voltage will continue to rise until there is just enough voltage put across the field coils to send that amount of current through them which will set up just enough field to produce the voltage generated.

The line R, called the **field resistance line**, shows the relation between the terminal voltage (voltage across the shunt field) and the current through the shunt field.

Suppose we ask why the terminal voltage does not stop at 100 volts. This is represented by point a on the magnetization curve. 100 volts would send $\frac{100}{178.6}$ or 0.56 ampere through the field, as shown by b on curve R. A current of 0.56 ampere through the field coils would set up 118 volts (point c). 118 volts would then send

$\frac{118}{178.6}$ or 0.66 ampere through the field (point d) which would set up 123.5 volts (point e), which would send $\frac{123}{178.6}$ or 0.69 ampere through the field (point f) which would set up 124.9 volts. This voltage sends $\frac{124.9}{178.6}$ or 0.699 ampere through the field which sets up practically 125 volts which sends 0.70 ampere through the field coils which sets up 125 volts, etc. The voltage cannot rise above 125 volts.

Assume that somehow a voltage of 130 was set up. This, according to curve A, would require a field current of 0.82 ampere but it would set up only $\frac{130}{178.6}$ or 0.728 ampere which would induce only 127 volts. This would in turn set up a smaller current and induce a smaller voltage until the terminal voltage becomes 125 volts. Thus the voltage at which the field resistance line crosses the generated voltage line (plotted to the same scale) determines the no-load voltage of a shunt generator.

If a self-excited generator fails to build up, it is usually because the field coils have been connected to the brushes in such a way that the current in them tends to set up a field in the opposite direction to the residual magnetism. Thus when the voltage first induced in the armature by the residual magnetism sends a current through the field coils, this current neutralizes the residual magnetism, rather than adds to it, and the induced voltage drops to zero instead of building up. The remedy is to reverse the field coil connections to the brushes. It may be necessary, however, to remagnetize the machine by lifting the brushes and sending a slight current, from some outside source, through the field. A few dry cells will supply all the current needed in this case.

125. Characteristics of a Shunt Generator. The characteristic curve of a shunt generator with commutating poles is represented in Fig. 181, curve A. Curve B is the external characteristic of the same generator operated separately excited. Note that when the machine is delivering no current, the terminal voltage is the greatest, both for a separately excited and for a shunt generator. As more and more current is delivered, the voltage drops further and further below the no-load voltage. In this respect, too, the shunt generator behaves like a separately excited generator, except that this falling off in voltage is even greater in a shunt generator than in a separately excited machine. This is because there is another factor

besides the armature resistance drop and the armature reaction tending to lower the voltage as the load increases.

This factor is a decrease in the field current, due to the above decrease in voltage which causes a decrease in the strength of the magnetic field, and still further decreases the terminal voltage.

Thus in Fig. 181, to construct curve *B*, the external characteristic of a separately excited generator, we subtract the armature resistance drop for the various armature currents from a constant

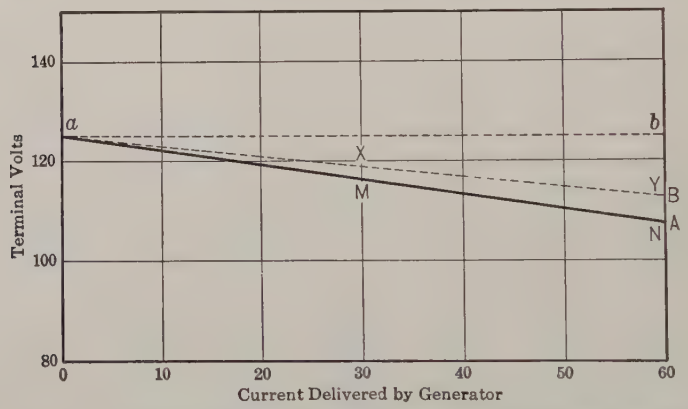


FIG. 181. Curve *A* shows the relation between the load and terminal voltage of a shunt generator.

generated voltage, 125 volts. The resistance of the armature is 0.20 ohm. Point *X*, the terminal voltage at 30 amperes armature current, is found by subtracting 30×0.20 or 6 volts from 125 volts. The point *Y*, the terminal voltage at 60 amperes armature current, is found by subtracting 60×0.20 or 12 volts from 125 volts.

The no-load voltage of the shunt generator was made to equal 125 volts by adjusting the resistance of the shunt field circuit so that the field resistance line *R* crossed the magnetization curve at the 125-volt point as shown in Fig. 180 and 182, which show the same curves. This gives us the zero-load voltage on curve *A*, Fig. 181, which is the external characteristic of a shunt generator. On this curve, point *M*, the voltage for a 30-ampere armature current, is found by subtracting 0.20×30 or 6 volts, **not from 125 volts,** but from the proper value of generated voltage obtained from curves *A* and *R* in Fig. 182.

The 6 volts are laid out as the line (*cd*) on the vertical edge of a strip of paper to the voltage scale of Fig. 182. This line (*cd*) is then

held vertical and the point (*d*) is moved along the resistance line *R* until the point (*c*) touches the magnetization curve *A*.

The ordinate of the point (*d*) is then the terminal voltage at the armature load. In this case it is 117 volts or 2 volts lower than the corresponding point on the external characteristic of this generator as a separately excited generator.*

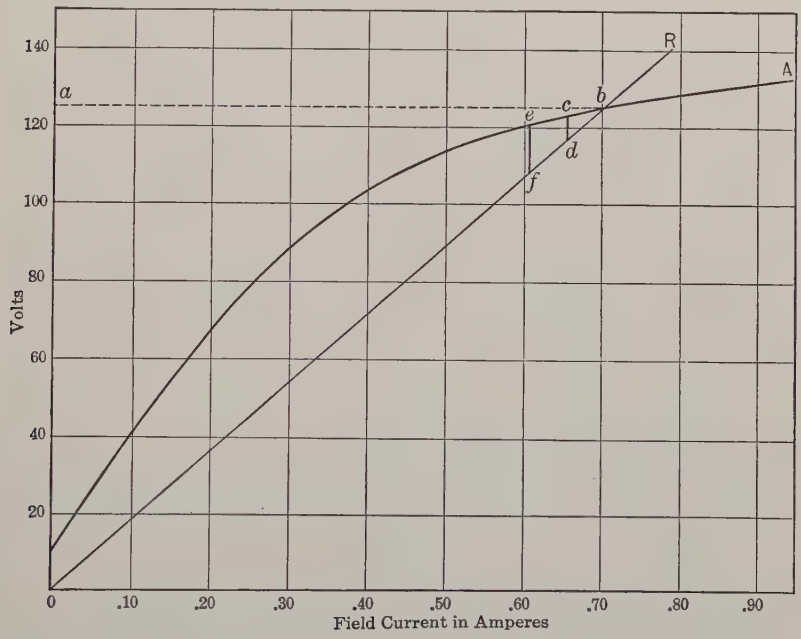


FIG. 182. A method of determining the terminal voltage of a shunt generator at any load.

That the point (*d*), Fig. 182, is the terminal voltage of the shunt generator at a 30-ampere armature current is shown as follows:

At point (*d*) the terminal voltage is 117 volts. The resistance of the shunt field is 178.6 ohms. Thus we have $\frac{117}{178.6}$ or 0.655 ampere field current. From the magnetization curve *A*, Fig. 182, 0.655 ampere field current will generate 123 volts as shown by point (*c*).

* The point *M* is the voltage for an **armature current** of 30 amperes, but for a **load current** of $30 - \frac{117}{178.6}$ or 29.3 amperes. Similarly point *N* is the terminal voltage for a line current $60 - \frac{108}{178.6}$ or 59.4 amperes.

The terminal voltage equals generated voltage of 123 volts minus the armature resistance drop of 6 volts or 117 volts, which we have seen is given by the point (*d*).

In the same way the point *N*, the terminal voltage at 60 amperes, armature current, is found by fitting in the vertical line (*ef*) equal to 60 × 0.20 or 12 volts, so that (*f*) is on the resistance line *R* and (*e*) is on the magnetization curve. The point (*f*), 108 volts, then, shows the terminal voltage at this armature current.

The external characteristic of a separately excited generator equipped with commutating poles is nearly a straight line. The external characteristic of a shunt generator equipped with commutating poles bends slightly downward.* As there is a maximum current which can be taken from the armature of a shunt generator, the external characteristic bends back toward the vertical axis as the voltage drops and finally reaches zero voltage on short circuit, but delivering an armature current equal to the quotient of the voltage due to the residual magnetism divided by the armature resistance.

Prob. 16–8. What is the steady short-circuit armature current of the shunt generator described in Art. 125?

Prob. 17–8. The data for the magnetization curve of a certain 3-kilowatt shunt generator with commutating poles, at 1500 r.p.m., are as follows:

Shunt field = 600 turns per pair of poles.
Shunt field resistance = 65 ohms.
Armature resistance including brush contact = 0.255 ohm.
Full-load armature current = 26.1 amperes.
Plot the external characteristic of the generator as far as the short circuit value of the line current. What is the voltage regulation of this generator?

Ampere-turns per pair of poles	Terminal volts at no load	Ampere-turns per pair of poles	Terminal volts at no load
0	7	750	112
50	13	900	122.5
150	27	1050	129.5
300	53	1200	133.6
450	78	1400	136
600	97.5	1700	139

* For a method of constructing the external characteristic of a shunt generator without commutating poles see " Industrial Electricity " by W. H. Timbie.

Prob. 18–8. Plot the external characteristic of the shunt generator of Prob. 17–8 as far as 150% full load when running at 1650 r.p.m. What effect does raising the speed have on the regulation of a shunt generator?

Prob. 19–8. Plot the external characteristic of the shunt generator of Prob. 17–8 as far as 150% full load when running at 1200 r.p.m. What effect does lowering the speed have on the voltage regulation of a shunt generator?

126. Voltage Control of Shunt Generators. While the characteristic curve of a shunt generator does not show an excessive drop in the terminal voltage until the full load of the machine has been exceeded, still the drop is usually much too large to enable incandescent lamps and other constant-voltage appliances to be operated by it. Lamps especially require a voltage which is practically unaffected by changes in the load.

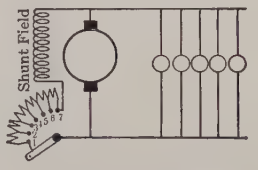


Fig. 183. The voltage of a shunt generator can be controlled by a resistance in series with the shunt field.

A shunt generator can be made to produce such a voltage in two ways which are in general use. One is by means of a field rheostat which controls the amount of current in the field coils. This is shown in Fig. 183. Of course, this rheostat cannot raise the voltage at full load, but is used to lower the voltage at light loads. At full load the rheostat is cut out. When, however, the load is decreased and the voltage tends to rise, enough resistance is introduced into the field circuit to reduce the voltage to full-load value. At light loads, therefore, the generator operates at low points on the saturation curve.

The other way is by means of a Tirrill Regulator which automatically cuts in and out a resistance in series with the shunt field. This latter method of voltage control is expensive while in the former case the rheostat changes must be made by hand. Shunt generators, therefore, are not in general use where close voltage regulation is required. It is customary in such cases to use compound generators, which can be built to deliver practically constant voltage.

Prob. 20–8. What resistance must be inserted in the field circuit of the shunt generator of Prob. 17–8 so that it shall have a no-load voltage of 80 volts at 1200 r.p.m.?

Prob. 21–8. How much of the resistance must be cut out from the field rheostat of Prob. 20–8 to produce a terminal voltage of 90 volts when half-load (13.0 amperes) is being drawn from the generator?

Prob. 22–8. Plot the external characteristic of the shunt generator of Prob. 17–8 when 12.3 ohms are inserted in the field and left unchanged. Speed 1400 r.p.m. What effect does extra resistance in the field have upon the voltage regulation of a shunt generator?

127. Compound Generators. A compound generator is primarily a shunt generator with a few series turns added to the field as shown in Fig. 184 and Fig. 185. The machine in Fig. 184 is said

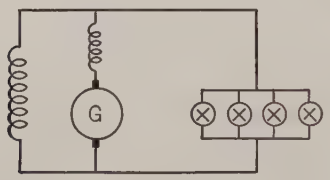


FIG. 184. A long-shunt compound generator.

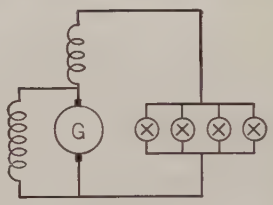


FIG. 185. A short-shunt compound generator.

to be connected long-shunt, because the shunt field is connected across the armature and series field. Figure 185 shows a short-shunt connection. All compound generators in this text are connected long-shunt. When no current is being delivered by the machine, the same current flows through the series and shunt coils. The shunt field then maintains the voltage at the normal value. But when any current is taken by the lamps, this current must also flow through the series field coils. The number of turns in these series coils is so gaged that whatever current flows through them, strengthens the field just enough to compensate for the drop in voltage due to the increased armature drop, and also for an increased armature reaction drop in case the machine has no commutating poles. The terminal voltage then is kept at practically a constant value from no load to full load. In practice, most compound machines have a characteristic curve like that shown in Fig. 186. The voltage is the same at no load and full load, but rises slightly for intermediate loads. Such a generator is said to be **flat-compounded.** Com-

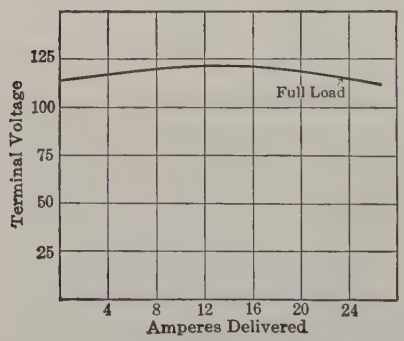


FIG. 186. The external characteristic of a flat-compounded generator.

mercial flat-compounded generators are often built to deliver rated voltage at $\frac{1}{3}$ load and at full load.

Generators are more often **over-compounded** so that the terminal voltage actually rises a little as the load increases. In this way a line drop of several volts may be compensated for and the voltage at the load be kept practically constant. Thus if a 112-volt generator were delivering 50 amperes to a load at the end of a line of 0.14 ohm resistance, there would be a line drop of 50×0.14 or 7 volts at full load. Even if the generator were flat-compounded and had the same terminal voltage, 112 volts, at full load as at no load, the voltage at the lamps would still be 7 volts lower at full load than at no load. Enough series turns, however, could be added to the field to cause the terminal voltage to rise 7 volts from no load to full load. This would maintain the same voltage at the lamps from no load to full load. Such a machine would be said to be $\frac{119 - 112}{112} = \frac{7}{112}$ or 6.25 per cent over-compounded.

Generators are often over-compounded to compensate for the decrease in speed of the driving engine which sometimes takes place as the load increases. The engine driving the generator may be so heavily loaded that its speed falls off from 3 to 5 per cent at generator full load, and unless the generator were over-compounded to this extent, a corresponding drop in the terminal voltage would result. Commercial generators of this type are generally over-compounded from 5 to 15 per cent, but machines may be purchased which have greater percentage of over-compounding.

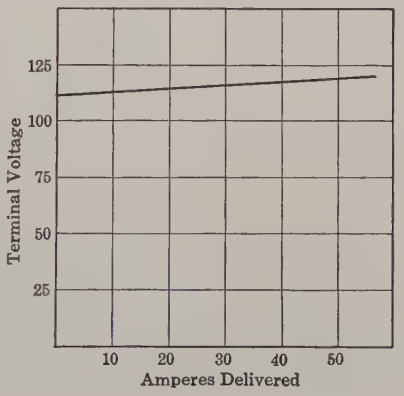


FIG. 187. The external characteristic of an over-compounded generator.

The characteristic curve of such a machine would have the appearance of that of Fig. 187.

Example 3. Suppose that we add 10 series turns per pair of poles to the shunt generator of Art. 125 which has the magnetization curve of Fig. 180 and 182, and note how the external characteristic differs from the shunt characteristic shown in Fig. 181. The other data for this generator are:

Shunt field has 2000 turns per pair of poles.
Resistance of shunt field = 178.6 ohms.
Full-load armature current = 60 amperes.
Armature resistance (including brush contact) = 0.20 ohm.
Resistance of series field = 0.04 ohm.
Long-shunt connection.

Solution.

Redraw the magnetization curve in Fig. 188 which is a copy of Fig. 180 and 182.

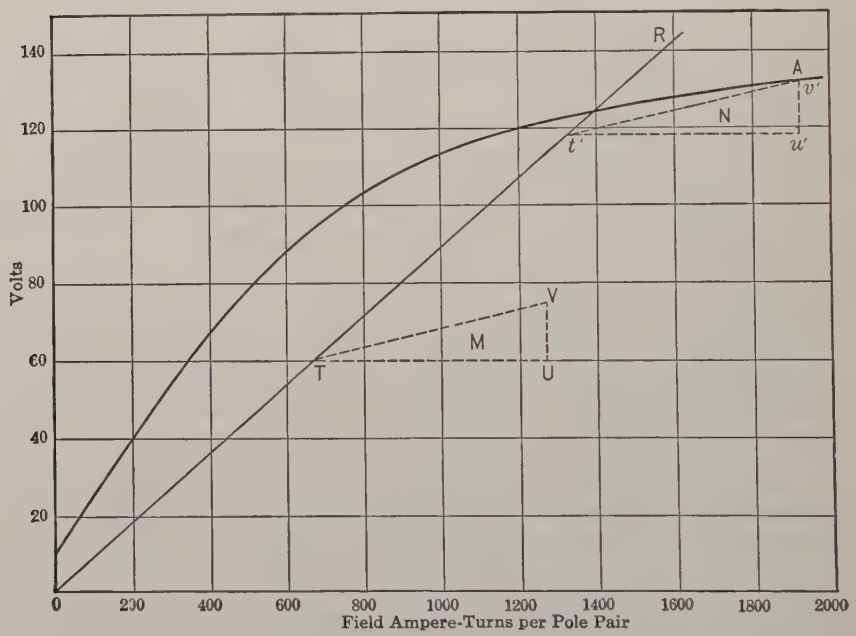


FIG. 188. A method for finding the terminal voltage of a compound generator at any load.

The no-load voltage will be 125 volts, as in the separately excited and shunt generator.

At 60 amperes, full-load armature current, the ampere-turns per pair of poles in the series field will be $60 \times 10 = 600$ ampere-turns.

The voltage drop due to the resistance of the armature and series field will be $60 \times (0.20 + 0.04) = 14.4$ volts. Cut out a paper right angle triangle TUV, making the vertical side UV equal to the voltage drop due to resistance, 14.4 volts according to the voltage scale, and the horizontal side TU equal to the series ampere-turns, 600 ampere-turns, according to the ampere-turn scale.

Keeping the side TU horizontal and the point T on the shunt field resistance line R, move this triangle up the sheet until the point V

touches the magnetization curve A, as it does in the position N. The ordinate of the point T is then the terminal voltage at this load. In this case it is 118 volts. Note that the 5 series turns per pole have raised the full-load terminal voltage from 108 volts (Fig. 181) to 118 volts, but not enough to flat-compound the machine. To do this, sufficient series turns must be added to bring the full-load voltage up to 125 volts. To prove that the ordinate of the point t' is the terminal voltage when the point V touches the magnetization curve, proceed as follows.

With a terminal voltage of 118 volts, the shunt field current is $\frac{118}{178.6}$ or 0.66 ampere, which produces 2000×0.66 or 1320 ampere-turns per pair of poles.

The series field ampere-turns $= 60 \times 10 = 600$.

Total ampere-turns per pair of poles $= 1320 + 600 = 1920$ ampere-turns per pair of poles.

The voltage generated by 1920 ampere-turns is 132.4 volts, according to the magnetization curve A, Fig. 188. The terminal voltage equals the generated voltage 132.4 volts minus the drop due to resistance 14.4 volts or 118 volts which is the ordinate of the point t' when the triangle is in the position of N.

In order to find the terminal voltage for any other armature current, construct another right-angle triangle making the vertical side equal to the voltage drop due to the resistance of the armature and series field, and the horizontal side equal to the series field ampere-turns per pole. Proceed with the triangle as above to find the terminal voltage.*

Prob. 23–8. Construct the external characteristic of the compound generator in Example 3 carrying the armature current values to 60 amperes.

Prob. 24–8. Construct the external characteristic of the compound generator in Example 3 using 5 series turns per pair of poles. Series field resistance = 0.02 ohm.

Prob. 25–8. Add 4 series turns per pair of poles (resistance 0.03 ohm) to the shunt generator in Prob. 17–8 and construct the external characteristic.

128. Number of Series Turns Necessary to Produce a Desired Compounding. To determine the number of turns necessary to produce a desired compounding, a value must be assigned to the resistance which the series turns are going to be allowed to have. This value can generally be adhered to very closely in constructing the series turns provided the resistance is chosen low enough to be sure the wire will carry the required current without overheating.

For example, suppose that it be required to determine the number

* For method of finding the terminal voltage of a compound generator without commutating poles see "Industrial Electricity," by W. H. Timbie.

of series turns per pair of poles to flat-compound the generator in Example 3.

The resistance of the armature is 0.20 ohm.

A value of 0.04 ohm for the series field would be practicable.

The voltage drop in armature and series field at full load = $60 \times (0.20 + 0.04) = 14.4$ volts.

Construct again, similar to Fig. 189, the magnetization curve of the machine. Draw line MN through the no-load voltage point,

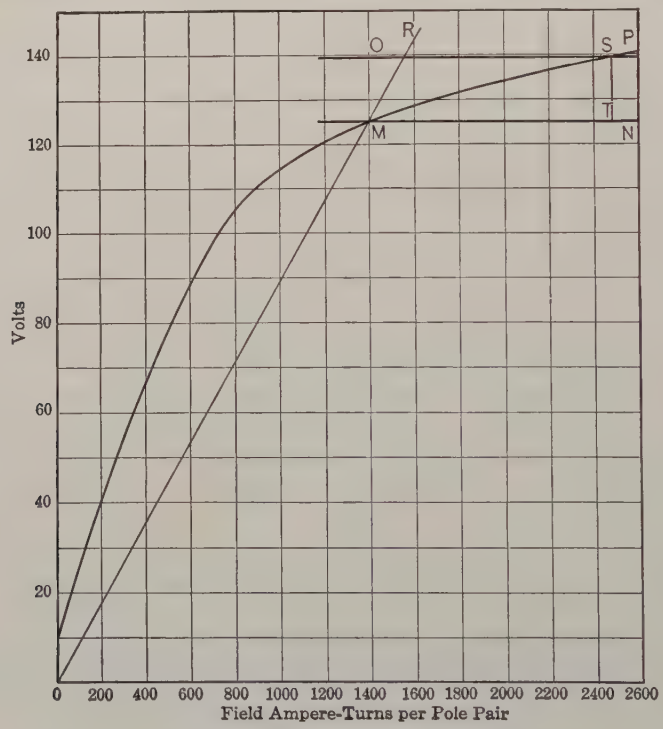


Fig. 189. A method for finding the series turns required on a compound generator.

parallel to the axis. Draw OP parallel to MN at a distance of 14.4 volts above it. Drop the line ST where OP crosses the magnetization curve.

The line MT represents the ampere-turns (1080) needed in series field to flat-compound the generator at 60 amperes armature current. The series turns per pair of poles then equal $\frac{1080}{60} = 18$ turns per pair of poles.

Prob. 26–8. Determine the number of series turns per pair of poles (connected long-shunt) to flat-compound the shunt generator of Prob. 22–8. Allow 0.03 ohm for resistance of series turns.

Prob. 27–8. How many series turns per pair of poles (connected long-shunt) would be needed to over-compound the shunt generator of Prob. 17–8, 6%? Use 0.040 ohm as the resistance of the series field. Shunt-field resistance is 88 ohms.

Prob. 28–8. Construct the external characteristic of the compound generator in Prob. 27–8, carrying the armature current values to 150% full-load values.

129. Adjusting the Compounding of a Generator. In order to adjust the amount of compounding of a generator, the series field is generally supplied with a shunt, the resistance of which may be varied. This arrangement is shown in Fig. 190. The shunt R around the series field in large machines is usually composed of grids. In smaller machines it is in the form of resistance ribbon. For a test machine, a piece of German silver wire is generally used, so arranged that any desired length of it may be connected across the series coils. If the machine is too highly over-compounded, it is merely necessary to cut down the length of the shunt wire and so decrease its resistance. This allows a greater part of the load current to flow through the shunt wire and less through the series field. The series field, therefore, will be just so much weaker, and the voltage correspondingly lower. The approximate resistance which a shunt wire must have may be computed as follows:

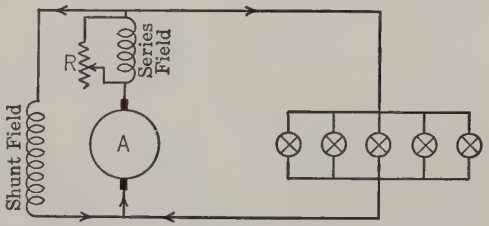


FIG. 190. The voltage of a compound generator can be controlled by a resistance R placed around the series field.

First. Compute the number of series ampere-turns needed by the method of Art. 128.

Second. Compute current needed in series turns to produce these ampere-turns.

Third. From the resistance of the series turns, series field current and total armature current compute resistance of shunt needed around series turns.

Example 4. What must be the resistance of a shunt around the series turns of the flat-compounded generator described in Art. 128, in order to cut down the full-load voltage to 120 volts?

Solution.

The resistance of the series field is 0.04 ohm. Assume that, when shunted, the resistance of the parallel combination of series field and shunt will be 0.02 ohm.

Voltage to overcome resistance of armature and series field $= 60 \times (0.20 + 0.02) = 13.2$ volts.

To have terminal voltage of 120 volts we must generate $120 + 13.2 = 133.2$ volts. From curve A, Fig. 188, this will require 2000 ampere-turns per pair of poles.

$$\text{Shunt field current at 120 volts} = \frac{120}{178.6} = 0.672.$$

Shunt field ampere-turns $= 0.672 \times 2000 = 1340$ ampere-turns per pair of poles.

Series field must furnish $2000 - 1340 = 660$ ampere-turns per pair of poles.

$$\text{Current in series turns} = \frac{660}{18} = 36.7 \text{ amperes.}$$

Current through shunt $= 60 - 36.7 = 23.3$ amperes.

Volts across shunt = volts across series field $= 36.7 \times 0.04 = 1.465$ volt.

$$\text{Resistance of shunt} = \frac{1.465}{23.3} = 0.0630 \text{ ohm.}$$

Prob. 29–8. A compound generator delivers 115 volts at no load and 125 volts at full load. With series field cut out, full-load voltage is 98 volts. It is desired to change the compounding so that it will deliver 120 volts at full load. The series field resistance is 0.018 ohm. What must be the resistance of the shunt to the series field (approximately)?

Prob. 30–8. Approximately what resistance must be used as a shunt around the series field of the generator in Prob. 29–8 to produce flat-compounding?

130. Parallel Operation of Shunt Generators. Series generators are rarely operated in parallel, but this is the common practice with shunt generators. Figure 191 shows the connections for two shunt generators in parallel to deliver power to a lamp load.

It is very important that the no-load voltages of both generators be adjusted to the same value and that their external characteristics be the same. If two shunt generators having different no-load voltages are connected in parallel, there will be a circulating current through the two machines, although no current is being delivered to the load.

If two shunt generators with different characteristics are connected in parallel each will fail to take its right fraction of the load as the load changes. This may result in the over-heating of the machine

with the better regulation. The two machines need not be the same capacity but they must have the same no-load voltage and the same full-load voltage.

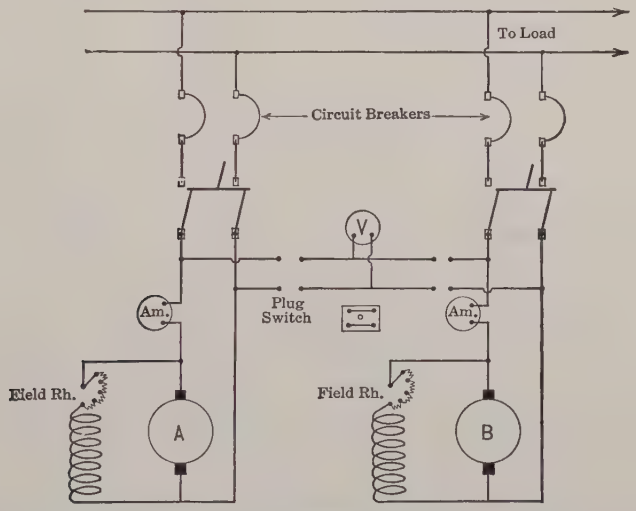


FIG. 191. Connections for the parallel operation of shunt generators.

Example 5. Let us consider the action of two parallel shunt generators which do not fulfill the conditions of the above article. Generator A has a rated full-load current of 48 amperes, a no-load terminal voltage of 125 volts and a full-load voltage of 110 volts. Generator B has a rated full-load current of 32 amperes, a no-load voltage of 120 volts and a full-load voltage of 100 volts. These two generators are joined in parallel to deliver a combined load of 50 amperes.

(a) At what voltage will they deliver this total current?

(b) What current will each deliver?

Solution.

Construct as in Fig. 192 the external characteristics of the two generators which will be straight lines approximately.

Lay off on a strip of paper the line XY equal to 50 amperes of the horizontal scale. Keeping the line horizontal, move it up until the end X just touches the external characteristic of B, and the end Y, the external characteristic of A. This is the position shown in Fig. 192. The point at which the line XY crosses the vertical axis is the terminal voltage, in this case, 113 volts. The line XZ measures the current delivered by B, in this case, 10.5 amperes. The line ZY measures the current delivered by A, in this case, 39.5 amperes.

Prob. 31–8. What current would each generator in the combination of Example 5 deliver at a terminal voltage of 120 volts?

Prob. 32–8. (*a*) What would be the terminal voltage of the parallel arrangements of the generators in Example 5 at no load?

(*b*) What current would each generator deliver under this condition?

Prob. 33–8. (*a*) At what voltage would the arrangement of Example 5 deliver 25 amperes?

(*b*) What current would each generator deliver?

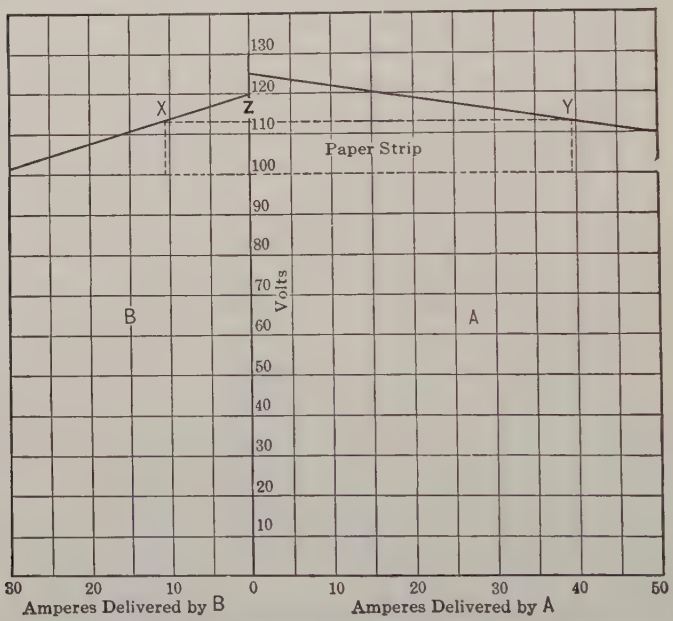


FIG. 192. A method for determining the current and voltage of shunt generators operated in parallel.

131. Parallel Operation of Compound Generators. If generators are under-compounded (having drooping characteristics) they can be operated in parallel just like shunt generators. But if they are over-compounded (having rising characteristics) the series fields must be connected with a low resistance connection called an equalizer, shown in Fig. 193. Without the equalizer it is impossible to operate two over-compounded generators in parallel. The generators must also have the same over-compounding and the resistances of the series fields must be inversely proportional to the capacities of the machines.

132. The Third-Brush Generator. The electric energy for starting and lighting an automobile is furnished by a storage battery. The generator which charges the battery is run by the gasoline motor

and hence varies greatly in speed. All the generator characteristics which we have studied are those of constant speed machines. The requirement of the charging generator on an automobile is that it shall reach a normal charging current when the speed of the car reaches about 10 miles an hour and shall not appreciably exceed this normal current at any higher speeds of which the car is capable. This characteristic is attained by means of the so-called "third brush," as illustrated in Fig. 194.

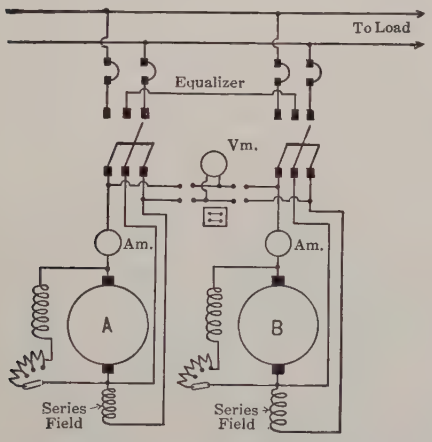


FIG. 193. Connections for the parallel operation of over-compounded generators.

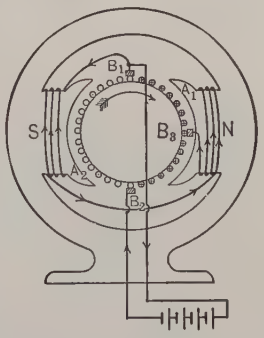


FIG. 194. The third-brush generator.

The current to charge the storage battery is drawn from the armature by the main brushes, B_1 and B_2. The current for the field coils is drawn from the armature by the main brush B_1 and the "third brush" B_3, placed midway between B_1 and B_2. The voltage across the field is therefore the voltage across the armature conductors between B_1 and B_3 only.

Assume that the car is running at 10 miles per hour and that the generator is charging the battery at the normal rate. Let the speed be increased to 20 miles per hour. The voltage induced between B_1 and B_2 would now be doubled if the field remained the same and a larger current would flow from the armature windings into the battery. But this larger current in the armature windings would create a greater cross magnetization of the armature and thus shift the field still further in the direction of rotation. This would weaken the pole tips A_1 and A_2. The weakened field from the pole tip A_1 would decrease the e.m.f. between B_1 and B_3 and lower the

current flowing through the field coils which would still further lower the field strength and decrease the voltage between B_1 and B_3, and also between B_1 and B_2. This would reduce the current delivered to the storage battery to very nearly the normal value. In fact, if we plot a curve between the speed and current delivered, we find the rise in current to be very slight after a speed of 10 miles per hour is attained by the car.

The third-brush generator represents an example where use is made of the shift in the field caused by the armature cross ampere-turns to regulate the voltage, and hence the current delivered by the generator. Some form of reverse-current cut-out is necessary to prevent the battery discharging through the armature when it is not rotating.

SUMMARY OF CHAPTER VIII

ARMATURE REACTION is a name given to the magnetomotive force produced by the armature current flowing through the armature turns. It is divided into two parts as follows:

CROSS AMPERE-TURNS are the product of the armature current times those armature turns outside of the double angle of brush shift. These turns produce field distortion and necessitate either a forward lead of brushes or commutating poles to get sparkless commutation. They also have a slight demagnetizing effect.

BACK AMPERE-TURNS are caused by the shifting of the brushes and are the product of the armature current times the armature turns inside the double angle of brush lead. These ampere-turns produce demagnetization and cause a drop in the terminal voltage. They are not present in a commutating pole generator.

D-C GENERATORS are divided into two classes: 1st, Separately Excited; 2nd, Self-Excited.

SEPARATELY EXCITED Generators have their fields excited by some outside source of current, and are used (1st) wherever the field strength must be independent of the terminal voltage; (2nd) wherever a given polarity is the essential factor. They have very limited use.

SELF-EXCITED machines are divided into three types, according to the method of connecting the field coils to the armature circuit:

(1st) Series.
(2nd) Shunt.
(3rd) Compound.

SERIES. All the current delivered to the line flows through field coils, which consist of a few turns of heavy wire. Voltage rises as load increases. Operates satisfactorily on a constant-current line.

SHUNT. Field is connected in parallel with line and only a small current goes through coils, which consist of many turns of fine wire.

Is a nearly constant potential generator, the voltage falling slightly as the load increases. Voltage may be controlled somewhat by field rheostat.

COMPOUND. The field consists of two sets of coils; one series and the other shunt. When enough series turns are wound on to offset exactly any fall in terminal voltage due to increased armature drop (and armature reaction), the machine is said to be Flat-Compounded. When terminal voltage rises slightly with load, it is said to be Over-Compounded.

THE PARALLEL OPERATION of shunt generators is satisfactory if the generators have the same voltage rating and regulation.

SATISFACTORY PARALLEL OPERATION of over-compound generators requires that the resistances of the series fields be inversely proportional to the capacities of the generators, and that an equalizer of low resistance connect the series fields of the two machines in parallel.

The THIRD-BRUSH generator is used for constant current regulation when the speed of the generator varies through wide ranges. As the armature current gets greater, the cross ampere-turns weaken the trailing pole tip, the flux of which supplies the voltage between a main brush and the third brush. This is the voltage across the field coils, and hence the field current falls as the armature current rises. This tends to keep the armature current constant.

PROBLEMS ON CHAPTER VIII

Prob. 34–8. A 2-pole shunt generator has an armature resistance of 0.32 ohm and a field resistance of 145 ohms. When this generator delivers 60 amperes to the line at 120 volts, what e.m.f. must it generate?

Prob. 35–8. What current flows in each armature path of generator of Prob. 34–8?

Prob. 36–8. A long-shunt compound generator delivers 100 amperes at 116 volts. Armature resistance = 0.038 ohm. Shunt field resistance = 82 ohms. Series field resistance = 0.024 ohm. Find:

(*a*) E.m.f. of generator at this load;
(*b*) Current in armature;
(*c*) Power lost in fields.

Prob. 37–8. If the generator of Prob. 36–8 were connected short-shunt, what resistance would be required in series with the shunt field in order to maintain the same current as in Prob. 36–8?

Prob. 38–8. A short-shunt compound generator is delivering power to the following pieces in parallel. One thousand 50-watt, 110-volt lamps and one 110-volt, 12-horsepower motor, with an efficiency of 85%. The resistance of the shunt field is 55 ohms; of the series field, 0.005 ohm; of the armature, 0.015 ohm; of the line wires, 0.035 ohm.

Find:

(*a*) The armature current;
(*b*) The shunt field current;
(*c*) The induced e.m.f. at this load.

Prob. 39–8. If the generator of Prob. 38–8 were connected long-shunt, what would be the answers to (*a*), (*b*) and (*c*)?

Prob. 40–8. A 6-pole, 6-path generator with commutating poles having the magnetization curve of Fig. 172 has the following constants:

Armature resistance (including brush contact)	= 0.180 ohm.
Number of active armature conductors	= 180
Full-load armature current	= 60 amp.
Resistance of shunt field	= 255 ohms.
Turns per pair of poles on shunt field	= 790
Speed	= 1100 r.p.m.

Construct the external characteristic of this machine operated as a shunt generator. Obtain sufficient points on curve to obtain maximum current.

Prob. 41–8. If 6 series turns per pair of poles are added to generator in Prob. 40–8 what amount of resistance must be shunted around the series field to produce flat-compounding? Resistance of series field = 0.04 ohm.

Prob. 42–8. What number of series turns must be added to the shunt generator of Prob. 40–8 to produce 6% over-compounding? Use 0.04 ohm as the resistance of the series field.

Prob. 43–8. Plot the external characteristic for the generator of Prob. 42–8.

Prob. 44–8. (*a*) If the generator of Prob. 40–8 is run at 1400 r.p.m., how much resistance must be inserted in the field to bring the no-load voltage down to 250 volts?

(*b*) By cutting out this resistance added to the field for how great an armature current can the terminal voltage be maintained at 250 volts?

Prob. 45–8. A group of motors situated 1000 feet from the power house require 50 amperes at 240 volts. Also the no-load voltage must not rise above 240 volts. The line wire is No. 2 B. & S. copper. It is proposed to use the generator of Prob. 40–8 with a fixed resistance in series with the shunt field to produce 240 volts at no load, and sufficient series turns (resistance to be 0.04 ohm) added to keep the full-load voltage at the motors 240 volts. Speed = 1320 r.p.m.

(*a*) What resistance must be added in the shunt field circuit?
(*b*) How many series turns are required?

Prob. 46–8. It is proposed to meet the conditions of Prob. 45–8 using the generator of Prob. 40–8 as follows: Instead of putting resistance in the shunt field to obtain a no-load voltage of 240 volts, the

speed is to be lowered and the shunt field left as in Prob. 40–8. The 240 volts at motors when drawing 50 amperes is to be secured as before by adding series turns to the generator field.

(*a*) At what speed must the generator be run?

(*b*) How many series turns must be added (resistance 0.04 ohm)?

Prob. 47–8. The generator described in Prob. 5–8 is to be used as a shunt generator using the same field coils. The no-load voltage desired is 300 volts.

What resistance must be inserted in series with the field coils?

Prob. 48–8. (*a*) What resistance must be inserted in shunt field circuit of Prob. 47–8 to produce a no-load voltage of 250 volts?

(*b*) How many series turns per pair of poles must be added to produce flat-compounding?

Full-load armature current 22 amperes. Series field resistance 0.10 ohm.

Prob. 49–8. Two shunt generators are operated in parallel. *A* has a full-load current of 60 amperes at 115 volts, with a 10% voltage regulation. *B* has a full-load current of 80 amperes at 113 volts and an 8% voltage regulation. (*a*) What current will each deliver to a load demanding a total of 120 amperes? (*b*) What will be the terminal voltage of the machines at this load?

Prob. 50–8. Shunt generator *A* has a no-load voltage of 230 volts and a full-load voltage of 220 volts at 50 amperes. Shunt generator *B* has a no-load voltage of 228 volts and a full-load current of 100 amperes. What must be the full-load voltage of generator *B* in order that the two generators may be operated in parallel and each take its proportional share of the load?

Prob. 51–8. The full-load voltage of each of two shunt generators is 115 volts. *A* has a voltage regulation of 5%; *B*, 10%. The full-load current of *A* is 100 amperes; of *B*, 60 amperes.

(*a*) What current will each deliver when a load of 160 amperes is demanded, if they are connected in parallel?

(*b*) When a load of 30 amperes is demanded?

(*c*) When a load of 10 amperes is demanded?

Prob. 52–8. Shunt generator *A* is rated as 100 kilowatts at 112 volts with a 6% regulation. Shunt generator *B* is rated as 200 kilowatts at 110 volts, with an 8% regulation. Consider their power-output characteristics as straight lines.

(*a*) If these two machines are operated in parallel, how much power will each deliver to a 113-volt load?

(*b*) What is the maximum power the combination can deliver without overloading either machine?

Prob. 53–8. Given the following magnetization data on a generator running at 1200 r.p.m.:

I_f	E_A	I_f	E_A
0.05	50	0.30	204
0.10	93	0.35	217
0.15	131	0.40	226
0.20	163	0.45	234
0.25	187	0.50	240

Two-pole machine having 960 conductors on the armature.
No. of shunt field turns, 6435 per pair of poles.
Total armature resistance, 1.19 ohms.
No-load voltage, 220 volts.
Full-load armature current, 12 amperes.
Speed 1200 r.p.m.

Compute the series turns necessary to flat-compound this machine, assuming that 10% of the current is to be shunted around the series field by a series-field shunt; that the arrangement is to be long-shunt and that the resistance of the series field and its shunt combined is to be 0.20 ohm.

Prob. 54–8. (*a*) Draw the external characteristic of the flat-compound generator in Prob. 53–8.

(*b*) Determine the greatest deviation from full-load voltage.

(*c*) What per cent deviation is this?

Prob. 55–8. Two 125-volt shunt generators A and B are operating in parallel at rated load and rated voltage. The generator A is a 100-kilowatt machine. The generator B is a 250-kilowatt machine. The voltage regulation of A at rated load is 8% and the voltage regulation of B at rated load is 4%. Assuming the voltage regulation curves of the two machines to be straight lines and the field rheostats not to be altered, what will be the terminal voltage and the current output of each generator,

(*a*) When the total current supplied is 2400 amperes,

(*b*) When there is no load on the bus-bars?

CHAPTER IX

ELECTRIC MOTORS

Every electric generator will run as a motor if supplied with the proper voltage, although a few minor changes have to be made to secure the greatest efficiency. To understand this motor action it is necessary to recall the effect of a magnetic field upon a conductor carrying a current.

133. Force of a Magnetic Field on a Wire Carrying a Current. It will be remembered that a wire carrying a current has a circular magnetic field around it like that in Fig. 195. If this wire is placed in a uniform magnetic field which is not parallel to the wire, the flux density will be increased on one side of the wire and decreased on the opposite side as in Fig. 196. The wire has a current flowing **in** and

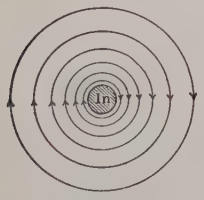


Fig. 195. The magnetic field about a wire carrying an electric current.

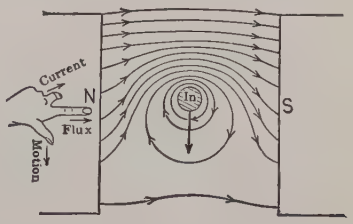


Fig. 196. The wire carrying a current distorts a parallel magnetic field.

a clockwise magnetic flux about it. When the wire is placed in a uniform left-to-right magnetic field, there are two magnetic fields in the same direction above it and in opposite directions below, hence it causes a concentration of the lines above the wire and a thinning out of the lines below. Since the magnetic lines act like stretched rubber bands and tend to straighten out, they exert a downward push on the wire. The direction of this force can be found most easily by the so-called " Left-hand Motor Rule."

" Left-hand Motor Rule." Extend the thumb and first two fingers of the **left** hand at right angles to one another. If the **forefinger** points in the direction of the **magnetic lines** of force and the **middle** finger in the **direction** of the **current** in the wire, the

thumb will point in the direction of the force on the wire. This is shown in Fig. 196.

134. Force of a Magnetic Field on a Coil Carrying a Current. Torque. In Fig. 197 is shown a cross-section of a single loop carrying a current in a magnetic field. The current is flowing **in** at A and **out** at B, as marked, and the flux is strengthened **above** A and **below** B. Thus there is a force pushing **down** on A and **up** on B, tending to cause the loop to rotate in a counterclockwise direction as marked. This tendency to revolve is measured by the force on both wires multiplied by the distance of each wire from the point around which the wire tends to revolve. In the case of a loop this point is the center of the loop.

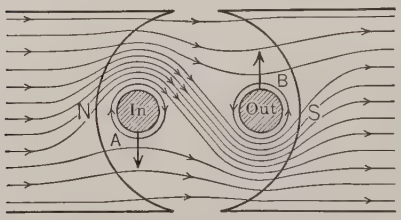


FIG. 197. The distortion of the magnetic field produces a tendency to turn the coil.

The force on the wire can be computed from the equation

$$F = \frac{BlI}{10} \text{ dynes}$$

but a more practical equation is

$$F = \frac{22.5\ BlI}{10^8} \text{ pounds}$$

where F = force on the wire in **pounds**
I = current in wire in **amperes**
B = flux density in air in **gausses**
l = length of wire in **centimeters**

Example 1. If each side of the coil in Fig. 197 is 65 cm. long and is carrying a current of 100 amperes and the magnetic field strength is 5000 gausses, what magnetic force is acting on conductor A to force it down?

Solution.

$$F = \frac{22.5\ BlI}{10^8} = \frac{22.5 \times 5000 \times 65 \times 100}{10^8}$$
$$= 7.31 \text{ pounds.}$$

The torque, or tendency to revolve, depends not only upon the force applied but also on the lever arm it has and is therefore found

by multiplying this force by the distance which the force is from the point around which the wire tends to revolve. This distance is called the **arm** of the force. In the case of the wire in Fig. 196 there is no revolving tendency, but the two wires in Fig. 197 tend to revolve about the center of the loop, and the torque can be found as follows.

Example 2. The width of the loop in Example 1 is 8 inches. What is the torque produced by the force on each wire when the loop is in this position?

Solution.

$$\text{Force on each wire} = 7.31 \text{ pounds}$$
$$\text{The arm of this force is } \tfrac{4}{12} = 0.333 \text{ foot}$$
$$\text{Torque} = \text{Force} \times \text{arm}$$
$$= 7.31 \times 0.333$$
$$= 2.44 \text{ pound feet.}$$

Torque is often expressed as the number of pounds at the end of an arm of 1 foot. Thus 2.44 pounds with a 1-foot arm would produce the same torque (2.44×1) as 7.31 pounds with a 0.333-foot arm (7.31×0.333).

Of course the torque on the loop is $7.31 \times \tfrac{8}{12} = 4.87$ pound feet.

Prob. 1–9. A wire 60 cm. long lies at right angles to the lines in a magnetic field. If the force on the wire is 1.6 pounds when 25 amperes flow through it, how strong is the magnetic field?

Prob. 2–9. A 100 cm. wire is urged perpendicular to lines in a magnetic field of 5000 gausses by a force of 2.5 pounds. What current is flowing in the wire?

Prob. 3–9. (*a*) In Fig. 198, indicate resultant field if poles and current are as marked.

(*b*) Indicate direction in which loop will tend to turn.

(*c*) If flux density is 2500 gausses, length of sides of loop 80 cm. each, current through loop 100 amperes, what force in pounds is there on each wire?

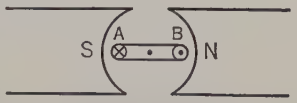


Fig. 198. A single loop in a magnetic field.

Prob. 4–9. The loop in Prob. 3–9 has a width of 16 inches. What torque acts on loop?

Prob. 5–9. If the field strength of Prob. 4–9 were doubled, what would be the torque?

135. Torque of a Drum-wound Motor. In the case of a drum-wound armature it is merely necessary to find the force on each wire and multiply it by the distance of the wire from the center of the shaft.

Thus in Fig. 199, note that the conductors opposite the South poles are carrying a current **into** the paper and according to the " left-hand rule " tend to turn the armature in a clockwise direction. The conductors opposite the North poles are carrying currents **up out** of the paper, and these, by the " left-hand rule," also tend to rotate the armature clockwise.

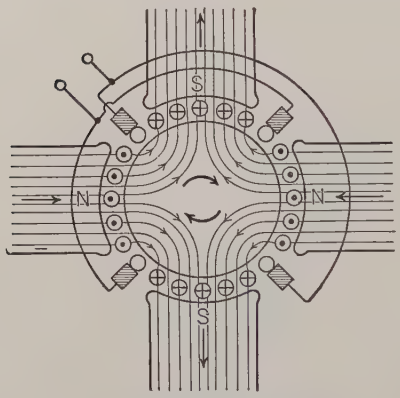


Fig. 199. The magnetic field of a four-pole motor.

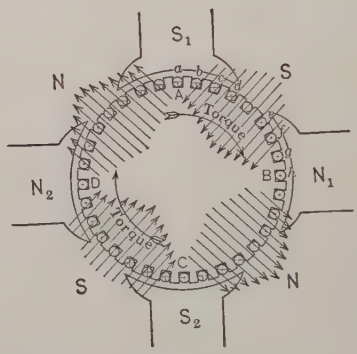


Fig. 200. Why a slotted-core armature tends to turn. Note the poles on the armature in spaces between the field poles.

Example 3. Assume that each wire in Fig. 199 is carrying 50 amperes and that the distance of the wires from the center of the armature is 6 inches. Let the length of the armature be 25 centimeters and the flux density in the air gap be 10,000 gausses. What is the total torque?

Solution.

$$\text{Force on each conductor} = \frac{22.5\ BlI}{10^8}$$

$$= \frac{22.5 \times 10{,}000 \times 25 \times 50}{10^8}$$

$$= 2.81 \text{ pounds.}$$

$$\text{Torque on each conductor} = 2.81 \times 0.50$$

$$= 1.41 \text{ pound feet.}$$

$$\text{Number of active conductors} = 20.$$

$$\text{Total torque} = 20 \times 1.41$$

$$= 28.1 \text{ pound feet.}$$

This is equivalent to a force of 28.1 pounds acting at the rim of a two-foot pulley, *i.e.*, acting one foot from the center of the pulley.

In the case of a slotted-core armature like that shown in Fig. 200

the force is not actually on the wires but on the iron between them. The wires lie in the slots where there is very little flux, most of the lines going through the projecting iron.

Note that the current in the windings between the points A and B on the armature tend to make a South pole on this quadrant of the surface of the armature. The current enters the conductors a, b, c and d and leaves by e, f, g and h. These conductors act then like a flat coil, and by applying the right-hand rule it can be seen that there is a tendency to set up a flux in the direction indicated, making this area a South pole, and causing it to be attracted by the North pole N_1.

Similarly, the current in the conductors lying between the points B and C on the armature tend to set up a North pole on this quadrant of the armature which is attracted to the South pole S_2 and repelled by N_1, etc.

It is fortunate that the force in large machines does not come upon the conductors but on the iron, as it would be very difficult to provide insulation which would not be crushed by the enormous forces developed by modern electric machinery.

In computing the torque of these machines, however, the equation that has been given can be used, if the value of the flux density B is taken as the average flux density in the air gap.

Prob. 6–9. A drum-wound motor armature is 60 cm. long and 25 cm. in diameter and has 196 active conductors under the pole faces. The average flux density in the air gap is 9500 gausses. If the current per conductor is 36 amperes, what force will be exerted at the rim of a 6-inch pulley on the motor shaft?

Prob. 7–9. A 6-pole, 6-path motor armature is 50 cm. long, 40 cm. in diameter and has 240 active conductors. The average flux density in the air gap is 8200 gausses. The pole faces extend over $\frac{3}{4}$ of the armature circumference. What armature current is required to produce a torque of 40 pound feet?

Prob. 8–9. A 4-pole, 4-path motor has an armature 12 inches in diameter and 10 inches long with 184 active conductors. The flux density in the air gap averages 68,000 lines per square inch. The armature takes 64 amperes. With what size pulley must it be fitted to exert a pull of 75 pounds on a belt? The pole faces cover $\frac{3}{4}$ of the armature surface.

136. Why it Requires Power to Drive an Electric Generator. The fact that a force is exerted on a wire carrying current and located in a magnetic field explains why it takes power to drive a generator when it is delivering current and why it **does not take**

power when the generator is not delivering current, even though a high voltage may be developed.

When the generator is not delivering current, practically no current is flowing in the armature windings; thus there is no force to be overcome. But as soon as a current flows through the armature, this force is present, as shown in Fig. 197, and must be overcome. In other words, the torque that we get in a motor when we send a current through the armature is exactly the torque we have to overcome when we desire to run the machine as a generator and draw this amount of current from it. This fact may be illustrated as follows:

Suppose we wish to use the machine in Fig. 199 as a generator to set up a current in the windings in the direction marked for a motor. By applying the right-hand rule for generated voltage we see that the direction of rotation must be counter-clockwise or the reverse to that in which it tends to run as a motor. Thus when we drive this machine as a generator and deliver current, the current in the windings acting on the magnetic field urges the armature to turn the other way. It is this urge that we have to overcome when we drive it as a generator.

Prob. 9–9. In what direction would the armature in Fig. 200 have to turn in order to generate currents in the armature in the directions marked on the conductors?

Prob. 10–9. The motor of Example 3 is operated as a generator delivering the same armature current as in the example. How many horsepower are required to rotate the armature at 1200 r.p.m.?

137. "Damping" of Electrical Instruments. This force of a magnetic field on a wire carrying a current explains also the "damping" of electrical measuring instruments. In the Weston Direct-current Ammeters and Voltmeters, for instance, the moving coil is wound on an aluminum bobbin. When this bobbin is moved in the magnetic field, currents are generated in it by the e.m.f. induced by the cutting of lines of force. These currents then produce a torque on the bobbin which opposes the motion, and thus the tendency to keep oscillating is constantly opposed by the torque of the currents induced in the bobbin.

138. Counter Electromotive Force. We have just been considering the "motor-effect" that is present in a generator when it is delivering a current. We have seen how this "motor-effect" is produced by the current flowing in the armature coils and that it is opposite to the motion of the armature of a generator.

Similarly, there is a "**generator effect**" **in a motor** which is called the **counter** or **back electromotive force.**

This will be clear when we consider that a motor consists of a number of conductors moving in a magnetic field. Now if conductors move in a magnetic field so as to cut lines of force, an electromotive force will be set up, whether we call the machine a generator, a motor, or anything else.

If we apply the **right-hand rule** to the motor coil turning as in Fig. 197, we find that there must be a voltage induced in A, which would tend to send a **current out.** The current is actually sent **in** at A to cause the coil to turn. By applying the rule to B, we find that here, too, there is a voltage induced which would tend to send a **current in,** while the current is **really flowing out of** B.

This electric pressure, which we find in each case opposes the flow of the current through the armature, is the **counter electromotive force.**

It is called the counter electromotive force because it is in the opposite direction to the electromotive force impressed across the armature to cause it to turn as a motor.

There are then two sources of electromotive force in the windings of a motor that is running: one, the voltage which is impressed on the terminals from an outside source; the other, the voltage which is set up in the windings by cutting lines of force. These two pressures are in opposite directions, and thus the current that flows must be proportional to their difference.

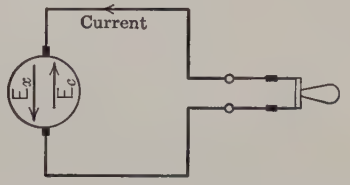


FIG. 201. The long arrow represents the impressed voltage, the short arrow the back e.m.f.

We might represent this condition graphically as in Fig. 201. The arrow E_x represents the impressed voltage across the motor. This voltage is in the same direction as the current. The counter voltage, generated when the motor is running, would be represented by a slightly shorter arrow, E_c, pointing in the opposite direction. The voltage E_x tends to send a current **down** through the armature, while the counter voltage tends to send a current **up.** The current will flow **in the direction** of the greater pressure E_x. The pressure under which it flows, however, will be the difference between the two forces $(E_x - E_c)$. This is analogous to the way any mechanical body behaves when acted upon by two opposing forces. It moves in

the direction of the greater force, and accelerates at a rate proportional to the difference between the opposing forces.

Thus if the impressed voltage $E_x = 110$ and the counter voltage $= 106$ volts, the resulting voltage, forcing the current through the armature of the motor, equals $110 - 106$, or 4 volts. And the result, as to current flow, is just as though there were but 4 volts drop across the armature. That is, if the armature resistance of this motor were 0.5 ohm, the flow of current would be $\frac{4}{0.5} = 8$ amperes.

Example 4. The armature resistance of a motor is 0.24 ohm. If it has a counter electromotive force of 108.8 volts when running on a 110-volt circuit, what current does it take?

Solution.

$$\text{Effective volts} = 110 - 108.8 = 1.2;$$

$$I = \frac{E}{R};$$

$$I = \frac{1.2}{0.24} = 5 \text{ amperes.}$$

Prob. 11–9. The impressed voltage across a motor armature is 115 volts; the counter e.m.f. = 112.4 volts; current = 19.5 amperes. What is resistance of armature?

Prob. 12–9. It is desired to find the counter e.m.f. in a motor armature at a certain speed. Armature resistance = 0.24 ohm. Impressed voltage = 115 volts. Current = 26.5 amperes.

Prob. 13–9. If the speed in Prob. 12–9 were reduced one-half and the field remained the same, what current would flow in the armature?

Prob. 14–9. Suppose the armature in Prob. 12–9 were stopped altogether, what current would flow?

139. Armature Reaction of Motors. Brush Lead. We have stated that any generator may be used as a motor. Assume the armature of a bipolar generator to run in a certain direction. Instead of taking a current from it, suppose we send the current through it, only **in the opposite direction** to the e.m.f. it is generating. The machine will then run as a motor and in the same direction in which it was being run as a generator. That is, if current in the machine as a motor flows in the opposite direction to its current as a generator, the machine turns in the same direction in both. Of course, the direction of the armature current in the motor being opposite to that in a generator, it will magnetize the armature core of a motor in the opposite direction. Thus the field of a motor will be distorted in the opposite direction to the field distortion in a generator, and

the axis of commutation will be shifted back **behind the neutral axis.** The brushes of a non-commutating-pole motor must therefore be shifted backward in order to maintain sparkless commutation.

A comparison of Fig. 202 and 203 will make this clear. Figure 202 represents a six-coil armature of a bipolar direct-current generator. Figure 203 represents a six-coil armature of a bipolar direct-current motor. Notice that the direction of rotation is the same in both machines. The direction of the current through the coils

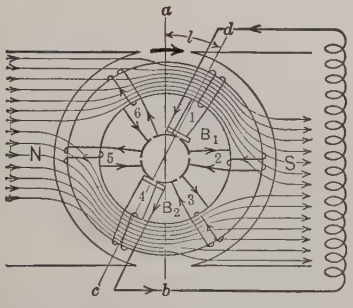


FIG. 202. The brushes of a generator are given a forward lead.

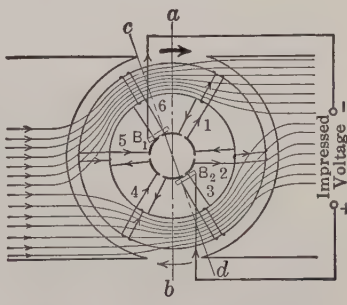


FIG. 203. The brushes of a motor are given a backward lead.

is **opposite.** The field distortion is **ahead** in Fig. 202, and **back** in Fig. 203. The axis of commutation *cd* of the generator (Fig. 202) is **ahead of the neutral axis.** The axis of commutation *cd* of the motor is **behind the neutral axis** *ab*.

The brushes of the generator are thus said to have a **forward lead.**

The brushes of the motor (Fig. 203) have a **backward lead.**

The brushes of the generator in Fig. 202 are set a little **forward even of** *cd*. This is in order to bring the current in the short-circuited coil to zero as soon as possible, by causing a slight voltage to be set up in it in the opposite direction to which the current is flowing. For a similar reason the brushes of the motor are set a little **behind** *cd*.

Consider in Fig. 203 coil No. 6 short-circuited at a certain instant by the negative brush B_1 of the motor. As the coil reaches the brush there must be a current flowing in it in the direction of the current in coil No. 5. In order that this current may stop flowing while the brush short-circuits the coil, the brush is made to short-circuit it just **before** it moves to a neutral spot and ceases to cut lines. Of course the cutting of the lines in a motor always sets up

a back e.m.f. which opposes the current actually flowing. So the e.m.f. now set up in this short-circuited coil No. 6 opposes the current that is flowing in it and quickly brings it to zero. Before this e.m.f. has succeeded in actually setting up a current in the opposite direction the brush has ceased to short-circuit the coil and therefore there is no current to be broken. Thus no sparking results as the segment passes from under the brush.

This shifting of the brushes in a motor not supplied with commutating poles, produces a demagnetizing effect just as it does in a generator. Note that the ampere-turns of coils 1 and 4 of the motor in Fig. 203 oppose the flux through the armature. The amount of this effect is measured for a motor just as for a generator by the product of the number of armature turns within the double angle of brush shift by the current in these turns. As nearly all direct-current motors are now equipped with commutating poles, the brushes are set on the zero axis and this demagnetizing effect is not present. It will not be considered further in this text.

140. Rule for Direction of Rotation of Motors. The rule for finding the direction of rotation of a motor is the same as that for finding the direction of force on a wire carrying a current in a magnetic field.

Point the **forefinger of the left hand in the direction of the flux, the middle finger in the direction of impressed voltage, and the thumb will point in the direction of rotation.**

To change the direction of rotation of a motor, it is necessary to change the **relation** between the direction of the flux and the direction of the armature current. If we change the direction of **both** the flux and the armature current we do **not change the relation** between them and therefore the direction of rotation remains as before.

The rule, then, for reversing the direction of rotation of a motor, is to reverse **either the field or the armature** connections, **not both.**

141. The Shunt Motor. Speed of a Shunt Motor. There are three general types of motors, as there are three general types of generators, **shunt, series** and **compound.** They are named as are generators according to the way the fields are connected. The shunt motor is by far the most common of the three types and will be studied first.

We have seen that the induced voltage of a generator depends upon the strength of the field and the speed of the armature. Similarly, the back voltage of a motor depends upon the strength of the

field and the speed of the armature. We make use of this fact in determining the speed of a motor at any given load.

The magnetization curve is drawn as in Fig. 204. The generated voltage is then divided by the speed and a second scale is added to the ordinates. This scale shows the relation of the volts generated at a speed of one revolution per minute to the field strength. We have now merely to find the back e.m.f. of a motor at any load and divide it by the volts per revolution per minute for the field strength of the motor. The result will be the speed at which the motor must run to generate the computed back voltage.

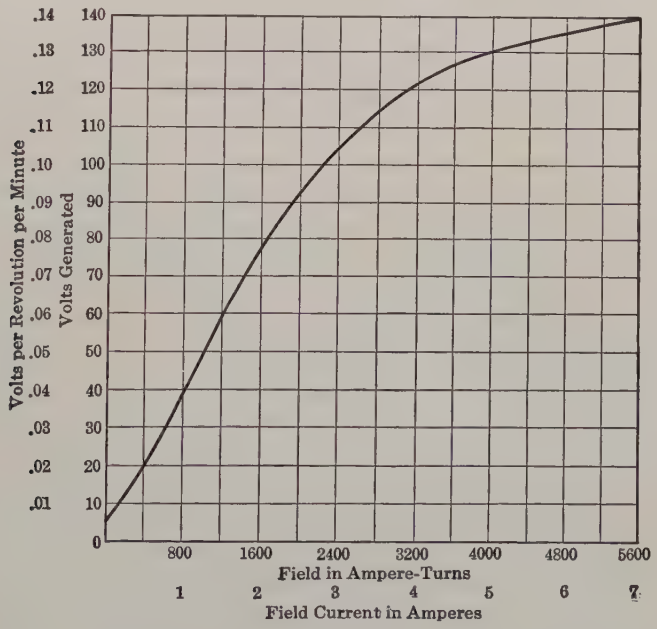


Fig. 204. The magnetization curve of a motor.

Example 5. The curve in Fig. 204 is the magnetization curve of a 2-path 110-volt commutating-pole shunt motor when running at a speed of 1000 r.p.m. The scale on the ordinates indicates the volts per r.p.m. The armature has 240 active conductors and a resistance of 0.12 ohm. Full-load armature current is 40 amperes.

The field including a rheostat has a resistance of 33.8 ohms, and 800 turns per pair of poles. At what speed will the motor run when drawing full-load armature current at 110 volts?

Solution.

The field current when 110 volts are applied $= \frac{110}{33.8} = 3.25$ amperes.

From curve in Fig. 204, we see that 3.25 amperes field current or 2600 ampere-turns generates 110 volts at 1000 r.p.m. or 0.110 volt per r.p.m.

Volts lost in armature $= 40 \times 0.12 = 4.8$ volts.

Back voltage $= 110 - 4.8 = 105.2$ volts.

If one revolution per minute generates 0.110 volt, it will take as many revolutions per minute to generate 105.2 volts as 0.110 is contained in 105.2 volts, thus:

$$\text{Speed} = \frac{105.2}{0.110} = 956 \text{ r.p.m.}$$

The speed regulation of this motor then would be

$$\frac{1000 - 956}{956} = 4.6 \text{ per cent.}$$

The speed at any smaller armature current would be greater than 956 r.p.m.

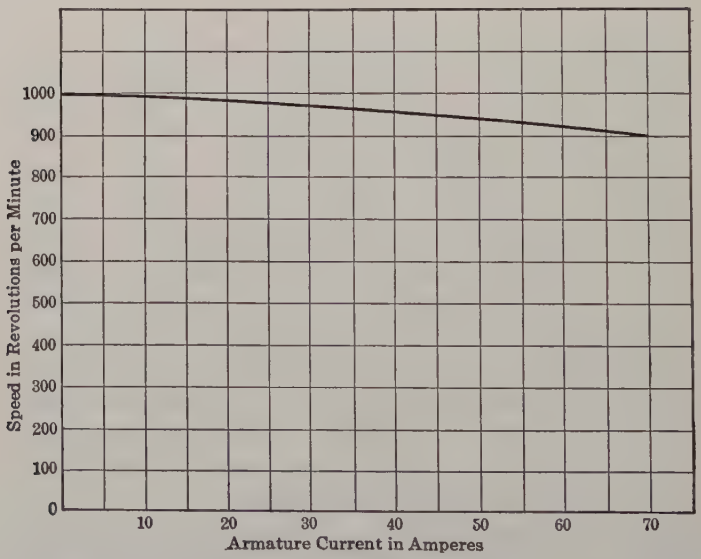


FIG. 205. The relation of the speed to the current of a shunt motor.

The plot between armature current and speed is called the speed-load characteristic of a motor. For a shunt motor the curve has the shape shown in Fig. 205. Note that the speed-load characteristic of a shunt motor is very similar to the voltage characteristic of a shunt generator. The speed of a shunt motor does not fall off quite as rapidly as the voltage of a shunt generator. For this reason a shunt motor is called a " constant speed " machine. Note how-

ever that it is not quite constant but falls off slightly. The curve in Fig. 205 should always be associated in the mind with a shunt motor.

Prob. 15–9. At what speed will the motor of Example 5 run when the armature is carrying 25 amperes?

Prob. 16–9. What is the no-load speed of the motor in Example 5?

Prob. 17–9. If the motor in Example 5 is overloaded so that the armature takes 60 amperes, at what speed will it run?

Prob. 18–9. At what speed will the motor of Example 5 run if 115 volts are applied to it and the armature draws 50 amperes?

Prob. 19–9. If an additional 6.4-ohm rheostat is inserted in the field circuit of the motor in Example 5, at what speed will it run on 110 volts when carrying full-load armature current? What effect has weakening the field upon the speed of a shunt motor?

Prob. 20–9. What resistance must be cut out of the field circuit of Example 5 so that the speed at full-load armature current will be 850 r.p.m. when operating on 110 volts?

142. Starting Box. We have seen that the current which a motor takes in the armature depends upon the difference between the impressed voltage and the back e.m.f. This back e.m.f. keeps the current from being excessive although the armature resistance is very low. We have seen that this back e.m.f. depends upon the speed, since it is caused by the armature conductors cutting lines of force.

When, therefore, the armature is not rotating, there is no back e.m.f. and the current through the armature depends upon the impressed voltage and armature resistance only. Suppose, then, that full voltage were thrown on to an armature which has not started to rotate. Since the resistance is small, the current through the armature would be excessive and might burn out the windings.

Example 6. The armature of a shunt motor contains 0.20 ohm resistance. The motor is to run on a 110-volt circuit. (*a*) Suppose that it is thrown on the circuit suddenly while the armature is standing still, what current will it take?

Solution.

$$I = \frac{110}{0.20} = 550 \text{ amperes.}$$

With the machine running at normal speed on 110 volts there is a back e.m.f. of 107 volts. (*b*) What current does it take at this speed?

Solution.

$$I = \frac{110 - 107}{0.20} = 15 \text{ amperes.}$$

In order to avoid this excessive current at starting, a **starting resistance** is introduced into the armature circuit, which cuts down the voltage across the motor armature at first, and allows but a small current to flow through it. By slowly cutting out this resistance as the motor speeds up and sets up its back e.m.f., it is possible to throw the full voltage across the motor. It is safe to have this high voltage across the motor as long as the speed is high enough to oppose it by a high back voltage, but at low speed (and thus a small back voltage) too much current is likely to be forced through the armature.

Figure 206 is a simple diagram of the starting resistance used with a shunt motor.

When the line switch is thrown and C is swung to the first point, it merely puts the shunt field F on the circuit, thus building up the field immediately. Then when the arm C is swung to the next contact point, the starting resistance SR is put in series with the armature across the line. The resistance SR prevents too large a current from entering the armature. As soon as some speed is acquired by the armature (and therefore a back voltage) the arm C is swung to the next contact point, cutting out some of the resistance. As the motor gets up its full speed the rest of the resistance SR is gradually cut out. Finally the armature is put directly across the line, by swinging the arm C to the point P.

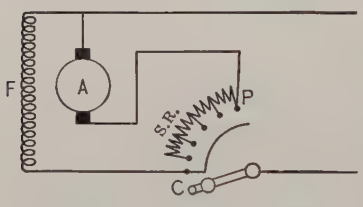


FIG. 206. A simple starting resistance.

The handle should not be held on any of the intermediate points more than a few seconds as the resistance is not designed to carry the necessary current for a longer period.

143. Speed Control of a Shunt Motor. From the two preceding articles it can be inferred that the speed of a shunt motor can be controlled as follows:

1. **An increase or decrease of voltage across the armature** will respectively **increase or decrease** the speed (the voltage across the field remaining constant).

2. **An increase or decrease of field flux** will respectively **decrease or increase** the speed.

144. Speed Control by Voltage Change across the Armature. The simplest way to change the voltage across the armature is to have a variable resistance in series with the armature as in a starting

box. In fact, if the starting resistance is designed to carry sufficient current continuously, the handle of the starting box may be held on an intermediate point and the motor will run at a lower speed.

This method has the advantage of being cheap as to apparatus and easily applied. It has the disadvantages of wasting energy and of producing a poor speed regulation in the motor, as illustrated in Example 7.

To avoid the poor regulation, a system may be used which subdivides the voltage of the bus-bars into two or three values, or an auxiliary motor-generator set with adjustable voltage may be employed, as in the Ward-Leonard system. These systems are expensive but produce good regulation over wide ranges of speed.

Example 7. It is desired to lower the speed of the shunt motor of Example 5 by inserting a resistance of 0.50 ohm in series with the armature as in Fig. 207. (*a*) How much will the speed be lowered at full load armature current of 40 amperes? (*b*) What will be the speed regulation of the motor under these conditions?

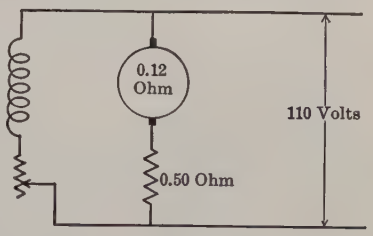


FIG. 207. The 0.5-ohm resistance is in series with the armature only.

Solution.

The field current is 3.25 amperes, as in Example 5, and the volts per r.p.m. are the same, 0.110 volt per r.p.m.

Voltage drop through armature and resistance $= 40 \times (0.12 + 0.50) = 24.8$ volts.

Back e.m.f. $= 110 - 24.8 = 85.2$ volts.

$$\text{Speed to generate 85.2 volts} = \frac{85.2}{0.110} = 775 \text{ r.p.m.}$$

Speed at no-load $= 1000$ r.p.m.

$$\text{Speed regulation} = \frac{1000 - 775}{775} = 29.0\%.$$

Without the 0.5-ohm resistance in series with the armature the regulation was only 4.6%.

Note also that the power consumed at full load by the 0.5 ohm resistance $= 40 \times 40 \times 0.5 = 800$ watts, about 17 per cent of the total power supplied to the motor.

Prob. 21–9. What will be the speed of the motor in Example 7 when it is drawing 25 amperes armature current?

Prob. 22–9. What would be the speed regulation of the motor in Example 7 if instead of the 0.50-ohm resistance being inserted in the armature circuit, a voltage of 90 volts was applied direct to the armature? The voltage across the field remains 110 volts.

145. Speed Control by Adjusting the Magnetic Field. The usual method of speed control of a shunt motor is by varying the resistance of a rheostat placed in the field circuit.

Suppose, by means of a variable resistance in the shunt field, we increase the resistance of the field coils. This causes a decrease of current through the coils, which in turn causes a decrease in the magnetization. Then fewer number of lines are cut per second by the armature coils, and thus the counter e.m.f. is lowered. This allows more current to flow into the armature coils.

This increase of current in the armature coils is much greater in proportion than the decrease in the strength of the magnetic field. Thus the actual force or the torque of the motor is greater, and the armature speeds up until a balance is again obtained. Of course, if the current increased only in proportion as the field decreased, the product of current times field strength (which always determines the force on the armature conductors) would remain constant and there would be no gain in torque, hence no increase in speed when the field strength was decreased.

Therefore, strange as it appears at first sight, the weaker the field, the faster a shunt motor tends to run.

When a slower speed is desired, some of the extra resistance is cut out and more current allowed to flow through the field coils. The armature coils then have to cut through a denser field, and a higher counter e.m.f. is developed, which cuts down the current in the armature and causes it to run at a lower speed.

The regulation of the machine with the speed adjusted by field control is not much inferior to what it would be without the extra field resistance.

There are limits to the changes in speed which can be effected by this means.

(1) It is not feasible to increase the flux density of the iron much beyond saturation, and thus the machine can be slowed down to a certain point only.

(2) On the other hand, when the flux density gets too low in the field, the magnetic disturbances caused by the armature are so great that serious sparking at the brushes results.

When a wide range of speeds is desired by means of field rheostat, the motors are usually equipped with commutating poles and compensating windings on the main poles. In this way a speed range of 6 to 1 can be obtained with practically sparkless commutation. The

action of commutating poles and compensating windings is explained later in this chapter.

Another scheme which takes advantage of this change in field strength to bring about a change in speed, is that used by the Stow Mfg. Co.

The cores of the fields are made of plungers which can be drawn away from the armature, thus increasing the air gap. This, of course, increases the reluctance of the magnetic circuit and thus the flux density decreases and an increase of armature speed results.

By this method most of the sparking at high speeds is avoided. The apparatus, however, for changing the position of the plungers in the field coils is complicated and expensive.

The same result is also obtained by the Lincoln motor by using a tapered armature and moving it axially. This changes the length of the air gap and thus modifies the flux density.

These two latter devices are rather expensive but produce a wide range of speed with sparkless commutation.

Example 8. In order to increase the speed of the motor of Example 5, a resistance of 10 ohms is introduced into the field. Other data as in Example 5. The impressed voltage is 110 volts.

(*a*) What will be the no-load speed?
(*b*) What will be the full-load speed?
(*c*) How much has the field resistance affected the speed regulation?

Solution.

(*a*) No-load speed.

$$\text{The field current} = \frac{110}{33.8 + 10} = 2.51 \text{ amperes.}$$

According to Fig. 204 the voltage per r.p.m. for this field current = 0.093 volt/r.p.m.

The back voltage is practically 110 volts, there being no appreciable armature IR drop.

$$\text{Speed} = \frac{110}{0.093} = 1182 \text{ r.p.m.}$$

(*b*) Full-load speed.

Armature IR drop $= 40 \times 0.12 = 4.8$ volts.
Back voltage $= 110 - 4.8 = 105.2$ volts.

$$\text{Speed} = \frac{105.2}{0.093} = 1131.$$

(*c*) $$\text{Speed regulation} = \frac{1182 - 1131}{1131} = 4.5 + \text{ per cent.}$$

Prob. 23–9. (*a*) What resistance must be inserted in the field circuit of the motor of Example 5 in order to produce a full-load speed of 1200 r.p.m. on a 110-volt circuit?

(*b*) What is the speed regulation of the motor under these conditions? Assume that the armature takes 1 ampere at no load.

Prob. 24–9. (*a*) What resistance must be cut out of the field circuit of the motor in Example 5 in order that it may run at a full load speed of 900 r.p.m. on a 110-volt line?

(*b*) What is the speed regulation of the motor under these conditions?

Prob. 25–9. At what speed will the motor in Example 5 run at full load if put on a 120-volt circuit?

Prob. 26–9. What resistance must be inserted or cut out of the field circuit of the motor in Example 5 if it is to run under full load at 1000 r.p.m. on a 115-volt circuit?

146. "No-Field" Release. Three-Point Starting Box. What would happen if the current in the field coils were reduced to zero, and only the residual magnetism were left in the fields?

We have seen that the armature increases in speed as the field decreases in strength. We would thus expect the armature speed to become excessive if the field became very weak. And this is just what happens to a shunt motor when the field circuit is broken, and the load is light. It immediately races and destroys the armature windings, the centrifugal force pulling them away from the core. Note in Fig. 204, which is the magnetization curve of a shunt motor, that if we take all the current from the field coils, the volts per r.p.m. become 0.006. To equal the impressed voltage of 110 volts the speed must be $\frac{110}{0.006} = 18{,}300$ r.p.m. Long before this speed is reached the windings would be pulled out of the slots in the armature.

Of course, if the motor is heavily loaded, it stops and the greatly increased current in the armature coils causes them to burn up. This event of "no field" must therefore be guarded against by some device, which will break the armature circuit automatically as soon as the field circuit is broken, and thus avoid the racing or the burning of the armature coils.

Figure 208 shows a device of this kind. The field current is led through a small electromagnet M on the starting box. The swinging arm C has a soft iron keeper K attached to it. When the arm has come into the running position and all the starting resistance SR is cut out, the keeper K comes in contact with the electromagnet M which holds the arm in this position, acting against the tension of a spring.

If anything happens to break the current in the field coils *F*, the current in the electromagnet is also broken, and the swinging arm is released and pulled away by the spring. This action breaks the armature circuit and thus stops the motor.

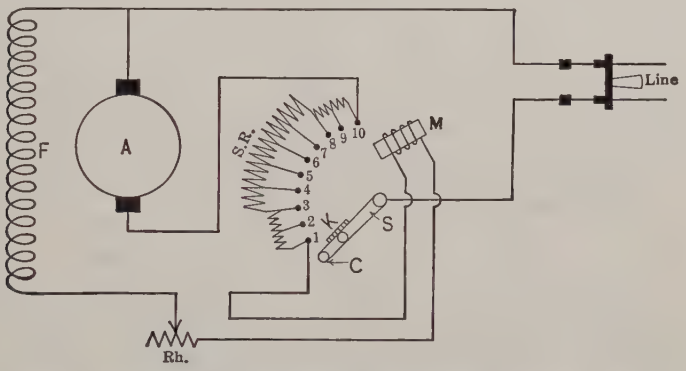


FIG. 208. Diagram of connections of a three-point starting box with a "no-field" release.

A starting box equipped with a "no-field" release is called a "**three-point box**" because it has three studs from which leads are to be carried to the line and to the motor. Figure 209 is a connection diagram of such a box. Note that one lead goes from the box to the **field** of the motor, another to the **armature** of the motor and the third

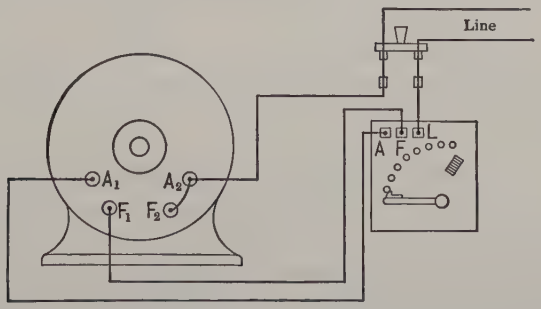


FIG. 209. A three-point starting box connected to a shunt motor.

to **one side** of the line. The other field and armature terminals are **connected together** and then connected to the **other side** of the line.

147. No-Voltage Release. Four-Point Starting Box. Besides the danger that the field of a shunt motor may be opened there is also the danger of the voltage going off the line, with the consequent slowing down and stopping of the motor. If the arm of the starting

box remains in the running position after the voltage has gone off, and the motor slows down or stops, the motor is likely to be burned up when the voltage is again thrown on, because all the starting resistance is cut out with the arm in this position. To take care of this emergency, the small retaining magnet instead of being in series with the field is connected directly across the line. When the voltage on the line lowers to a dangerous extent, the current through this magnet is diminished and the magnet so weakened that it releases the arm which is pulled back by a spring and cuts the motor from the line. This device is called a "**no-voltage release**" and the box is called a **four-point starting box.** Figure 210 is a diagram showing the

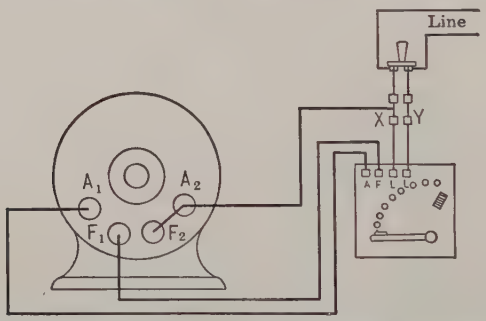


FIG. 210. A four-point starting box connected to a shunt motor.

connections from a four-point box with the no-voltage release. Note that one lead from the box is brought out to the **armature,** another to the **field** and the other **two** to the **line.** If the motor does not start when the arm is thrown to the first point, transfer the lead from the point marked *X* on Fig. 210 to the other side of the line marked *Y*.

Since the current in the retaining magnet is not affected by any changes in the resistance of the field, the no-voltage type of release is generally preferred. There are many types of automatic starting boxes in the market which automatically throw the field on and cut resistance into the armature circuit and then cut it out at the proper rate until the full voltage is across the armature. These are too complicated to be explained here. For more complete information, the student is referred to the various Handbooks on Electrical Engineering.

148. Overload Release. Many starting boxes are supplied with an **overload** release as well as a "no-voltage" or "no-field" release. This arrangement prevents putting too great a load on the

motor which might slow it down to such an extent that the armature current would become excessive. Figure 211 shows an overload release applied to a four-point box. When the current taken by the motor reaches a value which is likely to harm the armature windings, the plunger P is drawn up into the coil. This short-circuits the no-voltage release coil and allows the starter arm to be thrown to the "off" position by a spring. The same arrangement can be applied to a three-point box.

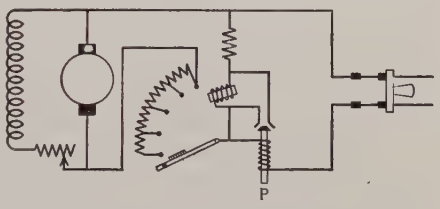


FIG. 211. Diagram of a starting box with no-voltage and overload releases.

149. Directions for Starting and Stopping a Shunt Motor. Start slowly, moving the starter arm slowly as the motor comes up to speed. Do not leave it on any intermediate point for more than 3 or 4 seconds, or the starting resistance may be burned out.

Always stop the motor by opening the line switch and allowing the starter arm to snap back when the electromagnet releases it. **Never** pull the starter arm back, as this causes bad arcing at the first contact point, which roughens and eventually destroys the copper contact.

150. Series Motors. No-Load Speed. Series motors, unlike shunt motors, have no definite "no-load" speed. When unloaded, a series motor will continue to increase in speed, until it destroys the armature by centrifugal force acting on the windings. Very small series motors ($\frac{1}{10}$ horsepower and under) usually have friction and other losses large enough at no load to keep the speed down to a safe limit.

The racing of a larger motor may be understood by considering the behavior of the field in an unloaded series motor. The motor starts up as the current is sent into the armature and field coils. There being no opposing torque, since the motor is unloaded, the speed tends to increase until the back e.m.f. equals the impressed voltage. But the ever increasing back e.m.f. decreases the current in the field and armature. As the field weakens, it requires an increased speed to set up the desired back e.m.f. The field continues to get weaker and the speed continues to increase in an effort to build up a back e.m.f. equal to the impressed voltage, until the centrifugal force wrecks the machine. Thus a series motor is used only where **the load is always attached** to the armature, as in a fan, a railway car, an

electric crane, etc. Series motors are sometimes equipped with " no-load " releases.

We have seen that an unloaded shunt motor tends also to speed up till its back e.m.f. becomes equal to the impressed voltage. But since the field remains constant, this equality is approximately reached and maintained at a certain definite speed.

Prob. 27–9. If the series generator of Prob. 10–8 were run as a series motor on a 735-volt line, what would be the speed when the current is 6 amperes?

Prob. 28–9. Draw the wiring diagram of a series motor equipped with a starting box, having a " no-load " and an " overload " release. Show inside wiring of box and motor.

151. Speed-Load Characteristic of a Series Motor. The relation between the speed and the load of a series motor is shown by Fig. 212. Note that the no-load speed is too high to be plotted, and that the speed falls rapidly with the first increases of load, and then more gradually as the load gets in the vicinity of full load. Figure 212 should always be associated in the mind in connection with series motors. This curve is obtained in the same manner as the speed-load curve for shunt motors.

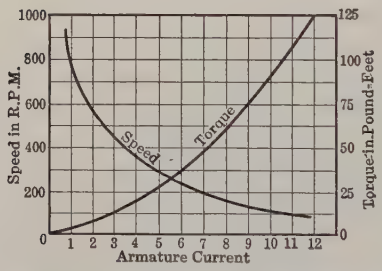


FIG. 212. Curves showing the relation of torque and speed to the armature current in a series motor.

Prob. 29–9. The magnetization curve of a certain series motor running at 1200 r.p.m. may be obtained from the following data.

Ampere-turns in Field per Pair of Poles	Generated Volts
0	5
2,400	46
4,800	85
7,200	121
9,600	140
12,000	150
14,400	155
16,800	159

Pressure is 125 volts.

Armature resistance including brush contacts = 0.048 ohm.

Resistance of series field = 0.012 ohm.

No. of series turns per pair of poles = 40.

(*a*) What is the theoretical speed at no load?

(*b*) Plot the speeds for armature currents of 100, 200, 300, and 400 amperes.

Prob. 30–9. Plot the speed-load characteristic for the series generator of Prob. 10–8 when run as a series motor on a 600-volt line.

Prob. 31–9. Plot the speed-load curve of the series motor of Prob. 30–9, run on a 500-volt line. What effect does the line voltage have on the speed of a series motor?

152. Comparison of Shunt and Series Motors. We have seen that the force on the armature conductors, and therefore the torque, of any motor is proportional to the current I in the armature times the average field strength B. Thus for any motor, we may write the relation:

Torque varies as BI.

In the case of a shunt motor operated on a constant-voltage line, the field current is constant, and the field strength is practically constant. Hence if the armature current is doubled the torque also doubles and we therefore write the above relation for a **shunt** motor:

Torque varies as I.

Figure 213 illustrates this relation for a shunt motor. The line marked "torque" shows how the torque varies as the armature current is changed. Since the torque-current line is straight, the torque is exactly proportional to the current.

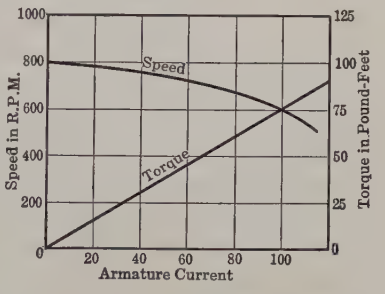


FIG. 213. Curves showing the relation of torque and speed to the armature current in a shunt motor.

Comparing this with Fig. 212 which is for a **series** motor, we see that the torque line is not straight, so the torque is not proportional to the armature current. At low values of armature current and torque the line curves upward very rapidly, but at higher values of current and torque the line becomes more nearly straight. We can explain this variation as follows:

Since the field and armature of this motor are connected in series, the same current passes through both. If the armature current increases, the field current also increases by the same amount. But this increase in field current causes an increase in the flux density B, so that in the general expression for the torque variation of any motor we see that both B and I increase. As a result, the torque

increases more rapidly than the current, instead of being proportional to the current as in the case of a shunt motor.

We also noticed in Fig. 212 that the torque line curved rapidly at low-current values but became more nearly straight as the current increased. This can be explained by referring to the magnetization curve of a motor (Fig. 204). The part of the magnetization curve which corresponds to low field current (hence low armature current in the series motor) is very nearly straight. That is, at low field current, the flux density is nearly proportional to the current. Since the torque varies as B times I, and B itself varies nearly as I for low currents, we see that in the first part of the torque curve in Fig. 212:

Torque varies nearly as I^2.

As we continue to increase I, and hence the field current, the flux in the magnetic circuit of the motor begins to saturate the iron. Thus we see that the magnetization curve of Fig. 204 bends over at larger values of current and the flux density tends to become almost constant. Since the torque in general varies as B times I, and B has approached a constant value, we conclude that at large current values the torque varies merely as I. This is shown to be true in Fig. 212 because the torque line at large currents is nearly straight, or, the torque is proportional to I.

The torque-current characteristic of a series motor curves upward rapidly at low currents because the flux density increases nearly as rapidly as the current and the torque hence varies nearly as I^2. At larger currents, however, the flux density becomes nearly constant and the torque increases only in proportion to the armature current, thus producing a straight line variation as in the shunt motor.

Figures 212 and 213 also show how the speed varies with load for the series and the shunt motor. In a shunt motor operating on a constant-voltage line, the magnetic flux is practically constant. If the speed should decrease, the back e.m.f. would decrease and hence allow more current to flow in the armature. The torque would increase in proportion to the increase in armature current, and due to the increased torque the armature would tend to increase its speed. Thus the shunt motor is essentially a constant speed device.

The speed of a series motor decreases much more rapidly than that of a shunt motor, as the load is increased (Fig. 212). In a series

motor, when the armature current increases, the field current increases the same amount. An increase in field current increases the magnetic flux, and if the speed should remain constant the back e.m.f. would also increase because it is proportional to both speed and average flux. But an increase in back e.m.f. would (if motor is connected to a constant-voltage line) decrease the armature current and hence decrease the torque. Due to this tendency to lose torque at constant speed, the speed must decrease until the increased armature current and flux density produce a torque sufficient to satisfy the load.

In the shunt motor, the torque to supply additional load is produced by an increase in armature current with only a slight change in speed. In the series motor both armature current and flux density increase as additional torque is required, and the speed drops rapidly.

153. Applications of Shunt and Series Motors. The shunt motor is especially applicable whenever fairly constant speed operation under variable loads is required. Of course, the motor must be large enough to take care of the heaviest load. A typical example of this application is in machinery drives. Machine tools such as lathes, milling-machines, etc., require a motor which will maintain an efficient cutting speed under widely variable load conditions, and this requirement is well satisfied by the shunt motor.

The horsepower output of any motor is proportional to torque times speed (see Art. 158). In the case of the shunt motor, speed is constant, so if the torque increases in order to carry additional load, the horsepower output of the motor increases. A series motor, on the other hand, decreases in speed under additional load. The torque increases, of course, but the product of speed times torque does not increase as rapidly for the series motor as it does for the shunt motor.

This is one of the important characteristics of the series motor which has led to its wide use in electric railway work. In ordinary starting, accelerating, and running on level track, the performance of both shunt and series motors is about the same, and there is no appreciable difference in size between the two types. When operating on steep grades, however, the horsepower output of the shunt motor must increase in proportion to the additional torque required, because the speed remains practically constant. A motor large enough to carry this temporary load is much larger than is needed for average conditions. On the other hand, the series motor carries this extra load by decreasing its speed when the torque increases and

thus working at a slower rate. Consequently a series motor which is adequate for a given railway application is in general smaller than the shunt motor which would be required.

There are other factors which favor the series motor for railway use. When the trolley or third-rail collector passes over a cross-over or section insulator with the controller "on," the power is shut off and suddenly reconnected. If a shunt motor is used, its field flux is maintained while the power is off, because it runs during this time as a self-excited shunt generator. When the power is restored, the armature current builds up very rapidly and the sudden application of torque thus produced causes a jerk which is unpleasant to passengers and harmful to gears, wheels, and couplings. When series motors are used, the additional inductance of the series field circuit prevents a rapid increase of current as the voltage suddenly comes on again, (see Chapter XII). Thus the torque is developed more slowly after power interruption and jerky operation is avoided.

It is common practice in railway work to have more than one motor driving a single car, or locomotive. These motors are mounted so that they drive different sets of wheels. If all the wheels are of the same diameter, the motors will all run at the same speed. Due to wear and repair work, however, it is frequently found that the wheels driven by one motor are of slightly different diameter from those driven by another motor. Since all the wheels are fastened to the car at the journals, they must all move the same distance in the same time. In order to do this, the smaller wheels must rotate faster than the larger, and the motors geared to the different sets hence run at slightly different speeds. We can see from the speed-current curve for the series motor, Fig. 212, that slight changes in speed produce very little change in armature current. When two series motors are used to drive a car with wheels of unequal diameter, both motors will take practically the same current from the line and each motor will do an equal share of the work. With shunt motors, we see from the speed-current curve of Fig. 213 that a small change in speed causes a much larger change in armature current. Since the slower motor generates a lower back e.m.f., it will draw a larger line current than the faster shunt motor. The motors hence do not divide the load equally, and the motor connected to the larger wheels must carry more than its share, because it runs at a slower speed.

Of course, when the armatures of the motors are connected in series for starting, as described in the next section, the same current

passes through both armatures, and either type of motor will divide the load equally. After starting, however, both motors are connected directly across the line, and when so connected unequal division of load is possible if shunt motors are used. This condition is undesirable because it requires that larger motors be used throughout so that in case the load divides unequally, any motor will be able to take more than its normal share of the total load.

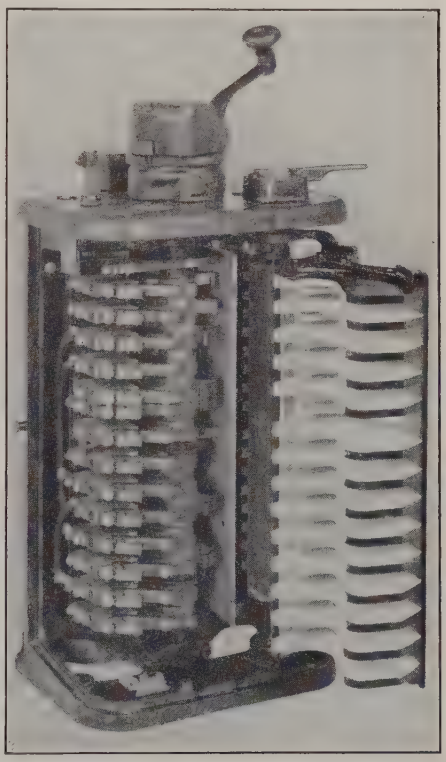


FIG. 214. A railway controller.

154. Series-Parallel Control for Electric Cars. Most electric cars have at least two motors. The controller shown in Fig. 214 is in the front of the car and is operated by the motorman as follows:

When the controller handle is advanced to the first notch, it places the two motors A and B in series with each other and with the starting resistance SR as in Fig. 215. As the handle is advanced, it gradually cuts out the starting resistance until the car gets up a speed of about 10 miles an hour on the level. Then the next notch puts the two motors in parallel with each other and again in series with the resistance SR as in Fig. 216. To reach the running position, the resistance is again cut out by advancing the handle.

The scheme of putting the two motors in series at the start, allows the car to be started on half the current it would take to start with them in parallel and thus preserves a more even distribution of current in the trolley system, and wastes much less power. It really makes one motor act as starting resistance for the other, at the same time helping it to supply tractive effort.

155. Caution in Use of Series and Shunt Motors.

Never open the field circuit of a shunt motor. A shunt motor races when the field circuit is broken and the armature is left on the line.

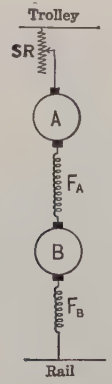


FIG. 215. Two series motors are in series with each other and resistance *SR* on starting.

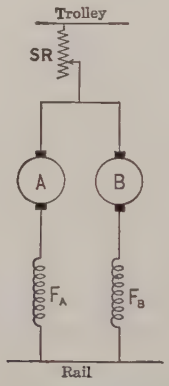


FIG. 216. When the car has reached a speed of 10 miles per hour the motors are thrown in parallel.

Never start an unloaded series motor, and never remove the load from a series motor while it is running.

A series motor races when unloaded.

156. Compound Motors. Differential and Cumulative. Where more nearly constant speed is desired under widely varying loads than can be obtained from a shunt motor, a **differential compound** motor may be used.

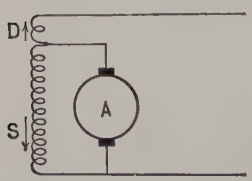


FIG. 217. A differential compound motor.

This motor has a set of series field coils "bucking" the shunt coils. The ampere-turns of the series coils *D*, Fig. 217, oppose the ampere-turns on the shunt coils *S*. (The arrows show direction of magnetomotive force, not current.) The field strength due to these series coils increases as the load increases and, acting against the field due to the shunt coils, weakens the total field enough to allow an extra current to flow through the armature. This current will create enough torque to overcome the extra drag on the pulley and keep the speed constant. Of course the shunt field *S* is much stronger than the series field *D*. It, however, is possible, by making the series

field large enough, to cause the speed to increase with the load. In the main, shunt motors furnish for most purposes a speed which is constant enough at all loads.

A differential compound motor then has exaggerated shunt characteristics; the starting torque is comparatively low (lower even than the starting torque of a shunt motor); the speed regulation is exceedingly good (being better than the speed regulation of a shunt motor).

As the starting torque of a series motor is greater than that of a shunt motor, series coils are sometimes added which "aid" the shunt coils instead of "bucking" them. The motor is then called a **cumulative compound motor.** The ampere-turns of the series coils *C*, Fig. 218, aid the ampere-turns on the shunt coils *S*. Such an arrangement gives the motor the advantage of the large starting torque of a series motor, without the disadvantage of the machine "racing" on no load, since the shunt coils maintain the field. The characteristics are somewhat like those of the series motor; the speed slowing down as the load increases, as in a straight series motor. To avoid this, the series field is often short circuited, after the motor has been started. The motor then starts as a series motor but runs as a shunt motor.

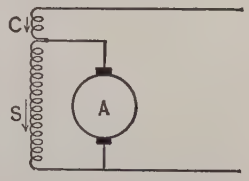


FIG. 218. A cumulative compound motor.

Prob. 32–9. Plot the speed-load characteristic of the compound generator described in Prob. 24–8, when run as a cumulative compound motor on a 115-volt line.

Prob. 33–9. Plot the speed-load characteristic of the generator of Prob. 24–8, when run as a differential compound motor on a 115-volt line.

Prob. 34–9. How many series turns per pair of poles must be added to the shunt motor in Example 5, Chapter IX, in order that it may run at the same speed at full load (40 amperes through armature) and at no load on a 110-volt line.

Prob. 35–9. If the series turns on the motor of Prob. 34–9 are reversed what will be the speed regulation?

157. Comparison between Generators and Motors. It is to be noted that generators and motors of the same type have very similar characteristics. We have merely to read **speed** in motors for **voltage** in generators. The following table is intended to bring out this relation:

Shunt

Generators. Constant **voltage**; good **voltage** regulation. **Voltage** controlled by field variation.

Motors. Constant **speed**; good **speed** regulation. **Speed** controlled by field variation.

Series

Generators. **Voltage** depends entirely upon load, **increasing** with the load.

Motors. **Speed** depends entirely upon load, **decreasing** with increasing load.

Compound

Generators. **Voltage** may be made practically uniform throughout wide range of loads.

Motors. (**Differential.**) **Speed** may be made practically uniform throughout wide range of loads.

158. Efficiency of Motors. Brake Horsepower. By the efficiency of a motor is meant the same thing that is always meant by efficiency, *i.e.*, the relation of the output to the input.

$$\text{Efficiency} = \frac{\text{output}}{\text{input}}.$$

Of course both output and input must be measured in the same units. As we usually measure the input of a motor in watts and the output in horsepower, we have to reduce either the watts input to horsepower or the horsepower output to watts. Thus, a motor which requires an input of 5 kilowatts in order to produce an output of 5 horsepower does not have 100 per cent efficiency, because the output and input must be computed in the same units.

Either both in horsepower

$$\text{Efficiency} = \frac{5 \text{ horsepower}}{5 \times 1.34 \text{ horsepower}} = 74.6 \text{ per cent},$$

or both in kilowatts

$$\text{Efficiency} = \frac{5 \times 0.746}{5} = 74.6 \text{ per cent}.$$

The input is measured by an ammeter and a voltmeter; the product of the amperes times the volts being the watts input. The output may be measured, if the motor is not too large (not over 200 horsepower), by a "Prony brake." A simple form of a Prony

brake is shown in Fig. 219. A rope is wound around the pulley and two spring balances A and B are attached, one to each end of the rope. The balances should be parallel to each other. The tension on the rope is adjusted by swivels, until the desired load is secured, the resistance to motion consisting of the friction of the rope on the pulley. This resistance, or force used by the motor to drive the pulley in the given direction, must be the difference between the readings of the two balances, because the force indicated by balance B aids in the turning and that indicated by balance A resists the turning.

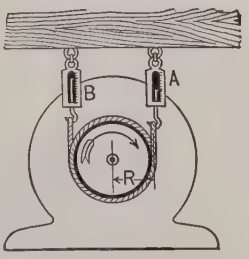


FIG. 219. A Prony brake for measuring the horsepower of a motor.

The speed of the pulley in revolutions per minute is taken, and the distance R (from the center of pulley to the center of rope) is measured. We measure to the center of the rope because it is there that the balances are measuring the forces. The work done in one revolution of the pulley is equal to the force exerted by the pulley times the distance around the pulley. Force is to be measured in pounds, distance in feet.

$$W = (A - B) \times (2\,R \times 3.14) = 6.28\,R\,(A - B).$$

The work done in one minute equals

$$6.28\,R\,(A - B) \times \text{r.p.m.}$$

$$\text{Horsepower} = \frac{6.28\,R\,(A - B) \times \text{r.p.m.}}{33{,}000}.$$

Example 9. A motor takes 20 amperes at 220 volts. It is rigged with a prony brake like that in Fig. 219, and runs at a speed of 1200 r.p.m. with a reading on balance B of 2.00 pounds, and on balance A of 50.0 pounds. The distance $R = 4.5$ inches. Find the efficiency.

Solution.

$$R = 4.5 \text{ inches} = 0.375 \text{ foot.}$$

$$\text{Output} = \frac{6.28 \times 0.375\,(50 - 2) \times 1200}{33{,}000}$$

$$= 4.11 \text{ horsepower.}$$

$$\text{Input} = \frac{220 \times 20}{746}$$

$$= 5.90 \text{ horsepower.}$$

$$\text{Efficiency} = \frac{4.11}{5.90} = 69.7 \text{ per cent.}$$

Prob. 36–9. A motor is fitted with a prony brake as shown in Fig. 219. When running at a speed of 1180 r.p.m. the balance A reads 32.6 pounds and B reads 3.7 pounds. R = 14 inches. The motor takes 35 amperes at 230 volts. What is its efficiency?

Prob. 37–9. (*a*) What torque has the motor in Example 9?

(*b*) What torque has the motor in Prob. 36–9?

(*c*) How can you determine the horsepower of a motor, from a given speed and torque?

Prob. 38–9. (*a*) Find the horsepower from the speed-torque curves of the shunt motor of Fig. 213 at an armature current of 90 amperes. (*b*) What is the efficiency of this motor at this load, if it is running on a 110-volt line? Field resistance = 82 ohms.

Prob. 39–9. What efficiency has the series motor of Fig. 212 when drawing a current of 10 amperes on a 220-volt line?

Prob. 40–9. What is the efficiency of the motor of Fig. 212 when drawing 5 amperes from a 440-volt line?

159. Losses in a Motor.

Copper Losses. The losses due to forcing the electric current through the windings of the armature and field are called the copper losses, because the windings are generally made of copper. These losses are proportional to the square of the current. Thus the copper loss in the field equals the square of the field current multiplied by the resistance of the field; and the copper loss in the armature equals the square of the armature current multiplied by the resistance of the armature. In the form of an equation,

$$\text{Copper loss in field} = I_f^2 R_f,$$
$$\text{Copper loss in armature} = I_a^2 R_a.$$

Hysteresis Loss. Since the armature is revolving in a magnetic field it becomes magnetized first in one direction through it and then in the other. As we have seen, this causes a loss called hysteresis loss, in the iron or steel of which the armature core is composed. The field poles and yoke, being magnetized always in the same direction and practically to the same intensity, do not have a hysteresis loss.

The equation for hysteresis loss per cubic inch of iron is

$$P = \frac{0.83\ K f B_{\text{max.}}^{1.6}}{10^7}$$

in which

P = power in watts.
K = Steinmetz hysteresis constant.
f = frequency, or double alternations per second.
$B_{\text{max.}}$ = maximum flux density in lines per square inch.

Wrought iron and annealed sheet steel of which armature cores are made have low hysteresis constants, so that the loss is usually low — less than one per cent.

Eddy Current Loss. We have seen that when any conductor cuts lines of force, there is always set up an induced electromotive force which tends to send an electric current along the conductor. This is the fundamental principle underlying the construction of the generator.

The copper wires of the armature which are to carry this induced current are wound on an iron core. This iron core is, itself, an electrical conductor, and when it revolves in a magnetic field, it cuts lines of force. This sets up an e.m.f. across certain portions of the core which tends to send an electric current through the core.

FIG. 220. A laminated pole core. *Westinghouse Electric & Mfg. Co.*

Currents thus set up are called **Foucault,** or **eddy currents,** and if they are allowed to flow around in the iron they will soon heat up the core and damage the windings on it. Also, it is always true that there is required an expenditure of power to set up an electric current. As we cannot use these eddy currents within the armature core, the power used in generating them is a dead loss. It is therefore necessary to reduce these eddy currents to as small a value as possible.

This is done by laminating the core; that is, building it up of thin sheets of iron, insulated from one another by japan or by a scale on the iron. This does not greatly affect the magnetic circuit,

but cuts down these electric currents, since the laminations are "transverse" to the direction in which they tend to flow. The thinner the laminations, the more effective is this method in decreasing the eddy current loss. Figure 220 is a photograph of a pole core laminated in this way.

There is no formula sufficiently accurate to determine the eddy current loss in the core of a given generator, largely owing to the impossibility of knowing the insulation resistance between the plates. For this reason the designer of a generator usually "estimates" rather than "calculates" the probable eddy current loss of his machine by means of tables made up from data measured on similar machines.

It is well to remember, however, that the eddy current loss is proportional to the **square of the speed,** to the **square of the flux density,** and to the **square of the thickness** of the laminations.

When slotted armature cores are used, it is customary to laminate the pole faces as well as the armature core. Figure 221 explains the reason for this.

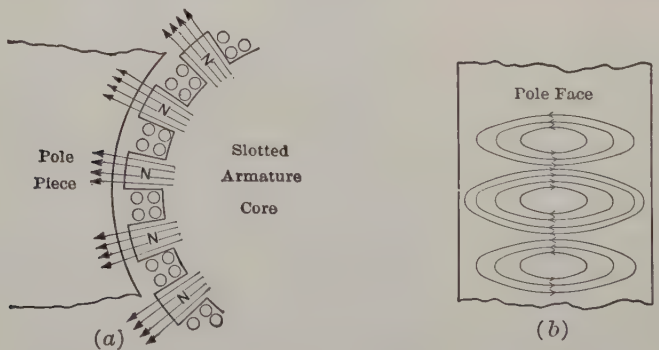


FIG. 221. Slotted-core armatures produce eddy currents in the pole shoes.

The teeth, between which the conductors lie, become more densely magnetized than the slots, and act like small poles. When they move across the pole face, therefore, the pole face is really cut by lines of force and small eddy currents are set up as shown in Fig. 221(*b*). Laminating the pole face breaks up the electric circuits and diminishes these currents.

Mechanical Losses. In addition to the above there are also certain losses such as the power used to overcome the brush friction, the friction in the bearings and the friction of the air. The sum total of these is usually spoken of as the mechanical losses.

160. Computing the Efficiency by Means of the Losses. Constant Speed Machines. In any machine

$$\text{The output} = \text{input} - \text{losses}$$

and

$$\text{The input} = \text{output} + \text{losses}.$$

The efficiency $\left(\text{which is always } \frac{\text{output}}{\text{input}}\right)$ therefore may be written,

$$\frac{\text{output}}{\text{input}} = \frac{\text{input} - \text{losses}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}}.$$

In some cases it is easier to measure the input and the losses. In this case the output is found by subtracting the losses from the input. Wherever the output and losses can be measured easily, they are merely added to find the input.

Certain losses in a generator or motor remain practically **constant** at all loads providing the speed does not materially change. The **constant losses** in a shunt motor or generator are

Field I^2R loss,
Eddy current loss,
Hysteresis loss,
Mechanical loss.

These may be found as follows:

Run the machine as a motor with no load; then all the power put into it must go to supply the above constant losses plus the I^2R loss in the armature. Knowing the current through the armature and the resistance of the armature we can compute the I^2R loss in the armature and subtract it from the total input into the motor at no load. The remainder will be the constant losses, which, of course, will remain practically the same at all loads.

The motor is now run at a given load. The constant loss remains as computed at no load and the new armature I^2R loss is computed from the new value of the armature current.

The sum of the constant losses and armature I^2R loss equals the total losses. This subtracted from the **input** gives the output. The efficiency therefore $= \frac{\text{input} - \text{losses}}{\text{input}}$.

Example 10. A 115-volt shunt motor has an armature resistance of 0.13 ohm, a field resistance of 84 ohms. The motor takes 5.2 amperes when running idle on a 115-volt line. What is the efficiency of the motor at full load when the current is 60 amperes?

Solution.

The field takes $\frac{115}{84} = 1.37$ amperes.
Armature current $= 5.2 - 1.37 = 3.83$ amperes.
Armature I^2R loss $= 3.83^2 \times 0.13 = 1.9$, approx. 2 watts.
Power taken at no load $= 5.2 \times 115$
$= 598$ watts.
Constant losses $= 598 - 2$
$= 596$ watts.
Total input at 60 amperes $= 60 \times 115 = 6900$ watts.
Armature current $= 60 - 1.37 = 58.63$ amperes.
Armature I^2R loss $= 58.63^2 \times 0.13$
$= 447$ watts.
Total loss $596 + 447 = 1043$ watts.
Output $= 6900 - 1043 = 5857$.
Efficiency $= \frac{\text{output}}{\text{input}} = \frac{5857}{6900} = 85\%$.

Prob. 41–9. What horsepower will the motor of Example 10 deliver when drawing 50 amperes on a 115-volt line?

Prob. 42–9. If the motor in Example 10 were used as a generator and delivered 55 amperes to the line at 115 volts, what efficiency would it have?

Prob. 43–9. A shunt motor on a 112-volt line takes 45 amperes at full load and 3 amperes at no load. The field resistance is 108 ohms and the armature resistance is 0.142 ohm. (*a*) What power does the motor deliver at full load? (*b*) What efficiency has the motor at full load?

161. Efficiency of Variable Speed Machines. The losses which are constant in a constant-speed machine (mechanical, hysteresis, eddy-current and field I^2R losses) are all variable in a variable speed machine, but they may be found as follows.

A series motor may be run at a given load on a given voltage and the speed and current noted. It can then be run unloaded as a separately excited motor with the field excited to value when loaded. Then enough voltage is applied to the armature to drive it at the given-load speed noted above. The input at no load is found from armature current and voltage, and field current and voltage. Subtract the I^2R loss of the armature from this value of the input and the result is the mechanical, hysteresis, eddy-current and field losses at the given load. The total loss at given load can be found by adding to the above the I^2R armature loss at the given load.

162. Commutation. In discussing the armature reaction caused by the current in the windings on the armature, we saw how part

of it consisted of cross ampere-turns which shifted the field and the neutral axis, forward in a generator, and backward in a motor. When the machine is not supplied with commutating poles, in order that no large electromotive forces be generated in the coils which must necessarily be momentarily short circuited by the brushes, we move the brushes to a new neutral axis. In fact we always move them a little beyond this new neutral axis so that the short-circuited coils actually cut flux lines in the opposite direction to that in which they have just been cutting, and thus tend to stop the current which has been flowing in the coils and set up a current in the opposite direction. This process is called **commutation,** the flux lines into which we move the short-circuited coils are called **commutation flux,** and the reverse voltage set up is called **commutation voltage.**

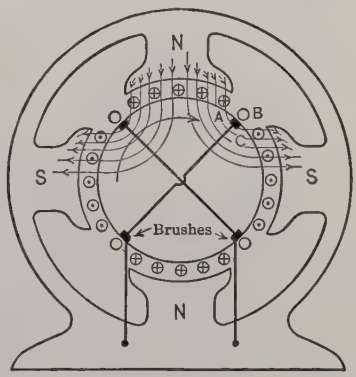


FIG. 222. A four-pole generator without commutating poles.

This is illustrated in Fig. 222. In the conductors opposite North poles an e.m.f. is induced which tends to send a current **into** the paper. In the conductors opposite South poles the induced e.m.f. tends to send a current **up out** of the paper. In passing through the space between a North pole and a South pole the currents must change from **in** to **out.** Thus in conductor A there is a current of **in.** When A reaches position occupied by B it will be short circuited by a brush, and when it comes out from under the brush it will be in the position occupied by C and will have a current of **out** as marked. During the time it was under the brush the current must have changed from **in** to **out.**

In order that this may be done, it is necessary to have the conductor cut flux from the South pole. So the brushes are not set as in Fig. 222, but are set a little nearer the following pole (in the direction of rotation). The conductor A would not be short circuited while occupying the position of B but when occupying a position nearer C, in the flux of the South pole, which would tend to reverse the current of **in** and set up a current of **out.**

This method of shifting the brushes forward to produce commutating voltage is not very satisfactory, for the following reason.

The more current the armature is carrying, the greater the voltage required to reverse it during the time the brush short circuits the coil. But the commutating flux actually gets smaller as the armature current gets larger because the larger armature current shifts the field farther away from the short-circuited coil. Thus the pole tip opposite conductor *C* in Fig. 222 gets weaker instead of stronger as the current in the armature becomes greater. In order to produce good commutation the commutating flux has to be greater as the current to be reversed becomes greater. To produce this result we could regulate the amount of the shift of the brushes according to the current the armature carries, pushing the brushes further and further forward as the load comes on. This is the way it was formerly done but it was a very inconvenient method. The modern way is to supply the generator or motor with commutating poles.

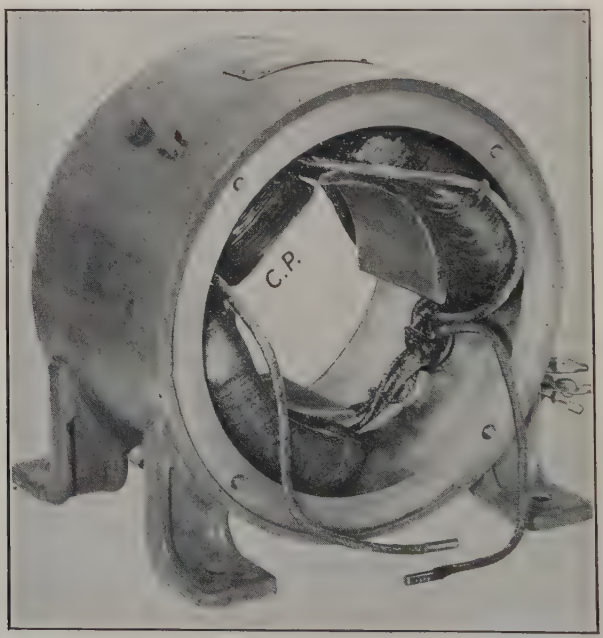


FIG. 223. A two-pole commutating-pole motor. *General Electric Co.*

163. Commutating Poles. Small poles, as the pole marked *C.P.* in Fig. 223 and 224, are set in the space between the main poles. In a generator, these commutating poles will be of the same polarity as the next pole ahead in the direction of rotation, but in a motor they

will have the polarity of the pole behind. Thus in Fig. 224, the commutating poles will be South poles if the machine is a generator and North poles if it is a motor. The reason is simple. In the case of a generator we shift the brushes forward to get the short-circuited coil into the flux of the following pole. The commutating pole in the case of a generator acts like a projection from the following pole. By being separate from the following pole it has the advantage that the winding on it can be put in series with the armature.

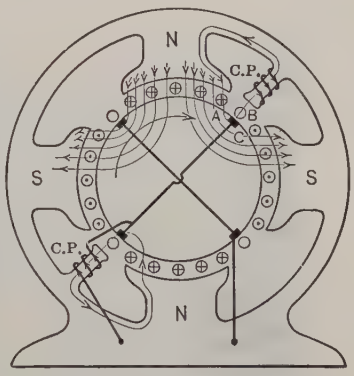


Fig. 224. Diagram of a four-pole commutating-pole generator.

The flux produced by it is thus nearly proportional to the armature current. Thus instead of getting a weaker commutating flux when the armature current increases, we get a stronger flux which sets up a greater commutating voltage in the short-circuited coil. This is the condition which makes commutating poles produce sparkless commutation without shifting the brushes. Note, in Fig. 224, that when the conductor A reaches the position of conductor B it will be in the field of a South commutating pole, which will tend to reverse the current of **in** and set up a current of **out**, so that by the time it reaches the position of C the current will have the proper direction.

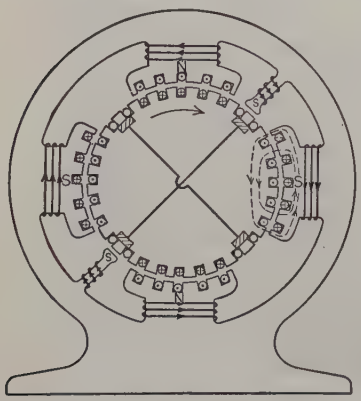


Fig. 225. A commutating-pole generator with compensating windings in the pole faces.

164. Compensating Windings on Field Poles. In motors where wide ranges of speed are desired, not only are commutating poles used but compensating field windings as well. These windings are set into the pole face as shown in Fig. 225, and are for the purpose of preventing any shift of the field by the armature cross ampere-turns. Since it is the cross ampere-turns which shift the field, the compensating ampere-turns are made equal and opposite to the cross ampere-turns of the armature.

Thus in Fig. 225, there are five conductors per pole which tend to produce a field across the poles. This is indicated on pole S. The main flux, of course, goes into the South pole S. The magnetomotive force, however, of the current in the armature windings opposite the pole tends to send a flux around in a counter-clockwise direction

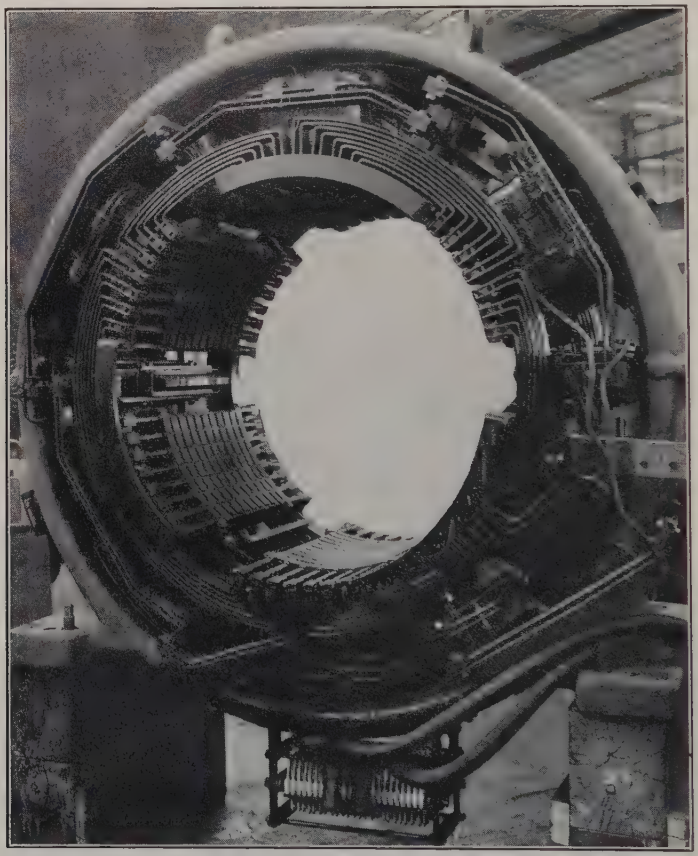


Fig. 226. Appearance of compensating windings. *Westinghouse Electric & Mfg. Co.*

as shown by the dotted line. This opposes the flux going in at the upper tip of the pole and thus weakens it. It aids the flux going in at the lower tip and thus strengthens it. This results in a shifting of the flux toward the lower or leading tip of this South pole. Now if five conductors are set in the face of this pole and the same amount of current led out of the armature through them as flows through the

armature windings (but in the opposite direction) an opposing magnetomotive force will be set up in the compensating windings which will neutralize the cross ampere-turns of the armature. Thus the shifting of the field will be prevented.

Note however that these compensating windings do not supply commutating flux to reverse the current in the short-circuited armature coil, and, therefore, the commutating poles are just as necessary as before. Figure 226 shows how compensating windings are applied.

SUMMARY OF CHAPTER IX

FORCE ON WIRE IN MAGNETIC FIELD. When a wire carrying an electric current lies in a magnetic field at right angles to the lines, there is a force on the wire due to the reaction between the circular field about the wire and the field in which the wire lies. The amount of this force in pounds can be found from the equation:

$$F = \frac{22.5\,BlI}{10^8}.$$

B in gausses
l in cms.
I in amperes

TORQUE. The torque of the armature of a motor is due to the above force, and is equal to the product of this force by the radius of the armature in feet.

MOTOR EFFECT IN GENERATORS. This is the opposing torque, which has to be overcome when the machine is running as a generator with same field strength and delivering same current. This helps us to understand how power is lost in producing eddy currents.

The opposing torque set up by Eddy Currents in the bobbin, on which moving coils of galvanometers, ammeters, etc., are wound, is used to damp the oscillations of the coil.

Eddy currents in motors and generators have a torque which opposes the motion and consumes power.

GENERATOR EFFECT IN MOTORS, COUNTER E.M.F. The armature conductors of a motor cut lines of force and set up an e.m.f. opposite in direction to the impressed e.m.f.

The current flowing through the armature is, then, under a pressure equal to the difference between the Impressed Voltage and Back E.m.f. The value of the current can be found from the equation:

$$I = \frac{E_x - E_c}{R}.$$

AXIS OF COMMUTATION. BACKWARD LEAD. The position of the brushes for a motor for sparkless commutation is on an axis back of the neutral axis, instead of ahead, as in a generator.

DIRECTION OF ROTATION. Extend the thumb, forefinger and middle finger of the left hand, at right angles to one another. When the middle finger points in the direction of the current flowing, the forefinger in direction of the flux, the thumb will indicate the direction of rotation.

Any motor must be started slowly by means of a starting box, which puts resistance in series with armature, to keep the current in the armature from becoming excessive, until a counter e.m.f. is set up by the motion of the armature.

Motors are divided into three general types.

(1) SHUNT; field shunted around the armature.
Nearly constant speed.
Low starting torque.
Races when field is broken, thus the necessity of "no-field" release on starting box.
Speed controlled by varying field strength.

(2) SERIES; field in series with armature.
Speed decreases with increase of load.
Large torque at slow speeds, thus large starting torque.
Races on "no load," thus the necessity of having load permanently attached.

(3) (a) COMPOUND (Differential); series field coils bucking shunt coils.
Exaggerated shunt characteristics.
Low starting torque.
Constant speed at all loads within limits.

(b) COMPOUND (Cumulative); series field coils aiding shunt coils.
Series characteristics.
Will not race at "no load."
Large torque at low speeds, hence high starting torque.
Speed varies with load.

The characteristics of motors and generators of same type are very similar if speed of motors is compared to voltage of generators.

BACK AMPERE-TURNS are produced by the armature current in a motor as in a generator and, per pair of poles, are equal to the current in each path of the armature winding multiplied by the turns included within twice the angle of brush shift.

CROSS AMPERE-TURNS are produced by the current in the remaining armature coils.

TO DETERMINE THE SPEED at which a motor will run under given load, add (or subtract) the ampere-turns in the series field to those of the shunt field. On the magnetization curve find volts generated by these resulting ampere-turns. Divide volts generated by the speed at which magnetization curve was taken. Result is volts per revolution per minute for this field strength. Subtract armature IR drop from impressed voltage to obtain generated voltage. Divide

generated voltage by volts per revolution per minute to obtain speed in revolutions per minute.

THE SPEED OF A MOTOR is increased by weakening the field.

THE HORSEPOWER OF A MOTOR can be found by means of a prony brake, and the speed. A prony brake measures the torque of the motor while running.

LOSSES IN MOTORS AND GENERATORS

Constant losses in a constant-speed motor.
- Copper loss in field = $I_f^2 R_f$.
- Eddy-current loss in core.
- Hysteresis loss in core.
- Windage, friction, etc.

Variable loss.
- Copper loss in armature = $I_a^2 R_a$.

Constant loss = power input to run idle as motor minus the corresponding armature copper loss.

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{\text{input} - \text{losses}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}}.$$

COMMUTATING POLES supply commutating flux which is nearly proportional to the armature current.

COMPENSATING WINDINGS set in the pole faces supply a magnetomotive force which is equal and opposite to that of the armature cross ampere-turns, and thus prevent the field from shifting.

WITH COMPENSATING WINDINGS and commutating poles a speed control of 6 to 1 can be obtained in shunt motors, with sparkless commutation.

PROBLEMS ON CHAPTER IX

Prob. 44–9. The armature of a 4-pole, 4-path motor has 240 active conductors, 80% of which lie in the magnetic field. Diameter of armature = 40 cm. Length of armature = 65 cm. Average field intensity is 6800 gausses. Armature current is 40 amperes. What is the force in pounds on each active conductor?

Prob. 45–9. (*a*) What is the total force on the active conductors in Prob. 44–9? (*b*) What torque is developed by the motor?

Prob. 46–9. (*a*) If speed of motor in Prob. 44–9 is 1200 r.p.m. what horsepower is transmitted to pulley, allowing 5% for constant rotational losses? (*b*) What voltage must be applied to motor, if efficiency is 85%? Field requires 1.5 amperes.

Prob. 47–9. In Prob. 46–9, the width of the pole face parallel to shaft is 65 cm. Assume width in the other direction to be 24 cm. What is the counter e.m.f. at speed given?

Prob. 48–9. A 2-pole, 220-volt shunt motor takes 60 amperes at full load. The field resistance is 100 ohms. Armature consists of 240 active conductors and has a resistance of 0.22 ohm. Flux per pole = 3,600,000 lines. At what speed does it run at full load?

Prob. 49–9. If the motor in Prob. 48–9 were a 4-pole machine with a 4-path, 4-brush armature, and all other data were the same, what would be the full-load speed?

Prob. 50–9. If the motor in Prob. 48–9 had 4 poles, and a 2-path armature and all other data were the same, what would be the full-load speed?

Prob. 51–9. The data for the magnetization curve of a 230-volt shunt motor operated at 1200 r.p.m. are as follows:

Field Current in Amperes	Generated Voltage	Field Current in Amperes	Generated Voltage
0.0	14	3.50	231
0.50	56	4.00	243
1.00	100	4.50	252
1.50	140	5.00	260
2.00	173	5.50	268
2.50	199	6.00	274
3.00	217	6.50	280

No. of active armature conductors = 250.
No. of turns per pair of poles on shunt field = 1000.
Resistance of shunt field = 43.1 ohms.
Resistance of armature (including brush contact) = 0.15 ohm.
Full-load armature current = 50 amperes.
No-load armature current = 4.5 amperes.
Paths through the armature = 2.

Compute the speed regulation of this motor when operated on a 230-volt line.

A rheostat resistance of 23.6 ohms is inserted in the field circuit.

Prob. 52–9. What would be the speed regulation of the motor of Prob. 51–9 if used on a 220-volt line without the field rheostat?

Prob. 53–9. What horsepower output will the motor of Prob. 51–9 deliver when carrying full-load armature current?

Prob. 54–9. What is the full-load efficiency of the motor of Prob. 51–9?

Prob. 55–9. How large a resistance would be required in series with the field coils of the motor of Prob. 51–9 for it to run at 1200 r.p.m. when carrying full-load armature current? Line voltage = 230 volts.

Prob. 56–9. What would be the horsepower output and efficiency of the motor operated as in Prob. 55–9?

Prob. 57–9. The magnetization curve of a certain shunt generator has the following coördinates, when the generator is run at 1200 r.p.m.

Field Current in Amperes	Terminal Volts	Field Current in Amperes	Terminal Volts
0	4	4	120
1	40	5	129
2	78	6	136
3	105	6.5	139

Armature resistance, including brush contacts = 0.124 ohm.
Full-load armature current = 72 amperes.
Field resistance including rheostat = 24.5 ohms.
No. of turns in field per pair of poles = 532.
No. of active conductors on armature = 120.
No. of paths through armature = 2.

Plot the voltage characteristic of this generator carrying the load current to 100 amperes.

Prob. 58–9. At what speed will the shunt generator of Prob. 57–9 run as a motor at full load on a 110-volt line? The full-load armature current as a motor is to be the same as the full-load armature current as a generator.

Prob. 59–9. What will be the no-load speed of the motor in Prob. 58–9, assuming the armature to have no current at no load?

Prob. 60–9. The motor of Prob. 58–9 takes 12.8 amperes at no load on a 110-volt line. What is the real no-load speed?

Prob. 61–9. What is the full-load efficiency of the motor in Prob. 60–9?

Prob. 62–9. What will be the efficiency of the motor of Prob. 60–9 at the load at which the constant losses equal the I^2R armature loss?

Prob. 63–9. What torque and horsepower does the motor of Prob. 60–9 deliver at full load?

Prob. 64–9. What horsepower does it take to drive the generator of Prob. 57–9 at full load? Use additional data of Prob. 60–9.

Prob. 65–9. A shunt motor on 125-volt mains has a speed of 1200 r.p.m. when the armature current is 10 amperes. The armature resistance is 0.5 ohm. What will the speed be if the field flux is decreased by 10%, a resistance of 1.2 ohms is connected in series with the armature, and the armature current is increased to 15 amperes?

CHAPTER X

BATTERIES AND ELECTROCHEMICAL ACTION

The greatest source of electric power is the generator, the action of which has been described in previous chapters. By making use of the laws of electromagnetic induction, this machine transforms mechanical power into electric power. The second source of electric power is the electric battery, which, using the laws of electrochemical action, transforms chemical energy into electrical energy.

165. Electric Battery. Primary Cells and Storage Cells. It has been found by experiment that when two electrical conductors of different materials are placed in a liquid so that they do not touch each other, and the liquid acts on one of them chemically, then an electric pressure is set up between the conductors. The liquid is called an **electrolyte.**

The conductor of the higher potential is called the **positive plate;** the other, the **negative plate.** If the two plates are then joined by a wire, an electric current will flow through it from the positive to the negative plate. The generated voltage, or pressure, between the plates depends entirely upon what the electrolyte is, and upon what the materials of the two plates are. The size, distance apart, etc., have nothing to do with the voltage of a battery, any more than the size of the conductors cutting lines of force affects the pressure across the terminals of a generator. The number of **lines cut per second** determines the voltage of a generator, and the **materials of the plates and electrolyte** determine the voltage of a battery cell.

When the liquid is dilute sulphuric acid, one plate is zinc and the other copper, the electric pressure set up is always about one volt. The chemical action takes place between the zinc and the acid. When one plate is zinc, the other carbon, and the electrolyte, a solution of sal ammoniac, the pressure is always nearly 1.5 volts, regardless of size of plates. The chemical action in this case takes place between the zinc and the sal ammoniac.

The important thing to remember is that, in order to have any voltage set up between the plates of a cell, the plates must be of different materials, and one of them must be acted upon chemically

by the electrolyte. Even a dry cell is not really **dry.** The liquid, however, is retained in plaster of Paris or in some filling, placed between the plates and in close contact with them. As long as the cell is delivering current, one of the plates (the negative) is being consumed, and the character of the electrolyte is changing. If this plate is replaced by another of the same kind when it is used up, and the electrolyte renewed by replacing it with fresh liquid, the cell is then called a **primary cell.**

If an electric current from an outside source be sent through the cell in the opposite direction, it will often cause the electrolyte to deposit the consumed material back on the plate. When this method of renewing the materials is used, the cell is said to be a **secondary or storage cell.**

166. Action of a Primary Cell. Ions and Electrons. Since the action of both Primary and Secondary cells is the same when delivering current, it is well to study a little more in detail the primary cell, it being the simpler.

Perhaps the very simplest cell is one composed of zinc, copper and sulphuric acid diluted with water. Figure 227 represents such a cell. **Cu** is the chemical symbol for copper and **Zn** for zinc.

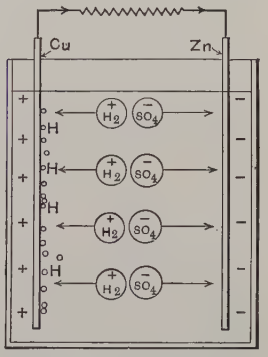


FIG. 227. Action in a simple battery cell.

Sulphuric acid is composed of two parts hydrogen, one part sulphur and four parts oxygen, and is represented by the symbol H_2SO_4.*

The sulphuric acid when diluted with water breaks up into two parts, as shown in Fig. 227, one part consisting of two atoms of hydrogen (H_2), the other part having one atom of sulphur and four atoms of oxygen (SO_4). These parts are called **ions** and are the carriers of electric charges. A negative electric charge is made up of small **negative particles** of electricity called **electrons.** The charge on the sulphate ion in Fig. 227 consists of two electrons. The sulphate part of the electrolyte is, therefore, **negatively** charged. But these electrons on the sulphate ion were taken away from the

* Each one of these parts is called an atom. Thus we say that the smallest possible quantity of sulphuric acid consists of two **atoms** of hydrogen, one **atom** of sulphur and four **atoms** of oxygen. This is called a **molecule** of sulphuric acid.

hydrogen ion. Thus the hydrogen ion is deficient in electrons, which are negative charges, and is, therefore, **positively** charged. In general, a body is **negatively** charged when it has a **surplus** of electrons, and **positively** charged when it has a **deficiency** of electrons. When it has its normal number of electrons, which varies with different substances, the body is **neutral** or **not charged.**

In Fig. 227 is represented the action of the acid breaking up into two oppositely charged ions and charging the plates. The positively charged ions of the electrolyte go **with the current** through the cell, while the negative sulphate ions go **against the current.** The fact that the charges on the plates are of opposite kinds of electricity causes a difference of potential of approximately 1.1 volts between the plates. Therefore if the copper plate is joined to the zinc by a wire outside the electrolyte, a current will flow through the wire from copper to zinc, and the chemical action will continue to go on. Thus the energy of the chemical combination is transformed into electrical energy in the form of an electric current. If the plates are not joined by an electrical conductor, the chemical action goes on until the plates are charged and then stops.

The action of all cells is similar to this. The electrolyte is always broken up into two oppositely charged parts, called ions. One part gives up its charge to one plate, the other part gives up its charge to the other plate. The difference of potential thus set up depends entirely upon the composition of the plates and electrolyte.

In a well-designed cell, the chemical action takes place only when the cell is delivering energy. The rate at which electrical energy is delivered by the cell, then, determines the rate at which the zinc is consumed, just as the rate at which steam energy is delivered by a boiler determines the rate of the coal consumption. Zinc may, therefore, be said to be a fuel, the chemical energy of which is turned into electrical energy by means of a battery cell, just as coal is a fuel, the chemical energy of which is converted into mechanical energy by means of oxygen, a furnace, water, and a boiler.

If energy could be obtained as cheaply by the consumption of zinc as by the consumption of coal, the combination of primary cells and electric motors would be in as great evidence as the combination of boilers and steam engines.

167. Dry Cells. The common dry cell has a zinc cup for the negative plate, on the inside of which is a layer of plaster of Paris saturated with a wet mixture of sal ammoniac and zinc chloride. The positive plate is a carbon rod placed in the center of the cup and

packed about with ground coke and manganese dioxide. The whole is sealed with asphaltum. The manganese dioxide is put in to clear the hydrogen bubbles away from the positive plate where, it will be seen in Fig. 227, they collect and raise the internal resistance of the battery.

Dry cells are not suited to deliver large steady currents because the manganese dioxide cannot clear up the hydrogen as fast as liberated. This hydrogen would raise the internal resistance and form a layer which would separate the carbon from the electrolyte and thus cut down the e.m.f.

For this reason dry cells are used for intermittent service only, or for small continuous currents. As will be explained later, a given dry cell can deliver a given number of ampere-hours depending on the weight of zinc in the zinc cup. But even if the cell is not used at all, it deteriorates so rapidly that it is worthless after from 12 to 15 months. If kept at a temperature above 70° F. the deterioration is even faster. To determine the condition of a cell, measure the momentary short circuit current with a low resistance ammeter. A good dry cell should give at least 18 amperes under this test.

168. Electrochemical Equivalent. Electrolysis. The rate at which the negative plate, generally made of zinc, is consumed, depends entirely upon how much current is being supplied by the battery. When no current is being supplied, all chemical action ceases if the metal is pure, and thus no material is consumed. The negative plate thus acts as fuel just as coal acts as fuel in the fire-box under a boiler. The rate at which coal is consumed depends upon how much heat energy must be converted into mechanical energy per hour. Similarly the rate at which the negative plate is consumed depends upon how much chemical energy must be converted into electrical energy per hour. When one ampere is drawn from the cell it consumes a definite quantity of zinc per hour. If two amperes are drawn from the cell, twice as much zinc is consumed per hour. Now this process is reversible. If an ampere of current were forced back through the cell in the opposite direction, this same amount of zinc per hour would be recovered from the electrolyte and deposited back on the negative plate again. If two amperes were forced through in this opposite direction, twice the amount of zinc would be deposited per hour on the negative plate. This reversed process, in which the liquid is broken up and the metal deposited, is called **electrolysis.**

The definite quantity of a substance which is consumed per hour

when one ampere is drawn from a cell, or which is deposited per hour by electrolysis when one ampere is caused to flow back through a cell, is called the **electrochemical equivalent** of the substance. The following table for the common elements is adapted from the "Handbook of Chemistry and Physics."

ELECTROCHEMICAL EQUIVALENTS OF ELEMENTS

Element	Symbol	Grams per Ampere-Hour	Ampere-Hours per Gram
Aluminum	Al	0.3370	2.967
Chromium	Cr	0.6466	1.546
Copper	Cu	1.1858	0.843
Gold	Au	2.4523	0.408
Hydrogen	H	0.0378	26.5
Iron	Fe	0.6944 1.0415	1.440 0.960
Lead	Pb	3.8632	0.259
Nickel	Ni	1.0944	0.914
Oxygen	O	0.2984	3.351
Silver	Ag	4.0248	0.248
Zinc	Zn	1.2193	0.820

By means of this table we can determine the amount of zinc, or of any other metal, necessary to produce a given current for a given length of time.

Example 1. How many ounces of zinc would be consumed in 8 hours by a battery delivering 20 amperes? 1 oz. = 28.4 gm.

Solution.

From table, 1.219 gm. of zinc are consumed in 1 hour by a current of 1 ampere.

$8 \times 20 \times 1.219$ gm. = 195 gm., or 6.87 oz. consumed in 8 hours by 20 amperes.

Also by means of this table, it is possible to find the amount of copper or other metal that would be deposited in a given time by a given current from a salt solution of the metal.

Example 2. How much copper would be deposited from a solution of copper sulphate in 12 hours by a current of 2 amperes?

Solution.

From table

1.19 gm. deposited in 1 hour by 1 ampere.

$2 \times 12 \times 1.19 = 28.6$ gm. deposited in 12 hours by 2 amperes.

Prob. 1–10. How many grams of silver are deposited per day from a solution of silver nitrate by a current of 10 amperes?

Prob. 2–10. The zinc plate of a cell weighs $\frac{1}{2}$ pound. How long will it last if the cell is required to deliver current of 3.2 amperes?

Prob. 3–10. If it is desired to deposit 100 lb. of copper in 24 hours, what current must be used?

169. Electroplating. In the process called electroplating, advantage is taken of the fact that if a current is passed through a solution containing the salt of a metal, the metal will be deposited on the negative plate. The positive plate of a vat used for this purpose is called the **anode**; the negative plate, the **cathode.** The action is as follows:

It is desired to copper plate a piece of iron. See Fig. 228. The iron B is placed in a solution of copper sulphate, $CuSO_4$, together with a piece of copper A. The copper sulphate is broken up into ions, some consisting of the ion (SO_4) being negatively charged, the others of copper (Cu) being positively charged. The current is now forced through the cell from A to B. The copper part, Cu, being positively charged, goes **with** the current, to the negative plate B and is deposited on it, thus giving it a coating of copper. The acid part, SO_4, being negatively charged, as explained in the action of a single cell, goes **against** the current, to the positive plate A and combines chemically with it, forming more copper sulphate, $CuSO_4$. Thus the copper plate is gradually dissolved and conveyed to the iron.

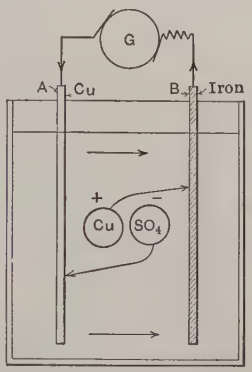


Fig. 228. Action in an electroplating vat.

If it is desired to nickel plate the iron, a piece of nickel is used for the **anode,** or positive plate A, instead of copper, and nickel ammonium sulphate for the electrolyte, instead of copper sulphate.

170. Electrotyping. The object of electrotyping is to reproduce the printers' set-up type, engravings, etc. A wax impression is first taken of the set-up type. Since wax is a non-conductor of electricity, a thin coating of graphite is now given the wax mold. The whole mold with its coating of graphite is immersed in copper sulphate together with a copper bar. An electric current is now sent through from the copper to the graphite. This causes copper to be deposited on the graphite just as it was deposited on the iron

in the cell of the previous paragraph. The current is allowed to run long enough to deposit a plate of sufficient thickness to be handled safely. Then the wax mold is removed and the remaining copper plate is an exact reproduction of the type.

171. Electrolytic Refining of Metals. If both electrodes are of the same metal, there is very little chemical voltage present. The only electrical energy used in passing a current through the combination is that lost in heating the electrolyte. There will always be a minute chemical voltage due to differences in purity of the metals and to differences in concentration in the various parts of the solution.

Since the metal, which is deposited on the negative plate, is remarkably pure, this method is often used to separate the impurities from a mass of metal. The impure mass is made the positive plate. When an electric current is passed through the cell, the pure metal is gradually dissolved by the electrolyte and carried over to the negative plate. At the end of the process, the negative plate is found to consist of very pure metal, since practically all the impurities remain at the positive plate or in the electrolyte.

For exact data and methods of procedure in the above processes, see "Electrochemical and Metallurgical Industry" and "Transactions of American Electrochemical Society."

Prob. 4–10. How long will it take to refine 1 ton (2000 pounds) of copper if 10 vats are in series and a current of 150 amperes can be used?

Prob. 5–10. An iron casting is to be copper plated and then nickel plated. The current in each case is to be 4 amperes. How long must it remain in each vat in order to have 3 lb. of copper and 6 oz. of nickel deposited on it?

Prob. 6–10. Two electroplating vats are arranged in series, one for gold plating and the other for silver plating. If a current flowing through the vats deposits 1.4 oz. of silver in a given time, how much gold is deposited at the same time?

172. Definition of Ampere. Voltameter. Since one ampere flowing for one hour through an electrolytic vat deposits a definite quantity of metal on the negative plate, we can determine very accurately the average value of an unknown current by weighing the plate before and after an accurately timed run. On account of this precise method of determining the current strength, the following definition of an ampere has been established:

An ampere is the rate of flow of a steady current which deposits 4.025 grams of silver per hour from a solution of silver nitrate diluted to a standard density with distilled water.

Prob. 7–10. An ammeter being calibrated by a silver voltameter registers 4.42 amperes for a current which deposits 39.2 grams of silver in 2 hours 10 minutes. What is percentage error of the ammeter at this reading?

Prob. 8–10. A copper voltameter, used to calibrate an ammeter, has 1.5 grams of copper deposited on the cathode in 1 hour and 25 minutes. What value should the ammeter indicate during the process?

173. Electrolytic Destruction of Metal Water Mains, etc. It is customary in electric railways to use the track as the return circuit. The rails, not being insulated from the ground, allow the current to leak into the ground and follow any low-resistance path it can find, such as a water or a gas main, back to the generator, which, of course, is also grounded.

This action is shown in Fig. 229. Where the current enters the pipe at A no harm is done. But at the point B, where the current leaves the pipe, generally near the generator station, there is all

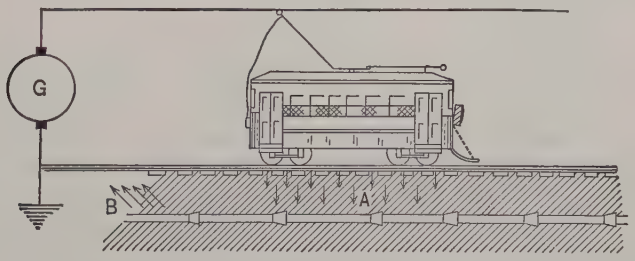


FIG. 229. A leakage current destroys a pipe at the point B where the current leaves the pipe.

the action that takes place at the positive plate in an electroplating vat. The pipe, being in moist ground, is in contact with water containing more or less salt, which causes it to act as an electrolyte. Thus, as the electric current leaves the pipe, there is a chemical action set up between the pipe and salt water, by which the iron of the pipe is eaten away in places, and carried by the electric current to some outside substance, which for the instant is acting as the negative plate.

In a short time heavy iron pipes have been eaten through by this action. Among the various precautions taken to avoid this destruction by electrolysis are the following: (1) The heavy bonding

of the rails to procure a return of low resistance. (2) The use of a second trolley wire for the return. (3) Laying in the ground a heavy copper return conductor to offer a low resistance path to the return current.

Method No. 2 is the most efficient, but also the most expensive and is very seldom used.

Stray currents must be kept out of the reinforcing rods in concrete structures for the failure of these rods is likely to cause the collapse of the whole structure.

There is electrolysis even when alternating current flows, but it is usually less than 1 per cent of what would be caused by the same amount of direct current. With alternating current the metal is dissolved on one half cycle and plated back on the next. Not all that is dissolved will plate back, even when the two half cycles of current are identical, due to diffusion of the salt formed and to secondary reactions.

174. Storage Batteries. Efficiency. In the ordinary primary cell, when the negative plate is nearly consumed, it is customary to replace it by another. If, instead of replacing the worn-out negative plate, we send a current from an outside source through the cell in the reverse direction, and deposit the metal back on the negative plate, the cell is then called a **storage battery.** When such a cell is delivering current it is said to be **discharging;** when receiving current, it is said to be **charging.** A storage cell differs fundamentally in no respect from a primary cell. Any primary cell could be used as a storage cell and have its negative plate restored by electrolysis as an ordinary storage battery does. It is not commercially economical, however, to do this in the case of any but a few cells, especially constructed for this purpose.

Remember that a storage cell does **not** store electricity. It stores nothing but chemical energy. In **charging,** electrical energy is transformed into chemical energy and stored in the cell; in **discharging,** this chemical energy is changed back again into electrical energy.

Of course there are losses in both transformations, which prevent an efficiency of 100 per cent. For one thing, the terminal voltage **applied to** the cell on charge must be higher than that **supplied by** the cell on discharge at same current rate, partly because, in each case, the internal resistance of the cell must be overcome.

Charging voltage = e.m.f. of cell + IR drop of cell.
Discharging voltage = e.m.f. of cell − IR drop of cell.

In charging a cell, the current must be sent in **against** the e.m.f. and the resistance of the cell, while on discharge, part of the e.m.f. of the cell must be used to overcome this internal resistance in forcing a current to flow through it in the other direction; so that only **part** of the e.m.f. is available as terminal voltage. In this respect it is merely like any other electrical machine. This difference between the terminal voltage on charge and discharge is further increased by the fact that the e.m.f. itself is less on discharge than on charge, due to the electrolyte changing in density.

Accordingly, good storage cells average only about 75 per cent efficiency at normal load.

175. Planté and Faure Types. Construction. The most common type of storage cell in use at the present time is the lead cell. In any cell, there must be two plates of **different** substances in an electrolyte. In the lead cell, the negative plate is pure spongy lead, Pb; the positive plate is lead peroxide, PbO_2; one part lead to two parts oxygen.

The electrolyte is sulphuric acid, H_2SO_4, diluted with water, H_2O.

The lead peroxide and spongy lead are poor conductors of electricity, and are not sufficiently hard to be made into plates; so it is necessary to attach them to frames of some harder material which is a good conductor of electricity. The material of this framework must also have the property of **not acting as a third plate,** otherwise the acid will produce **local action** between the framework and the spongy lead or between the framework and lead peroxide, wherever it touches them. The material usually chosen for this framework is an alloy of lead and antimony, which is mechanically strong and produces no local action in the presence of the other substances in the cell. The lead peroxide and the spongy lead are called the **active materials,** to distinguish them from the framework, and they form the real plates of the cell. The combination is called a **grid,** either positive or negative.

There are two methods of attaching the active materials to the framework, which give rise to two general types of lead cells: 1st, the Planté; 2nd, the Faure.

In the **Planté type,** the surface of the framework itself is roughened and then changed into the proper active material by a chemical process; that is, the surface of the negative grid is made into spongy lead; the surface of the positive grid, into lead peroxide.

In the **Faure type,** holes are punched in the framework. Lead peroxide is then forced into the holes of the positive grid, and

spongy lead into the holes of the negative. The Gould cell is an example of the Planté type, and the "Chloride Accumulator" of the Faure type. The Planté cell is heavier for the same capacity, but it is maintained that this method of attaching the active material to the framework holds it there more securely than does the other.

176. Chemical Action on Discharge. Consider Fig. 230. The sulphuric acid has broken up into positively charged (H_2) ions and negatively charged (SO_4) ions. The SO_4 ions unite with the lead plate forming lead sulphate ($PbSO_4$) giving up their negative charge to the lead plate. The hydrogen ions (H_2) carry their positive charges to the lead peroxide plate giving them up to the plate and uniting with the oxygen of the lead peroxide of the plate and forming water (H_2O). The hydrogen ions actually get electrons from the lead peroxide. This is equivalent to giving the lead peroxide a positive charge.

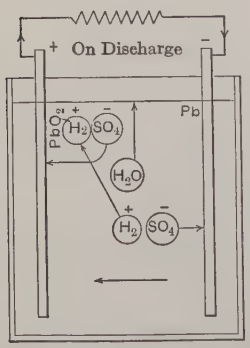


Fig. 230. Action of a lead storage cell on discharge.

The sulphuric acid in contact with the peroxide plate is also broken up into ions of H_2 and SO_4. These H_2 ions unite with the oxygen of the lead peroxide and form more water. The SO_4 ions instead of going over to the negative plate unite with the lead, Pb, of the lead peroxide plate and form lead sulphate, $PbSO_4$, on the positive plate. Thus both plates are being reduced to lead sulphate, $PbSO_4$. The cell continues to deliver current until the plates are entirely reduced to lead sulphate, when, of course, all action will cease, since there would be but one kind of material present, and a battery requires two kinds. The practical limit of discharge, however, is reached long before both plates are completely reduced to the same material.

Note **two things** which are taking place when a cell is discharging:

1st. The acid is continually growing weaker because of the formation of water. This results in a lower electromotive force.

2nd. The active materials, lead and lead peroxide, are being replaced by lead sulphate, which has a much higher resistance.

177. Chemical Action in Charging. Refer to Fig. 231. Assume both plates to consist of lead sulphate, $PbSO_4$. When a current from an outside source G is sent through the cell, it breaks up the water which has been formed during **discharge** into positively charged hydrogen ions (H_2) and negatively charged oxygen ions (O).

Part of the positively charged hydrogen ions are now attracted to the negative plate and unite with the SO_4 of the lead sulphate, forming sulphuric acid, H_2SO_4, and leaving pure spongy lead at the negative plate. The negatively charged oxygen ions (O) flowing against the current are attracted to the positive plate. Here they unite with the lead Pb of the lead sulphate $PbSO_4$ plate and form lead peroxide, PbO_2. The SO_4 part of the positive plate is finally united to the rest of the hydrogen ions liberated when the electric current broke up the water H_2O into H_2 and O. This action forms still more sulphuric acid, and a positive plate of lead peroxide. When all the lead sulphate has been changed over to lead peroxide and pure lead, the battery is restored to the state it was in before it was discharged, and is now ready to furnish current again.

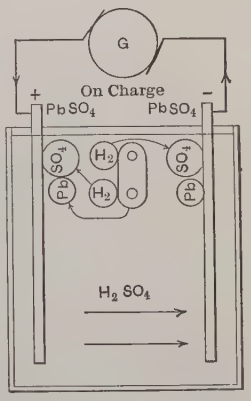


FIG. 231. Action of a lead storage cell on charge.

Note that the acid has been growing denser during charge, therefore the electromotive force must have been increasing as the charging continued.

The chemical equations for the different actions on discharge and charge may be written as follows:

DISCHARGE

$$PbO_2 + Pb + 2\,H_2SO_4 = 2\,PbSO_4 + 2\,H_2O.$$

CHARGE

or

DISCHARGING

Positive Plate		Negative Plate
PbO_2	←	Pb
+	Current ←	+
H_2	←	SO_4
+		
H_2SO_4		

Produces

$PbSO_4$ $PbSO_4$

\+

$2\,H_2O$

CHARGING

Positive Plate		Negative Plate
$PbSO_4$	→	$PbSO_4$
+	Current →	+
O	→	H_2
+		
H_2O		

Produces

PbO_2 Pb

\+

$2\,H_2SO_4$

178. Care of Lead Storage Cells. From a study of the chemical action of a storage cell and the physical results of this action, we can determine what treatment such a cell should receive in order to give the most efficient service.

The main points to be avoided are the following:

(1) Too rapid charging.
(2) Use of impure electrolyte.
(3) Use of too strong or too dilute electrolyte.
(4) Over-charging and over-discharging.

(1) **Too Rapid Charging.** The standard time of charge of storage batteries to be used in stationary work is 8 hours.

If charge or discharge is carried on more rapidly than this, the ampere-hour capacity of the cell is lowered somewhat. It is to be understood that the eight-hour rate is merely that rate for the longest life and the greatest efficiency. No great harm is done, however, if higher charging rates are used, provided the current in amperes at no time is greater than the number of ampere hours which can still be put into the battery. For instance, if a 100 ampere-hour battery is completely discharged, the charging rate might start at 100 amperes. But when 25 ampere hours had been put into the cell, the charging current should not exceed 75 amperes, the current continually decreasing as the battery becomes charged. When higher rates than these are used, the chemical action is violent, excessive gassing occurs, the temperature rises rapidly, and the active elements are not properly deposited on the plates and are likely to form a deposit in the bottom of the jar, which may eventually rise high enough to reach the plates and short-circuit them, if it is not removed.

(2) **Use of Impure Electrolyte.** This is the greatest source of trouble in a storage battery. Nothing but the purest sulphuric acid and distilled water should be used for the electrolyte.

Any metallic impurity in the acid or water is deposited on the plates as a pure metal and there causes local action, which consumes the material of the cell without producing any available energy.

Any acid impurity which will dissolve the lead of the framework is objectionable for very apparent reasons.

Too much care, therefore, cannot be exercised in selecting the materials which go to make up the electrolyte.

(3) **Too Dilute or Too Strong Electrolyte.** The specific gravity at which a cell works best depends somewhat upon the use to which it is to be put. If the cell is to be used continuously, it can work at a higher density than if it is to stand unused for any length of time.

It is of advantage to have as high a density as is practicable, because of the **decreased internal resistance** and the **increased electromotive force** which the cell has when the acid is of high density, as previously explained. But if the cell is to stand inactive, the strong acid tends to change the active material into lead sulphate more rapidly than if weaker acid is used. The specific gravity of the electrolyte also depends upon the amount of space available in the cell for electrolyte. When there is no space limitation, and plenty of electrolyte may be used, it is best to keep the specific gravity down to about 1.200 when fully charged. When there is little space and much less acid can be used, the specific gravity may run as high as 1.300.

When studying the chemical action of the cell we saw that the acid became lighter as the cell was discharged, and heavier as the cell was recharged. One method, then, of telling what condition a cell is in, with regard to charge or discharge, is to test its specific gravity with a hydrometer. When fully charged it should have a specific gravity of from 1.20 to 1.30, according to the conditions explained above. On discharge the specific gravity should never fall below 1.17.

For example, suppose a cell had a density of 1.22 when fully charged. If at a certain time it showed a density of 1.17, it would not be wise to discharge it any further.

(4) **Over-charging and Over-discharging.** When all the active material has been changed into lead peroxide and spongy lead, there is nothing gained in continuing the charging, and electric power is wasted.

There are three ways in which one may tell when a cell is fully charged:

(1) The plates when charging at normal rate begin to liberate gas in large quantities. The cell is said to "boil."
(2) The specific gravity has risen to the value which it should have when fully charged and has remained constant for an hour or so during the charging.
(3) The impressed voltage has been constant for an hour during the charge.

Note. The voltage of a cell on open circuit shows **practically nothing** about its condition as to charge or discharge.

The e.m.f. (voltage on open circuit) between lead and lead peroxide in sulphuric acid of 1.2 density is 2.05 volts (approximately). Since the e.m.f. of a cell does not depend upon the area of the plates, as long as there is **any** lead left on one plate and lead peroxide on the other, the e.m.f. will be about 2.05 volts. Therefore always take the voltage of a cell when it is either charging or discharging at its **normal** rate.

Over-discharge is sometimes a source of trouble in a storage cell. It weakens the acid and forms lead sulphate in excess, thus raising the internal resistance so that the terminal voltage becomes very low. It weakens the acid so that the e.m.f. is lowered, under working conditions, to a marked extent.

There are two ways in which one may tell when a cell has been discharged as much as is practicable:

(1) The specific gravity falls to a value depending upon the value when the cell is fully charged. (See previous page.)
(2) The voltage, **on discharge at normal rate,** falls to 1.75 volts.

Caution. Never continue discharging a lead cell after the voltage falls to 1.75 volts at normal discharge current rate.

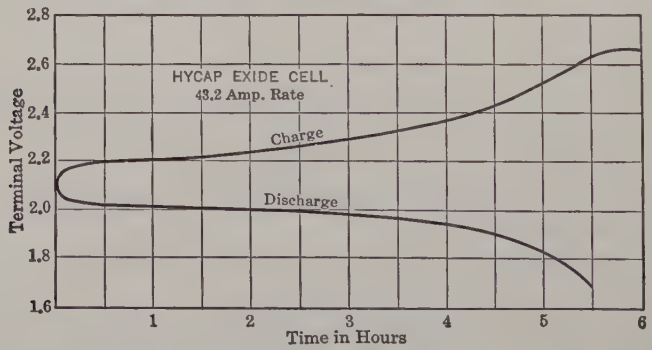


FIG. 232. Curves showing the terminal voltage of a lead storage cell on charge and discharge.

179. Curves of Charge and Discharge. The changes in the physical properties of a lead storage cell are best brought out by curves plotted from a series of readings. Figure 232 shows a set of curves plotted from data taken on an " Exide " cell.

The charge and discharge took place at the $5\frac{1}{2}$-hour rate. Note the rapid rise in voltage during the first and the last hour of charge.

During the intervening three or four hours the voltage was fairly constant.

On the discharge, the voltage falls rapidly during the last hour, but remains nearly constant during the remaining time. This makes the storage cell a first-rate source of constant potential.

In order to secure large plate area and sufficient active material, each grid usually consists of several plates joined in parallel as shown in Fig. 233.

Fig. 233. A lead storage cell. *The Electric Storage Battery Co.*

Prob. 9–10. What is the internal resistance of a cell which has a terminal voltage of 2.00 volts when discharging at a 12-ampere rate, and e.m.f. of 2.05 volts?

Prob. 10–10. What voltage would have to be impressed on cell of Prob. 9 in order to charge it at a 12-ampere rate?

Prob. 11–10. What steady current would the cell of Prob. 9–11 deliver with a short-circuit across its terminals?

180. Advantages in Use of Storage Cells. In general, the uses to which a storage battery may be put are three:

(1) To help generators carry the "peak" of a load. That is, the battery is charged when the load is light. Then, when the load is too heavy for the generators to carry alone, the battery is placed in parallel with the generators to take part of the load. This use allows the installation of much smaller generators, engines and boilers.

(2) To take care of a long-continued light load and allow the engine and generator to be shut down.

The battery is here charged while the generator is taking care of its minimum load, and only thrown in when the generator is shut down. The engine is run steadily at its most efficient load, but no fuel is wasted in keeping it running to supply a light load. Often the battery takes care of its load with no attendant present.

This use saves both in the cost of fuel and labor.

(3) As an auxiliary source of supply in case of accident. Many companies now supply power under heavy forfeits to keep the line "alive" at all times. This necessitates some reserve source of supply which can be drawn upon at a moment's notice. Storage batteries meet this demand wherever a source of direct current is required.

Prob. 12–10. A battery supplying a load is kept charged by a generator as in Fig. 234. Generator has a resistance of 0.2 ohm, e.m.f. of 125 volts. $R = 0.28$ ohm. B consists of 60 cells of 2 volts e.m.f. each. How many amperes must load use in order that the battery B may start to deliver current to the line? Neglect line resistances.

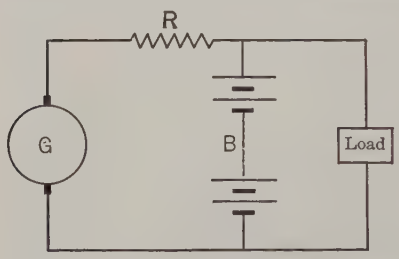


FIG. 234. The battery B makes the load on the generator more nearly uniform.

181. Edison Storage Battery. A storage battery invented by Thomas Edison and manufactured by the Edison Storage Battery Company uses an alkaline instead of an acid solution as the electrolyte.

The positive plate consists at first of nickel hydrate, $Ni(OH)_2$, held in steel tubes firmly attached to a steel grid.

The negative plate consists of iron oxide, FeO, held in pockets on a metal plate.

The electrolyte is a 21 per cent solution of caustic potash, KOH, in pure distilled water. Specific gravity 1.20.

The retaining jar, instead of being either glass or vulcanite as in the case of other storage cells, is made of nickel-plated sheet steel. The chemical actions which take place within the cell are not yet clearly understood. The following are supposed to approximate the reactions:

When the cells are **first charged,** the nickel hydrate, $NI(OH)_2$, is changed to nickel oxide NiO_2, and the iron oxide, FeO, is reduced to metallic iron, Fe. The nickel oxide NiO_2 soon decomposes to a lower oxide Ni_2O_3. The cell may thus be considered to consist of a positive plate of nickel oxide, Ni_2O_3, and a negative plate of pure iron, Fe.

182. Edison Storage Battery on Discharge. While the nickel oxide, NiO_2, is going to a lower oxide of nickel, Ni_2O_3, oxygen is liberated which is negatively charged and goes over to the iron plate,

gives up its negative charge, and unites with the plate to form iron hydroxide, $Fe(OH)_2$. The electrolyte grows lighter during charge which effect is opposite to that of the lead cell. The potassium hydroxide seems to act merely as a catalyzer or carrier.

183. Edison Cell on Charge. The iron hydroxide, $Fe(OH)_2$, is broken up and the negatively charged oxygen leaves the iron plate, travels back against the current through the cell and unites with the positive nickel oxide, Ni_2O_3, and forms a higher oxide, NiO_2.

The following equations show the chemical reaction:

DISCHARGE

$$5\,Fe + 5\,Ni_2O_3 \cdot 6\,H_2O + 9\,H_2O \rightleftarrows 5\,Fe(OH)_2 + 10\,Ni(OH)_2.$$

CHARGE

184. Physical Changes in the Edison Cell. As stated above, the specific gravity of the electrolyte lowers slightly on charge but the change is not enough to be used as an indication of the condition of the cell. Some other method, then, must be used to determine the condition of this form of storage cell as to charge and discharge.

Figure 235 shows some curves plotted from data taken on an Edison storage cell, type "A6."

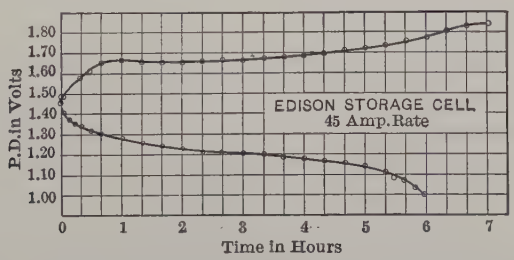


FIG. 235. Curves showing the terminal voltage of an Edison storage cell on charge and discharge.

Note that on discharge the voltage falls very rapidly at first as in a lead cell. Unlike a lead cell, however, there is no voltage at which it remains nearly constant, but has a general downward trend. Near the point of complete discharge the voltage falls off rapidly again, as in a lead cell. The condition may thus be determined closely by the voltage on discharge.

The average terminal voltage on discharge at normal current is 1.2 volts. The lowest voltage is 1.00 volt.

The voltage curve on charge is slightly irregular, but has the peculiar characteristic of reaching a maximum of 1.84 volts on normal current when fully charged. Average voltage on charge, 1.7 volts.

185. Comparison of Lead and Edison Cells. Average voltage per cell on discharge: Lead cell, 1.95 volts; Edison cell, 1.2 volts. Watt-hours per pound of cell complete: Lead, 8.5 watt-hours; Edison, 16.8 watt-hours.

Internal resistance: The Edison cell has a slightly higher internal resistance; the efficiency is therefore lower.

It is claimed for the Edison cell that it can stand for an indefinitely long time, charged or discharged, and not be injured. A lead cell should not remain any length of time **uncharged,** and only a few months **charged.**

The temperature range at which the Edison cell will work best is limited to the region around 70° F. Any condition of use which tends to lower greatly the temperature decreases the efficiency to a marked extent. The lead cells are not so limited in this respect.*

186. Other Electrochemical Processes. Electrochemistry is a profession by itself. It has rapidly become of fundamental importance in our complex life. A large number of chemical processes are now conducted by the aid of electricity and can only be given passing mention here.

Fused salts in place of salts dissolved in water may be decomposed by electrolytic action. This often requires the high temperature obtained from the electric furnace. In other cases, moderate temperatures may be used.

Aluminum is manufactured in this manner by electrically decomposing an ore of aluminum such as bauxite, H_3AlO_3. Aluminum requires the largest amount of electricity per pound of any metal obtained commercially by electrolysis. One company operates a battery of thirty-six 2500-kilowatt converters in a single plant to supply current for producing aluminum. The cost of aluminum is largely determined by the cost of electric energy. Metallic sodium and potassium are similarly produced, but by the use of much higher temperature.

* For further information concerning storage batteries, the student is referred to Vinal, " Storage Batteries."

Gaseous hydrogen and oxygen are commercially produced in one process by the electrolytic decomposition of water.

The fixation of atmospheric nitrogen for the production of fertilizers and explosives is of vast importance to the agricultural and military interests.

Carborundum, calcium carbide, fused quartz, silicon, graphite, phosphorus and steel alloys of various sorts are typical important electric-furnace products.

Immense quantities of caustic soda and chlorine are now produced commercially from the electrolysis of sodium chloride solution in specially constructed cells. A chlorine plant at Edgewood, Md., has a capacity of $12\frac{1}{2}$ tons of gaseous chlorine per day.

The growth of the electrochemical industries has kept pace with that of electric-power development. Because of the nature of the processes, power can often be obtained very cheaply by using power when other demands are low.

SUMMARY OF CHAPTER X

ELECTROLYTIC CONDUCTION is a name given to the carrying of electric charges through a fluid by the flow of ions.

Ions are molecules or atoms with more or fewer electrons attached than normal conditions demand. In a solid conductor the atoms and molecules do not flow, the current consisting of a flow of electrons only.

ELECTRIC BATTERIES are devices for transforming chemical energy into electrical energy. They require two unlike conductors, called positive and negative plates, and an electrolyte which acts chemically upon one of the plates.

A PRIMARY CELL is an electric battery in which the worn-out element is replaced by another. The negative element is generally of zinc.

A STORAGE CELL is an electric battery in which the worn-out element is replaced by the electrolytic action of an electric current forced through cell in reverse direction. The elements are generally of lead and lead compounds.

ACTION IN CELL. The electrolyte is broken up into two oppositely charged parts called ions. The positively charged ions give up their charges to positive plate, and negatively charged ions give up their charges to negative plate, part of which unites with the ions to form a new compound. This constitutes the chemical action in the cell. A difference in potential between the plates is thus produced.

Hydrogen bubbles collect on the positive plate and increase the resistance of cell, and lower the e.m.f. by setting up a counter e.m.f.

Some oxidizing agent is used as a depolarizer to remove the hydrogen by uniting with it to form water. Cells which are depolarized

rapidly are used for closed circuit work; those which are depolarized more slowly, for open circuit or intermittent work.

ELECTROLYSIS. When an electric current is sent through a solution containing a metallic salt, it will deposit the metal on the negative plate. If chemical action takes place between the electrolyte and positive plate, the positive plate is consumed.

ELECTROCHEMICAL EQUIVALENT. The amount of metal deposited on the negative plate, and the amount taken from the positive plate, by an ampere-hour of electricity is a constant, depending upon metals and electrolyte.

ELECTROPLATING. Use is made of electrolysis to plate conducting materials; to refine metals; to make electrotypes, etc., and to restore the active elements in a storage battery.

DESTRUCTION OF WATER MAINS, ETC., BY ELECTROLYSIS. The leakage currents from the return circuits of electric railways and other sources may travel along iron pipes and steel reinforcing of concrete, and are likely to eat away portions of the iron at the point where they leave the iron to return to the generator.

STORAGE CELLS. Do not store electricity, but chemical energy. Common types composed of lead peroxide, positive plate; and spongy lead, negative plate.

Action on charge and discharge:

Discharge

$$PbO_2 + Pb + 2\,H_2SO_4 = 2\,PbSO_4 + 2\,H_2O.$$

Charge

CARE OF STORAGE CELLS. Storage cells, to have longest life and maximum efficiency, should not be subjected to:

(1) Too rapid charging.
(2) Use of impure electrolyte.
(3) Use of too dense or too light electrolyte.
(4) Over-charging and over-discharging.

CONDITION OF CELLS. Can be ascertained:

(1) By color of plates.
(2) By specific gravity of electrolyte.
(3) By terminal voltage at normal charging or discharging rate (not by the e.m.f.).

USE OF STORAGE CELLS.

(1) To help carry "peak" of load.
(2) To carry all the "light load."
(3) As a reserve supply of electrical energy.

EDISON STORAGE CELLS. Positive element, nickel oxide, changes to lower oxide on discharge.

Negative element, finely divided iron, changes to iron hydroxide on discharge.

The electrolyte, 21 per cent solution of potassium hydrate, does not greatly change in specific gravity throughout charge and discharge, getting less on charge.

Edison cells are lighter per watt-hour than most lead cells, but have lower e.m.f. per cell. They can remain charged or discharged indefinitely without deteriorating. The efficient temperature range is limited.

A LARGE NUMBER OF CHEMICAL PROCESSES are now conducted by the aid of electricity. Aluminum, oxygen, caustic soda and chlorine are a few of the more common substances commercially produced by electrolytic action.

PROBLEMS ON CHAPTER X

Prob. 13–10. The zinc plate of a certain battery cell weighs 280 grams. How many hours will the cell last if it is required to deliver an average current of 3 amperes?

Prob. 14–10. A cell is desired which will deliver an average of 0.05 ampere for 2 months of 30 days. What weight should the zinc plates have?

Prob. 15–10. A 280-ampere-hour lead storage vehicle battery has an average voltage of 32 volts. How much energy can be drawn from it?

Prob. 16–10. If the average voltage required during charge of the battery of Prob. 15–10 is 40 volts and the time of charge 10% longer than the time of discharge at normal rate, what is the efficiency of the battery?

Prob. 17–10. How long must a current of 500 amperes run in an electro-refining vat to deposit enough copper to make one mile of No. 0000 wire?

Prob. 18–10. If the electric energy of Prob. 17–10 costs 1.25¢ per kilowatt-hour and the refined copper is worth 18¢ per pound, at what voltage must the refining vat be run to make the cost of energy equal 5% the cost of copper refined?

Prob. 19–10. If the internal resistance of a nickel plating bath is 0.15 ohm, and 1 volt is applied, what energy in kilowatt-hours is used in plating one pound of nickel?

Prob. 20–10. If the voltage in Prob. 19–10 is increased to 1.2 volts, what will be the new amount of energy per pound?

Prob. 21–10. Ten thousand articles each with a surface area of 1 square foot are to be plated with 0.003-inch thick nickel coat. Take

the weight of nickel as 0.311 pound per cubic inch. If 1 volt is used in the solution of Prob. 19–10 and the energy cost is 3 cents per kilowatt-hour, what is the energy cost per article?

Prob. 22–10. If the labor cost for plating bath attendance is 4¢ per article under the conditions of Prob. 21–10 and this can be reduced to 3¢ if the time of plating is halved, will it pay to use a 2-volt supply?

Prob. 23–10. Hydrogen under standard conditions of temperature and pressure weighs 0.0898 ounce per cubic foot. How long will it take 1000 amperes to liberate enough hydrogen by electrolytic decomposition to fill a balloon which is spherical and 40 feet in diameter?

Prob. 24–10. At a voltage of 1.5 and at 2¢ per kilowatt-hour, what is the cost of energy for filling the balloon of Prob. 23–10?

Prob. 25–10. Compare the energy costs of plating nickel and silver (assuming the resistance of the plating bath to be the same in each case) for equal weights deposited, and for equal thicknesses on an article. (Density of silver is 0.380 pound per cubic inch; of nickel is 0.315 lb. per cubic inch.)

Prob. 26–10. A dry cell delivering 1.5 volts will supply 1 ampere intermittently for a total of 50 hours. The cell costs 25¢ new. What is the cost per kilowatt-hour of power obtained in this way?

Prob. 27–10. An iron water pipe receives a stray current of 5 amperes from a trolley system. Where this current leaves the pipe, electrolysis occurs. How long will it take to remove 10 pounds of the iron from the pipe?

Prob. 28–10. An electroplating concern has an order to chromium plate 10,000 units per day of a certain article. The area to be plated measures 1.2 square feet and the thickness of the plate is 0.0006 inch. The vats are to be unloaded in the morning and reloaded with new work during the remainder of the day. What current will be required in order to complete the plating between 6 P.M. and 8 A.M. Weight of chromium is 368 pounds per cubic foot. There are 25 vats in series.

Prob. 29–10. A set of 90 lead storage cells, each having 2.0 volts e.m.f. and 0.004 ohm internal resistance, is to be charged from a 115-volt line. If each cell is to take its normal current of 20 amperes, what will be the best arrangement of cells in order to have least power lost in a series resistance in line?

Prob. 30–10. How could cells in Prob. 29–10 be arranged in order to deliver 160 amperes and not exceed normal current of each cell? At what voltage would they deliver this current?

Prob. 31–10. A storage battery of 250 cells in series is kept across one end of a 4-mile trolley line, the resistance of which (line and return) is 0.08 ohm per mile. At the other end of the line a generator maintains a voltage of 575 volts. Each cell of the battery has an

e.m.f. of 2.1 volts and an internal resistance of 0.001 ohm. What current will the battery supply to the line when there are 6 cars at the battery end of the line, each taking 60 amperes?

Prob. 32–10. What will be terminal voltage of the set of battery cells in Prob. 31–10?

Prob. 33–10. What current will cells in Prob. 31–10 take when there is no load on the line?

Prob. 34–10. What current will generator be delivering when the 6 cars of Prob. 31–10 are half way between station and battery?

Prob. 35–10. What current will battery be delivering to or receiving from line in Prob. 34–10?

Prob. 36–10. If there are but 2 cars on line in Prob. 34–10 each taking 75 amperes: (*a*) What power is generator delivering? (*b*) What power is battery receiving or delivering?

CHAPTER XI

SOME MORE DIFFICULT PROBLEMS

In order to solve readily the more difficult problems which present themselves in modern direct-current practice, the laws developed in the preceding chapters must be somewhat amplified.

187. More Difficult Applications of Ohm's Law. Kirchhoff's Laws. Several further laws have been deduced from Ohm's Law and called Kirchhoff's Laws, in honor of the man who first stated them. They are merely an extension of Ohm's Law and need not be learned separately. In fact, we unconsciously have been applying them to most of the problems dealing with the relation of current, resistance and voltage, in the more complicated systems.

It is well, however, to make a definite statement of the principles under the proper heading of **Kirchhoff's Laws,** in order to apply them more directly to certain complex problems in distribution.

Kirchhoff stated:

First Law. At any point in a circuit, there is as much current flowing **away** from the point as there is flowing **to** it.

Second Law. The sum of the several IR drops **around any one path** of an electric circuit equals the sum of the e.m.f.'s impressed on that same path. Care must be taken to get the algebraic signs of the e.m.f.'s correct. If there is no source of e.m.f. in any given path, then the sum of the IR drops in one direction equals the sum of the IR drops in the other direction.

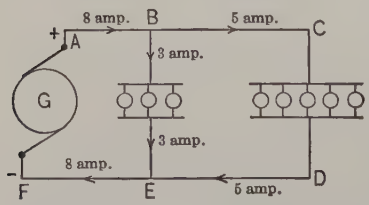


FIG. 236. A generator feeding two lamp banks.

First Law. This can best be explained by referring to diagram in Fig. 236. Assume each lamp to take 1 ampere and consider the point B.

Since there are $5 + 3$ or 8 amperes flowing away from B, there must be 8 amperes flowing to B along AB.

Consider point E.

Since there are 8 amperes flowing to point E, there must be 8 amperes flowing away from E.

We have unconsciously used this law again and again, considering that it was too obvious, from the very nature of an electric current, to need demonstration.

Second Law. This law is as obvious as the first, but more difficult to state clearly. Referring again to Fig. 236, the e.m.f. of the generator G must equal the IR drop in $AB + BE + EF +$ the IR drop of the generator itself.

Or considering another complete circuit on the diagram; the e.m.f. of generator must equal the IR drop in $AB + BC + CD + DE + EF +$ generator IR drop. Or again, considering the circuit $BCDE$, which contains no source of e.m.f., we can say that the IR drop of BE must equal the IR drop of $BC + CD + DE$, since the IR drop of BE is counter-clockwise in the circuit $BCDE$ and the IR drop of $BC + CD + DE$ is clockwise.

We have been accustomed to use this same principle under the two separate rules (1st) that the IR drop around any path in an electric circuit equals the sum of the impressed e.m.f.'s, and (2nd) that the voltage drop along parallel paths between two points is the same.

Thus we would have said that the voltage across BE equals the voltage across $BC + CD + DE$ because both the path BE and the path $BC + CD + DE$ are in parallel between the points B and E.

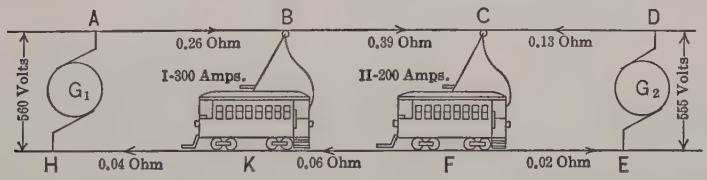


FIG. 237. The power is supplied by a generator at each end of the line.

There are, therefore, no new facts to be learned from Kirchhoff's laws, but slightly different viewpoints from which to regard familiar facts. The following example should make this clear:

Example 1. Assume trolley line, Fig. 237, to be fed by two generators, G_1 of 560 volts, and G_2 of 555 volts. Two cars are on the line; Car I taking 300 amperes, Car II, 200 amperes. The resistances of trolley and track are as marked.

Find:

(1) Voltage across each car.
(2) Efficiency of transmission.

Solution. Assume current to flow in direction indicated in different sections.

Note. If on solution any current values come out negative it merely means, as in solving force diagrams in mechanics, that the **direction** of the current should be the reverse of that indicated; the numerical result will be correct.

Let x = current in section AB.
Then x = current in section KH.
$x - 300$ = current in section BC and FK.
$500 - x$ = current in section DC and FE.

The above assumptions are made in accordance with the common-sense law that the amount of current which flows away from a point must be the same as that which flows to it. This, as we have seen, is called Kirchhoff's " First Law."

Voltage across Car II equals
$555 - 0.15\,(500 - x)$. (1)
Voltage across Car II also equals
$560 - 0.30\,x - 0.45\,(x - 300)$. (2)
Therefore
$555 - 0.15\,(500 - x) = 560 - 0.30\,x - 0.45\,(x - 300)$. (3)

These assumptions are made in accordance with the principle stated in Chapter II, that the voltage across a parallel combination is the same as that across any and all of the parallel paths between the same two points. This, as we have seen, is part of Kirchhoff's " Second Law." Thus the voltage across Car II must be the same whether computed from the voltage of G_2 minus line drop, or from the voltage of G_1 minus the line drop from that generator.

Solving (3) for x,

$x = 239$ amperes = current in AB and KH.
Voltage across Car I $= 560 - 239 \times 0.30 = 488.3$ volts.
Current in CD $= 500 - 239 = 261$ amperes.
Voltage across Car II $= 555 - 261 \times 0.15 = 515.8$ volts.
Current in $BC = 239 - 300 = -61$ amperes (the minus sign means it must be flowing in the direction opposite to that marked).
Power delivered by $G_1 = 560 \times 239 = 134{,}000$ watts.
" " $G_2 = 555 \times 261 = 145{,}000$ "
Total power $= 279{,}000$ kw.
Power used by Car I $= 488.3 \times 300 = 146{,}500$ watts.
" " II $= 515.8 \times 200 = 103{,}200$ "
Total power used by cars $= 249{,}700$ kw.

$$\text{Efficiency of transmission} = \frac{250}{279} = 89.6 \text{ per cent.}$$

By means of the principles illustrated above, all problems involving current and voltage relations in " mesh-work " may be solved.

Prob. 1–11. In Example 1, what current must be taken by Car II in order that no current shall flow in line BC? The current in Car I remains at 300 amperes.

Prob. 2–11. Under the conditions of Prob. 1–11, find:

(a) Voltage across each car.

(b) Efficiency of transmission.

Prob. 3–11. In Fig. 238, each lamp takes 10 amperes. Resistance of $AB = BC = 0.08$ ohm. Resistance of $CD = MK = 0.04$ ohm.

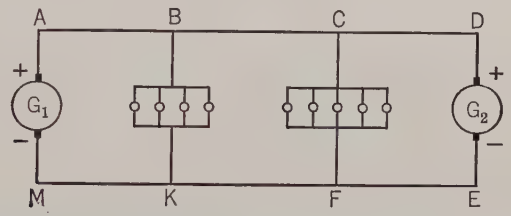


FIG. 238. Two generators supplying two groups of lamps.

Resistance of $EF = KF = 0.03$ ohm. What is the voltage across CF and BK, if terminal voltage of $G_1 = 120$ volts and of $G_2 = 125$ volts?

Prob. 4–11. What is the efficiency of transmission of system in Prob. 3–11?

188. Parallel Combinations of Unlike Generators or Battery Cells. When battery cells, of either primary or storage type, are used in series and parallel combinations, they are ordinarily considered to have the same internal resistance and e.m.f. This is approximately true, and the process of finding resulting voltage and current is a matter of simple addition. Even if the cells are unlike and are joined in series, the e.m.f.'s and resistances merely add together. This case need not be considered.

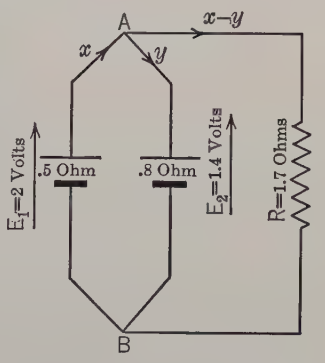


FIG. 239. Two parallel cells for supplying current to the resistance R.

But it is interesting to see what will happen if we have two or more cells of unlike e.m.f. and resistance, joined in parallel and feeding a line. Such combinations represent actual conditions in many telegraph and telephone circuits, and afford excellent practice in the application of Ohm's Law as extended by Kirchhoff.

Example 2. Consider Fig. 239. Cell E_1 has 2 volts e.m.f. and 0.5 ohm internal resistance. Cell E_2 has 1.4 volts e.m.f. and 0.8 ohm internal resistance. When they are joined in parallel to feed a load of 1.7 ohms resistance:

Find

(*a*) Amount and direction of current through each cell and through the line.

(*b*) Terminal voltage (across AB).

Solution. Assume current flows as marked. There is probably a reversed current through E_2 on account of the higher e.m.f. of cell E_1. We will assume this, and if the current value comes out a negative quantity, we have merely to reverse the arrow head.

Let x = current through E_1.
Let y = current through E_2.
Then $x - y$ = current in line.

The voltage drop across the points AB will be equal along the three paths.

Thus voltage across $AB = 2 - 0.5\,x$
$= 1.4 + 0.8\,y$
$= 1.7\,(x - y)$.

Therefore

$$2 - 0.5\,x = 1.4 + 0.8\,y,$$

$$y = \frac{0.6 - 0.5\,x}{0.8}, \quad \ldots\ldots\ldots\ldots \quad (1)$$

and

$$2 - 0.5\,x = 1.7\,x - 1.7\,y,$$

$$y = \frac{2.2\,x - 2}{1.7}. \quad \ldots\ldots\ldots\ldots \quad (2)$$

Since (1) = (2)

$$\frac{0.6 - 0.5\,x}{0.8} = \frac{2.2\,x - 2}{1.7};$$

$$1.02 - 0.85\,x = 1.76\,x - 1.6;$$

$$x = 1.00 \text{ ampere};$$

$$y = \frac{0.6 - 0.5\,x}{0.8};$$

$$= \frac{0.6 - 0.5}{0.8};$$

$$= 0.125 \text{ ampere}.$$

$$x - y = 1.00 - 0.125$$

$$= 0.875 \text{ ampere}.$$

Thus current through $E_1 = 1.00$ ampere.
" $E_2 = 0.125$ "
" $R = 0.875$ "
Voltage across $AB = 2 - 0.5\,x$;
$= 2 - 0.5$;
$= 1.5$ volts.

Check
Voltage across AB = (current through R) $\times$ (resistance of R);
$= 0.875 \times 1.7$;
$= 1.49$ volts.

From the results of our computation we see that the current was backing up through cell E_2 on account of its low e.m.f. This shows what is likely to happen in a battery of storage cells when one cell or set of cells, joined in parallel with others, becomes worn out before the rest.

It is common practice, as we have seen, to operate both shunt and compound generators in parallel. The necessity for using machines of approximately equal e.m.f. and resistance is apparent from an inspection of this example. The distribution of current in line and generators so used is similar to that in the case of battery cells. Thus, if, in the above example, the expression **" generators "** be substituted for **" battery cells "** the method, computation, and results would be the same.

Prob. 5–11. What resistance should be placed in series with E_1 in Fig. 239 in order that the current in E_2 be zero?

Prob. 6–11. What resistance is required in series with E_1 in Fig. 239 to cause each cell to supply the same current? What current will each cell supply?

Prob. 7–11. Assume the following values for Fig. 239 and find current through cells and load.

E_1 has 2.2 volts e.m.f. and 0.40 ohm internal resistance.
E_2 has 1.8 volts e.m.f. and 0.75 ohm internal resistance.
R has 0.12 ohm resistance.

Prob. 8–11. A series set of 5 cells each having 1.40 volts e.m.f. and 0.25 ohm internal resistance, is joined in parallel to a series set of 6 cells of 1.3 volts e.m.f. and an internal resistance of 0.20 ohm each. The parallel combination feeds a load having a resistance of 0.48 ohm.

Find:

(*a*) Current through 0.48 ohm resistance.
(*b*) Voltage across 0.48 ohm resistance.

189. Feeders. In order to avoid a heavy line loss in both power and voltage, it is customary in many low voltage systems, *i.e.*,

600 volts or under, to parallel the main conductor with a second conductor called a **feeder.**

Such a feeder is often run alongside of a trolley wire and joined to it at regular intervals.

This arrangement is merely a case of parallel conductors, but it presents some interesting problems in current, voltage, and power distribution.

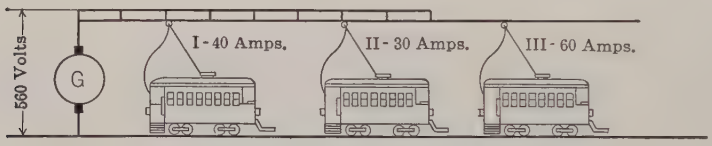


FIG. 240. A trolley line supplied with a feeder.

Example 3. Trolley wire in Fig. 240 is No. 0, hard-drawn copper 0.530 ohm per mile. Feeder is No. 0000 annealed copper 0.258 ohm per mile. Track resistance is 0.04 ohm per mile.

Car No. I is 1 mile from Generator station.
" II is 2 miles from Car I.
" III is 2 miles from Car II.

Feeder extends 4 miles from Station and is tied to trolley every half mile.

Find:

(*a*) Volts lost between Generator and I.
" " " I " II.
" " " II " III.

(*b*) Voltage across each car.

(*c*) Power lost in each section of line.

(*d*) Efficiency of transmission to three cars.

Solution. The feeder and trolley may be considered as two parallel circuits with a combined resistance per mile, found as follows:

Resistance per mile	Conductance per mile
0.530 ohm	1.89 mhos
0.258 ohm	3.88 mhos
	5.77 mhos $= \frac{1}{5.77}$, or 0.173 ohm.

If the trolley and feeder had both been of the same kind of copper, it would have been possible to have considered them as one wire, the cross-section area of which was equal to the sum of the cross-section areas of each. Then the resistance per mile could have been found from the equation $R = \frac{Kl}{A}$ as explained in Chapter III.

The trolley resistance between Gen. and Car I
$= 1 \times 0.173 = 0.173$ ohm.
" " " " Car I and Car II
$= 2 \times 0.173 = 0.346$ "
" " " " Car II and Car III
$= 0.173 + 0.530 = 0.703$ "
" total " " Gen. and Car I
$= 0.173 + 0.04 = 0.213$ "
" " " " Car I and Car II
$= 0.346 + 0.08 = 0.426$ "
" " " " Car II and Car III
$= 0.703 + 0.08 = 0.783$ "

Current between Car II and Car III $= 60$ amp.
" " " I and Car II $= 60 + 30 = 90$ amp.
" " Gen. and Car I $= 90 + 40 = 130$ amp.

Drop in line from Gen. to Car I
$= 130$ amp. $\times 0.213$ ohm $= 27.7$ volts.
" across Car I $= 560 - 28 = 532$ volts.
" in line from Car I to Car II
$= 90$ amp. $\times 0.426$ ohm $= 38.3$ volts.
" across Car II $= 532 - 38 = 494$ volts.
" in line from Car II to Car III
$= 60 \times 0.783 = 47$ volts.
" across Car III $= 494 - 47 = 447$ volts.

Power lost between Gen. and Car I $= 130^2 \times 0.213 = 3600$ watts.
" " " Car I and Car II $= 90^2 \times 0.426 = 3450$ watts.
" " " Car II and Car III $= 60^2 \times 0.783 = 2820$ watts.
Total power lost in line $= 9870$ watts.
Power delivered by Gen. $= 560 \times 130 = 72{,}800$ watts.
" " to Cars $= 72{,}800 - 9870 = 62{,}930$ watts.

or, as a check

Power delivered to Car I $= 532 \times 40 = 21{,}280$ watts.
" " " II $= 494 \times 30 = 14{,}820$ watts.
" " " III $= 447 \times 60 = 26{,}820$ watts.
" " to Cars $= 62{,}920$ watts.

Efficiency of transmission $= \frac{62{,}930}{72{,}800} = 86.4$ per cent.

Prob. 9–11. An 8-mile electric railway has a No. 00 trolley wire of hard-drawn copper. A feeder, No. 0000 annealed copper wire, extends from the station 5 miles along the trolley wire and is tied to it every half mile. At a certain instant there are 4 cars on the line. Car I is 2 miles from station, taking 30 amperes, Car II is $3\frac{1}{2}$ miles from station, taking 50 amperes. Car III is 6 miles from station, taking 60 amperes. Car IV is $7\frac{1}{2}$ miles from station, taking 40 amperes. Track resistance is 0.05 ohm per mile. What must terminal voltage of generator be in order that Car IV may have a pressure of 550 volts?

Prob. 10–11. If the resistance of generator in Prob. 9 is 0.06 ohm, what e.m.f. must it generate? Shunt field = 150 ohms.

Prob. 11–11. When the cars do not happen to be exactly at a junction of the feeder and trolley, the problem becomes a little more involved.

Assume, in Fig. 241, the hard-drawn trolley wire has 0.530 ohm per mile and feeder has 0.258 ohm per mile, extends outwards 4 miles from station and is joined every **mile** to the trolley. Find the voltage across each car.

Distance between Gen. and Car I = 1 mile.
" " Car I and Car II = $1\frac{1}{2}$ miles.
" " Car II and Car III = 2 miles.
Track resistance = 0.04 ohm per mile.

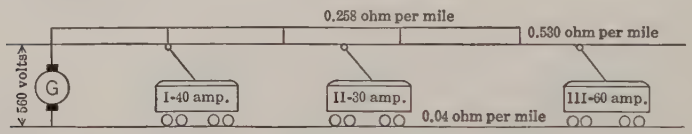


FIG. 241. Car II is not at a junction point of trolley line and feeder.

190. Relation of Voltage to Efficiency of Transmission. One of the most noticeable features of present day electrical engineering is the utilization of extremely high voltages in transmission systems and the planning of even higher voltages for projected lines. The reason for this is the fact that for the same amount of available power, the line loss is less and the efficiency higher as the voltage is raised.

This is strikingly brought out by inspection of the table on the next page, in which the line loss and efficiency at various line voltages have been worked out for the particular case where 30,000 watts are generated and it is desired to transmit as much as is possible over a 3-ohm line.

Note that the efficiency increases very rapidly with the voltage until about 1500 volts are reached. From there on, the increase is much slower, though the efficiency continues to rise a little with each increase of voltage. If the efficiency were the only condition affecting the operating cost, it would be advisable to use indefinitely high voltages. The difficulties in the insulation of the machine and line limit the voltage at the present time to some more moderate value. A rough rule for the most efficient voltage is " 1000 volts per mile." Yet the fact is evident that as far as the line loss (I^2R) is concerned, the higher the voltage used, the higher the efficiency of transmission.

Prob. 12–11. Construct a table as below for an 80-kw. generating station and a 2-ohm line, starting at 100 volts and gradually increasing it to 10,000 volts. Plot curve between volts and efficiency of transmission.

Prob. 13–11. Repeat Prob. 12–11, using an 8-kw. generator.

30,000 WATTS GENERATED AT E VOLTS

Volts E	Amperes I	Line Loss in Volts $R=3$ ohms IR	Line Loss in Watts I^2R	Volts Left for Motor V	Watts Transmitted to Motor IV	Efficiency Per Cent
100	300	900	Impossible			
200	150	450	"			
300	100	300	30,000	0	0	0
400	75	225	16,875	175	13,125	43.8
500	60	180	10,800	320	19,200	64
600	50	150	7,500	450	22,500	75
800	37.5	112.5	4,220	687.5	25,780	85.9
1,000	30	90	2,700	910	27,300	91
1,200	25	75	1,875	1,125	28,125	93.8
1,500	20	60	1,200	1,440	28,800	96
2,000	15	45	675	1,955	29,325	97.8
3,000	10	30	300	2,970	29,700	99
5,000	6	18	108	4,982	29,892	99.6
10,000	3	9	27	9,991	29,973	99.9

191. Relation of Line Loss to Voltage: Output Constant. On inspection of the above table, it will become evident that the line loss in watts varies inversely as the square of the voltage of the generator (assuming a constant watt output).

For instance, the line loss at 500 volts is 10,800 watts, while at 1000 volts it is only $\frac{10,800}{4}$ or 2700 watts. By doubling the voltage we have quartered the line loss.

Prob. 14–11. (*a*) If the line loss on a 110-volt system is 1.4 kw. what will it become if the voltage of the system is changed to 220 volts? (*b*) To 550 volts? Assume the same power delivered to the line.

192. Relation of Size of Conductor to Voltage (Line-Loss Constant). In the above paragraphs we have been considering a means of decreasing the operating expenses by decreasing the power

lost in the line. We have seen that the power lost varies inversely as the square of the voltage of transmission. It is also important to consider the effect on the original cost of transmission if we raise the voltage. In analyzing this point, we will assume that we have fixed upon an amount of power which we are willing shall be lost in the line, and that we keep this constant.

By referring again to the above table, we find that we have a loss of 1.2 kw., when the generator voltage is 1500 volts. Let us assume this to be the line loss we are willing shall be kept constant for economical reasons. By doubling the voltage to 3000 volts, we of course decreased the line loss to $\frac{1.2}{4}$ or 0.3 kw. But since we are allowed a line loss of 1.2 kw., we may now increase our line resistance by using a wire of a diameter enough smaller to make up this line loss.

The resistance of the line has been kept at 3 ohms. At 1500 volts, the line current was 20 amperes; line loss $= 20^2 \times 3 = 1.200$ kw.

At 3000 volts, line current was 10 amperes; line loss $= 10^2 \times 3 = 0.300$ kw.

But being allowed 1.2 kw. line loss, if we use the 3000 volts, and 10 amperes line current, we may use a wire of 12 ohms, instead of 3 ohms. Then the line loss will become $10^2 \times 12 = 1.2$ kw., the same value it had at 1500 volts with a 3 ohm line. That is, by doubling the voltage we are able to use a line wire of four times the resistance, and not increase the line loss.

This may be stated in more general terms as follows:

The line resistance for a constant line loss varies directly as the square of the voltage of transmission.

Since the resistance of a line varies inversely as the cross-section we may say that the size of a wire required for transmission varies inversely as the square of the voltage used. That is, we may use wires one-quarter as large if we double the voltage.

It can be seen from this that in transmitting electric power between any two points at given line loss, a great saving in the cost of copper can be made by transmitting at a high voltage. In fact, since the cost of installed moderate-size copper wire is almost directly proportional to its cross-section or weight, we may say that as far as the cost of the line wire is concerned, the initial expense varies inversely with the square of the voltage of transmission.

In choosing the line wire to be used in the installation of a system,

such a size must be used as will maintain a fair balance between the initial cost and the cost of running, *i.e.*, the line loss. The initial cost **increases** as the size of the wire increases, while the cost of running (line loss) **decreases** as the size increases.*

The choice of voltage is more or less arbitrary, — the engineer always trying to use as high voltage as conditions permit. Better methods of insulating both line and machine are continually raising the voltage at which power may be most economically transmitted over given distances. The expense of insulating the line and the apparatus connected to it increases rapidly as we choose higher voltages. Of course the cost of the copper in the circuit goes down rapidly at the same time, the high voltage enabling us to transmit the same power with the same loss over **smaller wires.** Above a certain voltage, the difficulty and expense of insulation increases faster than the cost of the copper decreases, so that the total cost of transmission would be increased by raising the voltage any further. The limiting pressure at which this occurs is being continually raised by improvements in the manufacture of insulation, which make good insulation cheaper, or insulation of a given cost much stronger. At the present time this economical limit is about 275,000 volts, but there are prospects that it will go higher.

Prob. 15–11. A 110-volt line is to transmit the same power at the same line loss as a 220-volt line of equal length. How will the size of the line wires of the two systems compare?

Prob. 16–11. If No. 00 (B. & S.) wire is used for the 110-volt line of Prob. 15–11 what size wire would be necessary for the 220-volt line?

193. Three-Wire System. The great saving of power or copper in the line by merely doubling the voltage has led to the wide establishment of 220-volt circuits. But lamps of about 110-volt rating are the most common type of incandescent lamp because they are the cheapest and most durable, and if they are to be used on a 220-volt circuit, they must be put two in series. This would compel a customer always to burn at least two lamps at once. If he needed three lamps, he would have to use four, two parallel sets of two in series.

* Lord Kelvin deduced the following rule with regard to the **most economical area of conductor** for transmitting a given amount of power at a given voltage:

The conductor should be of such an area that the value of power lost per year in the line, equals the interest cost per year of the money invested in the line.

To avoid this feature and still retain the advantage of transmitting at 220 volts, a third wire, called a neutral, has been added to the usual arrangement, which produces the **three-wire system** as shown in Fig. 242.

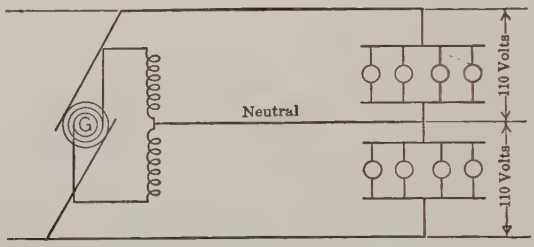


FIG. 242. A three-wire system.

This neutral wire is usually the same size as each of the other two in order to be able to operate one side of the system at full load even if the other side is out of commission. So if we allow the same watts lost in the line, the total amount of copper is a little greater than $\frac{1}{4}$ that of a two-wire system. In fact, there is just $\frac{3}{8}$ as much copper in the three-wire system.

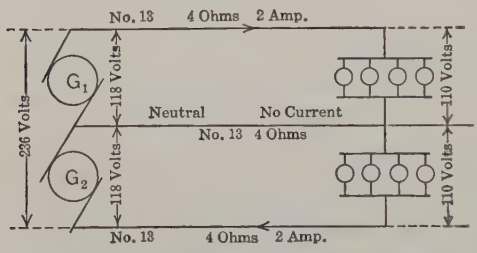


FIG. 243. A balanced three-wire system.

The generator also has to have a special device to which this third wire is attached. It is much simpler, however, to consider the generator as divided into two generators in series as in Fig. 243, with the neutral coming to the junction of the two.

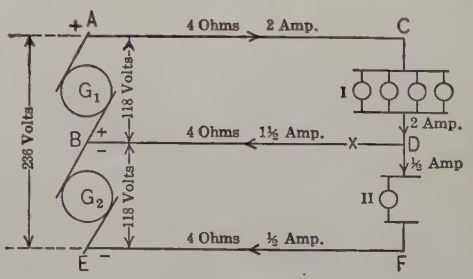


FIG. 244. An unbalanced three-wire system.

194. Balanced and Unbalanced Three-Wire System. When there are the same number of lamps burning on each side of the neutral, as in Fig. 243, the neutral carries no current, and the system is said to be **balanced.** It is only when the system is **unbalanced** that the neutral is of use and carries current, as in Fig. 244. The

system may be said to be unbalanced, then, when the appliances on one side of the neutral are carrying more current than those on the other side, thus compelling the neutral to carry the surplus. Thus, if four $\frac{1}{2}$-ampere lamps were turned on, as in Fig. 244, on the (+) side of the neutral, and only one on the other side, the neutral would have to carry $1\frac{1}{2}$ amperes back to the generator G_1 which is supplying the greater part of the power.

195. Voltage Distribution in Three-Wire System. In using the three-wire system, every effort is made to keep it balanced. In this case, the voltage, current and power distribution differ in no respect from that of a two-wire system, with lamps, or other electrical appliances, paired off, two in series.

Of course on a large system any slight deviation from a balance makes no noticeable difference, but it is instructive to see what happens to the voltage distribution in a system unbalanced as much as that of Fig. 244.

The voltage across Group I is very easy to find.

To force 2 amperes through the line out to the lamps over a 4-ohm wire requires

$$2 \times 4 = 8 \text{ volts.}$$

To force 1.5 amperes through the return line to the generator over a 4-ohm wire (the neutral) requires

$$1.5 \times 4 = 6 \text{ volts.}$$

$$\text{Line drop} = 6 + 8 = 14 \text{ volts.}$$

$$\text{Voltage across Group I} = \text{voltage of } G_1 - \text{line drop}$$

$$= 118 - 14 = 104 \text{ volts.}$$

$$\text{Voltage across Group I} = 104 \text{ volts.}$$

The voltage across Group II is a little more difficult to compute. The easiest way is to draw a "voltage diagram," as in Fig. 245 which represents the voltage diagram of the circuit for Fig. 244.

Let the vertical lines AB and EB represent the 118 volts of the generators. Then draw the line wire AC sloping in the direction in which the current flows. Since the voltage drops 8 volts along the top wire, the line AC falls 8 volts from the level of A. In the same way the line DB slopes in the direction in which the current flows and falls 6 volts from the level of D, representing the 6 volts lost in the neutral wire. The rest of the vertical distance, from the level of A to the level of B, is the voltage across the points C and D, where the lamps of Group I are located. This is 104 volts, which agrees with the result found by computation.

Another way of stating it is that the distance between the horizontal lines is the voltage of the generator. Thus the line A_1B_1 would represent 118 volts. The line CD is 14 volts shorter than A_1B_1 and therefore represents 104 volts, which is the voltage across the lamps, because they are across the points C and D.

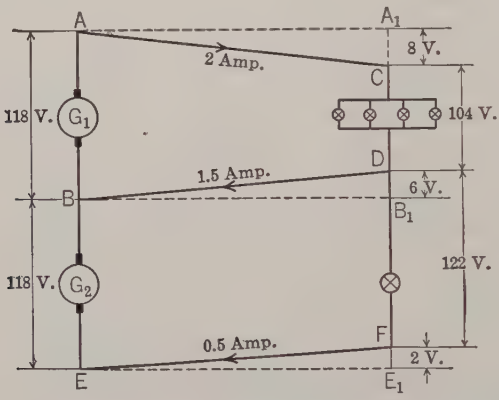


Fig. 245. Voltage diagram of an unbalanced three-wire system.

If, now, we should consider the circuit of Group II, DB is the neutral sloping toward the generator representing a fall of 6 volts toward the generator as a current is flowing in this direction. Line FE represents the wire EF in which a current of 0.5 ampere is flowing to generator II. Therefore the line FE slopes toward the generator. Since the wire FE has a resistance of 4 ohms, it requires 0.5×4, or 2 volts to force the 0.5 ampere through it. Therefore the line FE falls 2 volts from the level of F. Group II is connected to the line at points D and F, thus DF must represent the voltage across Group II. The distance between the horizontals is always the same as the voltage of the generator. The line B_1E_1, therefore, represents 118 volts. But the line DF, which represents voltage across Group II, is 2 volts shorter than B_1E_1 at one end, and 6 volts longer at the other; thus it must be, in all, 4 volts longer. It therefore represents $118 + 4$ volts, or 122 volts. Since Group II is connected in at points D and F, it must have 122 volts across it. Thus the voltage across Group II is 122 volts.

The voltage across Group I was found to be only 104 volts.

The lamps of both groups were made to run on 110 volts, and would probably be ruined by 122 volts. We can say, then, that the result of extreme unbalancing is to disturb the voltage distribution

to such an extent that appliances built for special voltages would not operate satisfactorily on the system.

Prob. 17–11. Each lamp in Fig. 246 takes 1.5 amperes. Find:

(*a*) Current in *AB*, *CD*, *EF*.

(*b*) Line drop in *AB*, *CD*, *EF*.

(*c*) Voltage across *BD* and *DF*.

(*d*) Resistance of each set of lamps.

(*e*) Power loss in lines.

(*f*) Efficiency of transmission.

(*g*) Output of each generator.

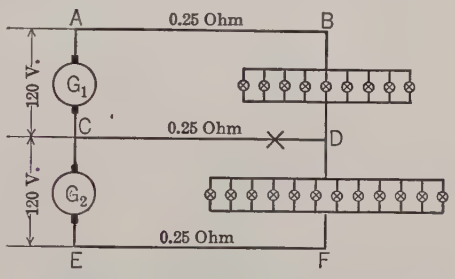


FIG. 246. An unbalanced three-wire system.

Prob. 18–11. Assuming that the resistance of the lamps in Fig. 246 remain constant, what voltage would there be across *BD* and *DF* if the neutral wire broke at point *X*?

SUMMARY OF CHAPTER XI

KIRCHHOFF'S LAWS. (An extension of Ohm's Law.) (1st) Sum of currents flowing away from a point in a circuit equals sum of currents flowing to the point.

(2nd) In any circuit, algebraic sum of e.m.f.'s equals sum of IR drops. In a circuit containing no source of e.m.f., sum of IR drops clockwise around the circuit, equals sum of IR drops counter-clockwise.

These laws are applied to find current distribution in "mesh work" and in systems of generators or battery cells operated in parallel.

ELECTRICAL TRANSMISSION. Is convenient and efficient over long distances.

LINE LOSS. Varies inversely as the square of the voltage of transmission. For a given line loss, the size of the wire, either by cross-section or by weight, varies inversely as the square of the voltage.

FEEDERS are used in low-voltage systems to increase size of conductors. Power, voltage and current distribution are found by Ohm's Law.

THREE-WIRE SYSTEM. Neutral wire is intended to carry very little current.

Requires three-eighths as much wire as a two-wire system of same capacity, and of same voltage as one leg.

Affords two standard voltages.

PROBLEMS ON CHAPTER XI

Prob. 19–11. An electric railway in the form of a rectangle 8 miles long and 3 miles wide is supplied by a generator located at the middle of one of the long sides. The trolley wire is No. 0 hard-drawn copper; the track resistance is 0.04 ohm per mile. A No. 0000 annealed-copper feeder runs from the generator directly across the rectangle to the midpoint of the other side where it joins the trolley wire. At a certain time, there is a car in the middle of each short side; one takes 90 amperes, the other 60 amperes. What is the voltage across each car if the generator delivers 600 volts?

Prob. 20–11. If there were 4 cars on the line in Prob. 19–11, one at each corner of the rectangle, each taking 60 amperes, what voltage would be across each car?

Prob. 21–11. In Prob. 19–11 what would be the voltage across each car if no feeder were used?

Prob. 22–11. What is the efficiency of transmission in Prob. 19–11? In Prob. 21–11?

Prob. 23–11. A 12-mile trolley line is supplied by a generator at one end delivering 610 volts. The trolley wire is No. 0 hard-drawn copper; the track resistance is 0.03 ohm per mile. There are two separate feeders from the generator to the trolley wire. One feeder of No. 0000 annealed-copper extends for 8 miles and is tied to the trolley every 2 miles. The other feeder is of 400,000 cir. mil hard-drawn copper and extends 4 miles out where it is tied to the trolley wire. There are three cars on the line, as follows: Car I, 4 miles from generator, takes 110 amperes. Car II, 7 miles from generator, takes 60 amperes. Car III, 11 miles from generator, takes 80 amperes. (*a*) What is voltage across each car? (*b*) What is efficiency of transmission?

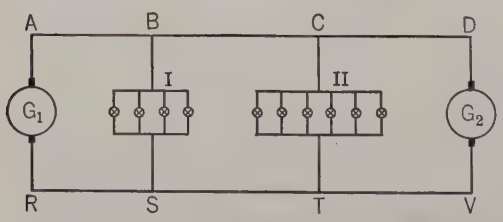


FIG. 247. Lamps fed by two generators.

Prob. 24–11. If the trolley wire in Prob. 23 breaks 1 mile from the generator, what will voltage across each car become?

Prob. 25–11. In Fig. 247, each lamp takes 10 amperes. The resistances of the different sections of the line and of the generators are as follows:

AB = 0.04 ohm	ST = 0.04 ohm
BC = 0.03 "	TV = 0.04 "
CD = 0.06 "	G_1 = 0.025 "
RS = 0.05 "	G_2 = 0.025 "
E.m.f. of G_1 = 132 volts.	E.m.f. of G_2 = 136 volts.

Find: (*a*) Voltage across each lamp group.
(*b*) Terminal voltage of each generator.

Prob. 26–11. What is the efficiency of transmission in Prob. 25–11?

Prob. 27–11. Each of two storage batteries has an e.m.f. of 8.0 volts and an internal resistance of 0.0120 ohm. If the two batteries are joined in series across a load of 1.6 ohms resistance, what current will flow?

Prob. 28–11. If the batteries of Prob. 27–11 are joined in parallel across same load, what current will flow?

Prob. 29–11. Assume one of the batteries in Prob. 28–11 had an e.m.f. of 7.6 volts and an internal resistance of 0.0060 ohm, and other remained as in Prob. 28–11. What current would flow through the load if joined as in Prob. 28–11?

Prob. 30–11. If batteries of Prob. 29–11 were joined as in Prob. 27–11, what current would flow?

Prob. 31–11. It is desired to light a building with 300 incandescent lamps, in parallel. The lamps have a hot resistance of 200 ohms each and operate on 115 volts. A battery of storage cells is to be used, each of which has an e.m.f. of 2.20 volts, an internal resistance of 0.0040 ohm. Normal discharge rate of each cell is 25 amperes. How many cells are required and how should they be arranged?

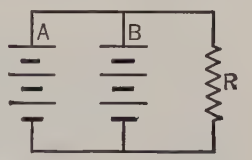


FIG. 248. A resistance fed by batteries connected in parallel.

Prob. 32–11. The series-parallel group of six cells in Fig. 248 is discharging through a resistance of 1.5 ohms. Normally, each cell has an e.m.f. of 2.1 volts and internal resistance of 0.018 ohm. One cell in set A is defective and has an e.m.f. of 1.7 volts and an internal resistance of 0.094 ohm. What current is supplied to R by each set of cells?

Prob. 33–11. If the resistance of R, Fig. 248, is reduced to 0.02 ohm, with other data as in Prob. 32–11, what current will flow through each set of cells and in what direction?

Prob. 34–11. If no current is supplied by the cells of set A in Prob. 32–11, what is the value of the resistance R? What current is flowing in the load?

Prob. 35–11. If the resistance of R, Fig. 248, is increased to 2.6 ohms and the other data left as in Prob. 32–11, what current will flow through each set of cells and in what direction?

Prob. 36–11. Four storage cells in parallel supply a coil of 2 ohms resistance. Each cell has an e.m.f. of 2.15 volts and an internal resistance of 0.022 ohm. What current is supplied to the coil, and what is the terminal voltage of the battery?

Prob. 37–11. One of the cells in Prob. 36–11 becomes defective. Its e.m.f. drops to 1.96 volts and the internal resistance increases to 0.074 ohm. The four cells are still used as in Prob. 36–11. (*a*) What current is supplied to the coil? (*b*) What is the terminal voltage of the battery?

Prob. 38–11. A storage battery of 240 cells in series is maintained at the end of a 4-mile trolley line, the resistance of which (line and return) is 0.16 ohm per mile. The generator voltage is 550 volts; each cell has an e.m.f. of 2.1 volts and an internal resistance of 0.001 ohm. (*a*) What current will the cells supply to the line, when there are 5 cars at the battery end of the line, each taking 80 amperes? (*b*) What will be the terminal voltage of the set of battery cells?

Prob. 39–11. What current will the cells in Prob. 38–11 take when there is no load on the line?

Prob. 40–11. What current will the generator be delivering when the 5 cars of Prob. 38–11 are half way between station and battery? Cars are carrying the same current as in Prob. 38–11.

Prob. 41–11. What current will the battery be delivering to or receiving from the line in Prob. 40–11?

Prob. 42–11. How far must the 5 cars of Prob. 38–11 be from the generator in order that the battery shall neither receive nor deliver current?

Prob. 43–11. Prove the statement that a three-wire system takes $\frac{3}{8}$ as much copper as a two-wire system to transmit a given amount of power a given distance with the same line loss. The voltage between one side and neutral is to be the same as the voltage of the two-wire system.

Prob. 44–11. A 500-kw. generator is to be selected to transmit power 5 miles over a 2-wire line No. 00 copper wire. Plot a curve showing relation of voltage to efficiency of transmission for above power, ranging from 0 per cent efficiency to 99 per cent. Have at least 12 points about equally spaced along the curve.

Prob. 45–11. State what voltage you would select to transmit power in Prob. 44–11. Using 3 times this voltage, what size wire could be used with the same line loss?

Prob. 46–11. Allowing 5 per cent of the power generated to be lost in the line, plot a curve between size of the copper wire (in circular mils) and voltage of transmission in Prob. 44–11. Plot at least 8 points.

Prob. 47–11. According to Kelvin's Rule for most economical size conductor, what size copper wire should be used under the following conditions?

Cost of installed copper wire per lb., \$0.20.
Cost of generating power \$0.015 per kw.-hr.
Probable rate of interest, 5 per cent.
Power to be transmitted, 100 kw.
Voltage of transmission, 550 volts.
Power to be used 10 hrs. per day throughout year.
Copper weighs 555 lb. per cu. ft.

Prob. 48–11. Compute size of aluminum wire that should be used under conditions of Prob. 47–11, assuming that aluminum costs twice as much per lb. installed as copper. Aluminum weighs 168 lb. per cu. ft.

Prob. 49–11. If double the voltage of Prob. 47–11 is used what size copper conductor is the most economical?

Prob. 50–11. Assume that each lamp in Fig. 249 takes 3.2 amperes and that the resistance of the lamps remains constant. The brush potential of the generator is 220 volts. Each small machine of the balancer set is so regulated as to maintain 110 volts across it. Find:

(*a*) Line drop in each section.
(*b*) Voltage across each set of lamps.
(*c*) Efficiency of transmission.

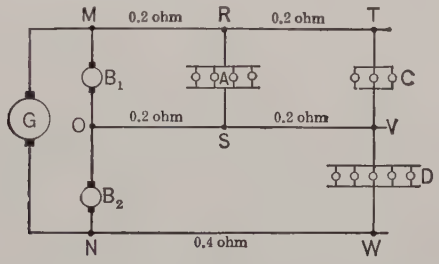


FIG. 249. An unbalanced three-wire system.

Prob. 51–11. If a break occurred in the neutral between *O* and *S*, Fig. 249, what would be the values of (*a*), (*b*), and (*c*) of Prob. 50–11? The lamp resistances are to remain as in Prob. 50–11.

Prob. 52–11. If a break occurs between *S* and *V*, Fig. 249, what will be the values of (*a*), (*b*), and (*c*) of Prob. 50–11? Lamp resistances remain as in Prob. 50–11.

Prob. 53–11. If a break occurs in the outside wire between *R* and *T*, Fig. 249, what will be the values of (*a*), (*b*), and (*c*) of Prob. 50–11? Lamp resistances remain as in Prob. 50–11.

Prob. 54–11. An electric railroad is supplied with power by sub-stations 8 miles apart. The trolley wire is connected at frequent intervals with a heavy copper conductor called a "feeder." The two may be considered jointly as equivalent to a single conductor whose resistance is 0.18 ohm per mile. The return path from the cars to the sub-stations consists of the track which is well bonded at the joints and has a resistance of 0.032 ohm per mile. At a certain time two cars are running in the section between these sub-stations, one taking 185 amperes at a distance of 3.5 miles from one end, the other taking 240 amperes at a distance of 2.5 miles from the other end. The voltage at the busbars of each power station is 680. What is the voltage between trolley and track at each car?

Prob. 55–11. An electric circuit consists of the armature of a separately excited generator, two connecting wires and a storage battery, all in series. The armature has a resistance of 0.28 ohm and generates an e.m.f. of 118 volts by its rotation. Each of the connecting wires has a resistance of 0.2 ohm. The battery has a resistance of 0.16 ohm and generates by chemical action an e.m.f. of 108 volts opposed in direction to the e.m.f. of the generator. (*a*) Find the terminal voltage of the generator. (*b*) Find the terminal voltage of the battery.

Prob. 56–11. A single-track railroad line is 20 miles long and is supplied with power from power stations at its two ends. The positive conductor consists of a third rail reinforced by a copper cable in parallel, the combination having a resistance of 0.026 ohm per mile. The negative conductor consists of a pair of bonded track rails in parallel, the combination having a resistance of 0.035 ohm per mile. The station at one end maintains a potential difference of 600 volts and that at the other end 575 volts between positive and negative conductors.

At what point along the line would an electric locomotive requiring 1200 amperes receive a minimum voltage between third rail and track? What would this voltage be, and what current would each station supply when the locomotive was at the point in question?

Suggestion: Make determination from plot of voltage and distance from one station.

Prob. 57–11. Figure 250 represents a method of raising line voltage. *T* is the trolley wire and *R* the rails. *F* is a feeder which is connected to the trolley wire at a distance of 4.0 miles from the power station.

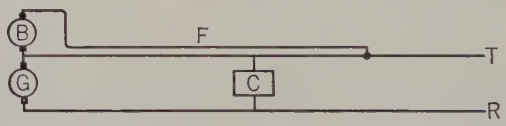


FIG. 250. The booster *B* raises the voltage at the load *C*.

B is a booster, *i.e.*, a generator used to increase the voltage applied to the feeder. The following conditions exist. The terminal voltage of the generator *G* is 650. The trolley wire has a resistance of 0.064 ohm per 1000 feet. The rails form a return conductor whose resistance is 0.0061 ohm per 1000 feet. The booster armature has a resistance of 0.024 ohm and generates an e.m.f. of 100 volts. The total resistance of the feeder is 0.82 ohm. *C* is a train requiring a current of 580 amperes. What is the lowest voltage which the train can have in the region between the power station and the point of connection between the feeder and trolley?

Suggestion: Plot curve for points at various distances from the power station and determine lowest voltage from the curve.

CHAPTER XII

INDUCTANCE

In the previous chapters on magnetism and electricity, we have dealt almost entirely with steady electric currents and magnetic fluxes. We must now consider the phenomena occurring when we start or stop an electric current or a magnetic flux.

196. Induced E.M.F. Mutual Inductance. Suppose we have two coils of wire, A and B, Fig. 251. Coil A is placed within coil B, but has **no electrical connection to it.** Across the terminals of B a voltmeter is placed. When switch S is closed, a current rushes into coil A and flows around the coil counter-clockwise, looking at top of coil. But also, strangely enough, a **momentary** current flows in coil B, and through the voltmeter, **but in the opposite direction.** This momentary current in B lasts for but an instant, and dies out.

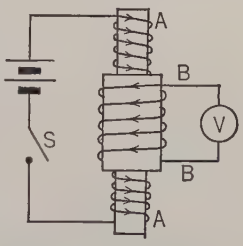


FIG. 251. A change of current in coil A sets up a voltage in coil B.

If now, we suddenly open the switch S, in order to stop the current in A, we notice that another **momentary** current is set up in B. This time the **momentary current in** B is in the same direction as the current we are stopping in A. This momentary current, like the first one, dies out almost instantly. When we closed the switch and started a current in A, we noticed that a **momentary** current was set up in B, opposite in direction to the current we were starting in A. On the other hand, while we were stopping the current in A, we noticed a momentary current set up in B, in the same direction as the one we were stopping. These momentary currents set up in B, whenever there is any **change** in the current in A, are called **induced currents.** The e.m.f. tending to cause them to flow is called an **induced e.m.f.** and the two electric circuits are said to possess the property of **mutual inductance.**

Note that, although in one case the induced current was in the opposite direction to the current in A, and in the other case in the same direction, in both cases it **opposed the change taking place** in the current in A. It **opposed** the **setting up** of a current in A,

when we closed the switch, by setting up one in the opposite direction. When we opened the switch, it **opposed** the **stopping** of the current in A, by setting up one in the same direction as the one we were stopping. In each case as soon as the **change** in the current in A was over, the induced current in B died out.

Let us consider also the magnetic field set up by these induced currents in B. When we sent the current into A, we were setting up a field within the coil making the top end a north pole. Note that the induced current in B **opposed this setting up** of a north pole by setting up a field with a south pole at this end, in order to neutralize the field being set up. But after the field was once set up, the current in B ceased trying to neutralize it and stopped flowing. As soon, however, as we began to destroy this field by stopping the current in A, a current was induced in B, which **opposed the dying out of the field,** by setting up a field of its own in the same direction as the one we were destroying. In each case the field of the induced current **opposed the change** that was taking place in the field within the coils.

All these phenomena are merely manifestations of a great law first stated by Lenz, and called Lenz's Law, part of which is:

An induced current is always in such a direction that its field opposes any change in the existing field.

It remains to be shown that it is really the magnetic field set up by an electric current which causes these phenomena of induction; that the magnetic field contains the kinetic energy of a circuit, just as a fly wheel and other moving parts contain the kinetic energy of a machine in motion.

In getting a machine up to speed, we must overcome not only the force of friction but also the **inertia reaction** during the process. In setting up a current in a circuit, we must overcome not only the resistance (friction) of the circuit, but also the **inductance** (inertia) reaction of the magnetic field. In trying to stop a machine which has a large fly wheel, we feel conscious of some reaction tending to keep the machine turning, *i.e.*, an opposing force, which increases enormously if an effort is made to stop the wheel suddenly. This is the reaction due to the inertia of the machine. Similarly, we find that there is a reaction which tends to oppose the sudden stopping of an electric current which has a strong magnetic field, *i.e.*, there is a force which tends to keep such a current flowing. This is the reaction due to the **inductance** of the circuit. The larger the **moment**

of inertia of the fly wheel, the larger is the force opposing any effort to stop it in a given time. The larger the **inductance** of the electric circuit, the larger the force which opposes the stopping, in a given time, of the current flowing through it.

Just as the reaction due to inertia opposes the increasing or decreasing of the speed of a fly wheel, so the reaction due to inductance opposes the increasing or decreasing of the current in an electric circuit. Let us investigate the causes and the practical results of this property of a magnetic field.

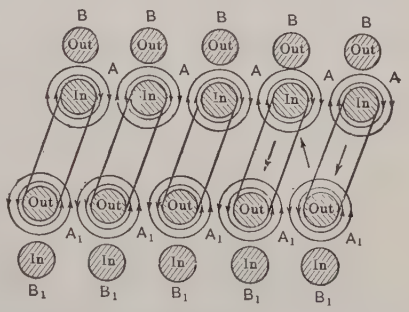


FIG. 252. A cross-section of the coils in Fig. 251.

197. Induction Coils. In Fig. 252, the inner coil A is composed of a few turns of coarse wire. If a current were sent into coil A, let us consider what would happen while the current was building up to its maximum value.

Assume the current enters coil A at the right-hand end and flows **in** at the top of the loop A and **out** at the bottom of the loop A_1. This causes a clockwise field to grow around the top wires of coil A and a counter-clockwise field around the bottom wires A_1. This field spreads out in ever widening rings as the current is increased and cuts across the sides of the coil B. This is shown in Fig. 253 (1), (2), (3), where the wire A represents the end of a wire in coil A, and B represents the end of a wire in coil B. It shows that, as the current in A grows, the magnetic field caused by it spreads out and cuts across the wires of coil B. The wires on the top of coil B are cut by the lines as they move upward. This is equivalent to the wires of B moving downward and cutting the lines. By applying the right-hand rule, we see that there would be a voltage induced in B tending to send a current **out** of B. **This is in the opposite direction to the inducing current in A.**

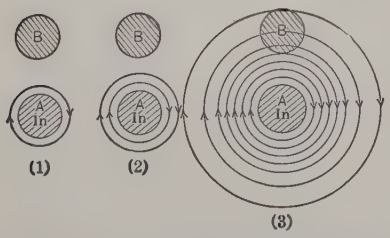


FIG. 253. The growing flux around wire A cuts wire B and induces a voltage in B.

If we apply this to the lower sides of the coil, Fig. 252, we find that the inducing current in coil A is **out** in the wires A_1 at the bottom, and that the wires B_1 of coil B are cut by the growing field in such a way as to induce a voltage which tends to send a current **in** at B_1. Here again we see that the **induced voltage** in coil B **is opposite** in direction **to the increasing current** in coil A. If we look at the left-hand end of the coil, Fig. 254, we see that the inducing electric current in A is in a counter-clockwise direction. As long as it is increasing, it is causing an induced current to flow in B in a clockwise direction.

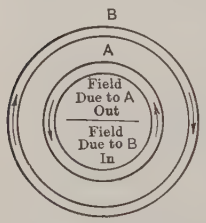


FIG. 254. End view of coils in Fig. 251.

The magnetic field in the air core, due to the current in B, is opposed to the building up of the field due to the current in A.

The current in A is trying to build up a field **out** (Fig. 254), while the current induced in B tends to oppose this building up of a field **out** by neutralizing it with a field **in.**

There is no electrical connection between coil A and coil B. They are very carefully insulated from each other. Yet all the time that a **current is increasing** in one direction in A, it causes a current to build up in B. The field of this induced current in B **opposes the building up** of a field by the current in A. As soon as the current in A reaches its constant value, the lines of force around the wires no longer will be spreading out and there will be no lines cutting the sides of B, and thus the induced current in B will die out.

The induced current thus lasts **only as long as the "primary" current** (the current in A) **is growing.** The opposition to the building up of the field is due to the **inductance** of the coil. Since this opposition exists in another coil not connected electrically to the first coil, the two coils are said to possess **mutual inductance.**

As long as a force is trying to speed up a machine, it has to overcome a certain opposition to the building up of speed. As soon as a constant speed is reached, the opposition to building up ceases. This opposition to building up the speed is due to the **moment of inertia** of the moving parts, as we have stated before.

By comparing the two statements, it can be seen that there is a close analogy between **inertia** and **inductance,** in that they both oppose an **increase.**

But inertia reaction not only opposes any increase in speed, but also opposes any **decrease** in speed. Let us now consider how in-

ductance reaction opposes the **decrease** of the current. Figure 255 (1), (2), (3), represents the current in A dying out. The field around each wire now contracts and in so doing, again cuts the wires of coil B. But this time the cutting is in the opposite direction to what it was when the field was spreading out. As the field was spreading out, there was a current of **out** induced in the coil B, which was **opposite** to the current of **in** in A. Now, by applying the right-hand rule, we find that when the field is contracting, there is a current of **in** induced in B, which is in the **same direction** as the dying current of **in** in A.

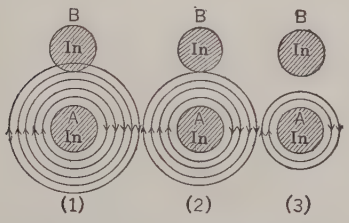


FIG. 255. The contracting field about A cuts across the wire B.

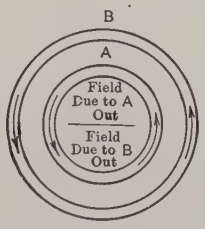


FIG. 256. End view of coils in Fig. 251 when field in A is decreasing.

If we look again at the end view of the coil (Fig. 256) as the current is dying out in A, instead of increasing, we see that the field due to the dying current in A is **out** and the field due to the induced current in B is also **out.**

This shows that the field due to the induced current B **opposes the decreasing** of the magnetic field of current in A by building up one in the same direction as the dying field.

This is as we should expect, since we have seen that inductance is analogous to inertia.

Inertia reactions oppose any increase or decrease in the existing speed.

Induced currents oppose any increase or decrease in the existing magnetic field.

198. Direction of E.M.F. due to Inductance. Lenz's Law. The first part of Lenz's law is:

The direction of the induced e.m.f. is such that it tends to set up a current the magnetic field of which always opposes any change in the existing field.

The important word in this law is the word **change.** The field of the induced current does not always oppose the existing field, nor

does it always aid it, but it always opposes any **change** in it. Thus if the existing field is zero, and we send a current through the primary coil to set up a field, there will be induced in the secondary coil a current the field of which will **be opposite** to the growing field, and thus will tend to keep the condition of the field as near zero as possible. But if there is a field already within the coil, and we try to weaken it, say by decreasing the current in the primary, there will be induced in the secondary coil a current the field of which will **aid** the existing field and tend to keep it at the same strength. The induced current thus is always such that its field **opposes any change** in the existing magnetic field.

The second part of Lenz's law is:

The amount of the induced e.m.f. is equal, in volts, to one hundred-millionth of the number of cuttings per second of the magnetic field by the circuit in which the e.m.f. is induced.

Thus the amount, in volts, of the induced e.m.f. in each turn of coil B is equal to one hundred-millionth of the flux lines per second that cut across the turn every time the current in coil A is changed. The total voltage induced in coil B is this number multiplied by the number of turns in coil B.

Example 1. Assume that there are 50 turns in coil B and that when the current is turned on in A, 40,000 magnetic lines cut across each turn in 1/100 of a second. What e.m.f. will be induced in coil B?

Solution.

Lines cut per turn per sec. $= 40{,}000 \times 100 = 4{,}000{,}000$ lines/sec.

Total cutting per sec. $= 4{,}000{,}000 \times 50 = 200{,}000{,}000$ lines/sec.

Voltage induced $= \dfrac{200{,}000{,}000}{100{,}000{,}000} = \dfrac{2 \times 10^8}{10^8} = 2$ volts.

Prob. 1–12. If the field in Example 1 were produced in 0.0025 second, what voltage would be induced in B?

Prob. 2–12. In a circuit similar to Fig. 251, an induced e.m.f. of 540 volts is produced. If the field is cutting each turn of coil B 450,000 times in 0.0001 second, how many turns are there in the coil?

Prob. 3–12. When the switch of coil A in an arrangement like that of Fig. 251 is opened, 800,000 magnetic lines sweep across each of the 7200 turns of coil B. In what time do these lines cut the various turns of coil B if 30 volts are induced in B?

199. Induction Coils: Ignition System of Gas Engine. One type of induction coil often used for igniting the charge in a gas engine, depends upon this induced voltage in the secondary coil.

The primary has comparatively few turns of heavy wire; the secondary has many turns of fine wire. A core of soft iron is put within the coils to strengthen the magnetic field. The secondary and the primary windings are so close together that practically all the flux set up by the primary cuts the secondary. The secondary circuit B, Fig. 257, has an air gap in it. This gap is between two points (R and R_1) which are inside the cylinder of the gas engine. The primary circuit A is connected to a set of battery cells. The cam first closes the primary circuit and causes a field to be set up in the coil. Then, as the cam opens the primary circuit, the field is suddenly diminished as the current in the primary dies out. This causes a high induced e.m.f. to be set up in the secondary, because the field on dying out cuts the large number of turns in the secondary at a rapid rate. This induced e.m.f. is high enough to cause a spark to jump between the points R and R_1, and ignite the charge of gas in the cylinder. There may be a small spark when the circuit in the primary is made, because the **growing** field cuts the wires of the secondary coil. But the field **grows** more slowly than it **dies out** and thus the induced e.m.f. is not as great and the spark is much more feeble.

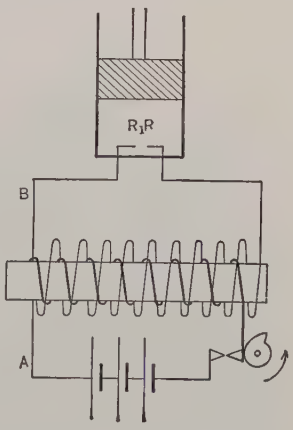


FIG. 257. Jump-spark coil for gas-engine ignition.

200. Ruhmkorff Coils. Radio Vibrator. A Ruhmkorff coil is built on the same plan as the jump-spark coil. The number of turns in the secondary is many times the number in the primary, and the voltage across the secondary is many times that across the primary. The voltage of the secondary will be in about the same ratio to the voltage of the primary as the number of turns in the secondary is to the number of turns in the primary. Of course what is gained in voltage is lost in amperage.

The Ruhmkorff coil has an interrupter in the primary, which works very rapidly, sometimes as high as 200 times a second. This will cause a continuous discharge of sparks to pass across the spark gap in the secondary when the current is broken. The current for the primary is usually supplied by battery cells. This type of coil is in general use with a gas engine, in which case, when the cam in Fig. 257 closes the primary circuit, the interrupter makes and breaks

it very rapidly, causing a series of sparks across the gap in the secondary.

Figure 257(*a*) shows a type of induction coil now widely used for operating automobile radio sets from the car battery. A primary coil A of relatively few turns of heavy wire is wound on an iron core, and supplied from the storage battery through the contact S_1. The secondary B is wound on the same core, but has many turns of fine wire. It is connected to a load circuit through the contact S_2. Both S_1 and S_2 move together and are operated by the magnetic field set up in the core. Normally, S_1 is closed and S_2 is open. When current flows in A, a magnetic field is set up in the iron core, and the device is built so that this field opens the contact S_1. The current in A then decreases, and S_1 is returned to the closed position by a spring. The action then repeats, and the contact S_1 simply vibrates just as an electric bell or buzzer.

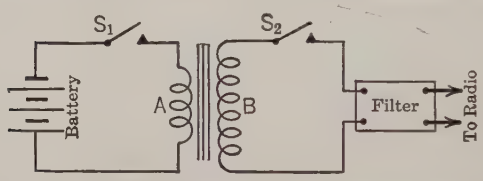


FIG. 257(*a*). An induction coil used to supply high-voltage direct current for automobile radio sets.

As S_1 vibrates, the current in the primary increases and decreases very rapidly and thus induces a high voltage in B which changes in direction at the same frequency as the vibration of S_1. The contact S_2 is attached mechanically to S_1 and vibrates with it, but is so arranged that it closes the secondary circuit only during the time that the voltage in the secondary coil is in one direction. That is, S_2 closes just before S_1 opens, and then S_2 opens just before S_1 closes. Thus the 6-volt storage battery of the car is used to produce a much higher d-c voltage in the secondary coil B.

The voltage in B is not steady, and if it were used directly in the radio set, nothing would be heard but noise. It is therefore passed through a " filter " as shown in Fig. 257(*a*), where it is " smoothed." The type of circuit used in this filter is described in Art. 290.

201. Transformers. Transformers are mutual induction coils used on **alternating current** lines for raising or lowering the voltage. Each transformer consists of a primary coil and a secondary coil wound on a core of soft iron or annealed steel.

We have seen that it is much more efficient to transmit power at high voltages. Accordingly, the coil of few turns is connected to the generator and the coil of many turns to the line. The voltage

of the line is then approximately as many times higher than the voltage of the generator as the number of turns in the line coil is greater than the number in the coil connected to the generator. A transformer so connected is called a "**step-up**" transformer. Since it is difficult to use this high voltage for driving motors, etc., it is "stepped down" wherever it is to be used. This is done by connecting in the same kind of transformer, putting the high-voltage side (the one with the large number of turns) on the line, and connecting the low-voltage side to the motor. Thus, by means of two transformers of the same kind, alternating current power may be generated and used at low voltage, yet transmitted at high voltage. The transformers of the present date being remarkably efficient, the loss in transmission is thus reduced to a minimum.

FIG. 258. A distribution transformer. *General Electric Co.*

Figures 258 and 259 show the appearance and construction of a typical distribution transformer. Figure 258 illustrates the appearance of the transformer in its steel case. The four leads at the front come from two low voltage coils which can be connected either in series or parallel to give two different voltages. One of the primary leads is shown on the right-hand side. Note in Fig. 259 that the core is laminated to reduce eddy currents to a minimum.

In drawing circuit diagrams, a transformer is usually represented as shown in Fig. 260.

The action of a transformer follows the same principle as the action of a jump spark coil except that the current in the primary instead of being periodically **broken,** is periodically **reversed** by being connected to a source of alternating current.

The current in the secondary thus opposes not only the dying out of a magnetic field in the core, but also an actual setting up of a field in the opposite direction. Since the current in the primary coils alternates, the field in the core continually changes from a

maximum in one direction to a maximum in the other, and there is thus induced in the secondary coils an alternating e.m.f. which tends to oppose this **continuous change,** according to Lenz's law.

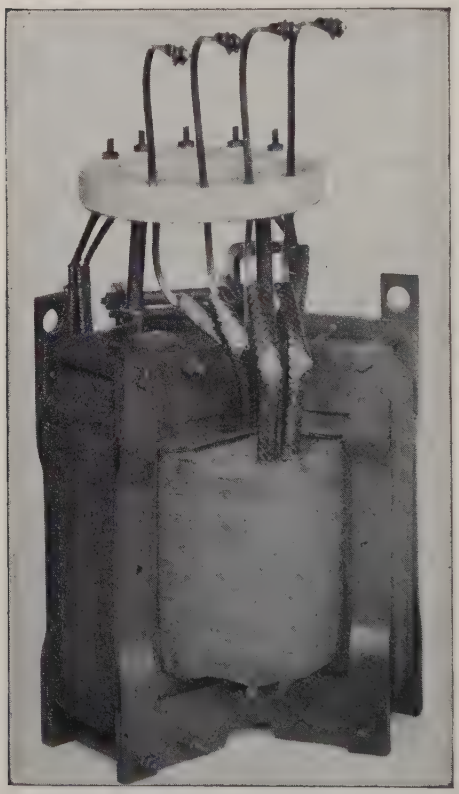


Fig. 259. Contents of a distribution transformer. *General Electric Co.*

The method of transmission by step-up and step-down transformers is shown in Fig. 261. Notice the peculiar circumstance that the alternating current motors M_1, M_2 and M_3 have no electrical connection with the generator G, yet they draw all their power from it. Note also that the transformers T_1, T_2 and T_3 are connected in parallel.

202. Self Induction. Suppose we have a piece of wire 1000 feet long and of 11 ohms resistance. If we stretch this wire out double, "line and return," for transmitting power 500 feet, and impress 110 volts across its terminals, a current of 10 amperes will flow, according to Ohm's law. Practically the instant we close the switch the current rises to a steady value of 10 amperes and remains so until we break the circuit. On breaking the circuit, a very slight spark may be noticed.

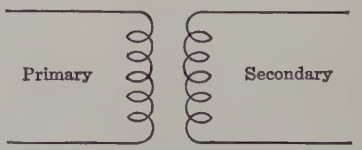


Fig. 260. Conventional diagram of a transformer.

Now suppose we wind this same wire into a single coil as in Fig. 262. The coil still has 11 ohms resistance. When a pressure of 110 volts is now put across its terminals, the ammeter in series with the coil will register 10

amperes, in obedience to Ohm's law. But it will not register the 10 amperes the instant that the switch S is thrown. It will re-

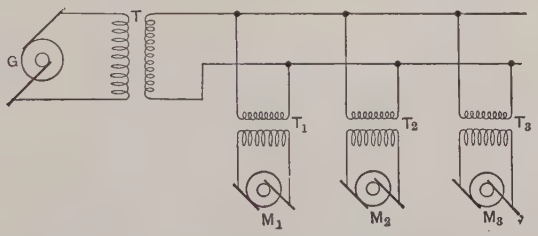


FIG. 261. Generator G supplies the three motors with power through transformers and line.

quire some time to acquire the steady value of 10 amperes. If we should take data concerning the current and the length of time

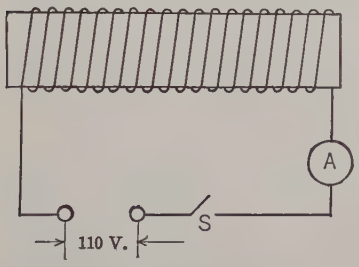


FIG. 262. A coil having self-inductance.

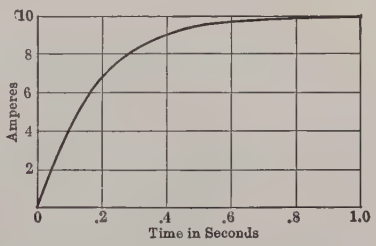


FIG. 263. Curve showing growth of current in an inductive circuit.

since the switch was closed, we should secure some such results as the following:

Time Elapsed since Switch was Closed, in Seconds	Current in Amperes
0	0
.1	4.2
.2	6.6
.3	8.0
.4	8.9
.5	9.3
.6	9.6
.7	9.8
.8	9.9
.9	10.0 (practically)
1.0	10.0 (practically)

Plotting these data, for current strength and time, we should get a curve as in Fig. 263.

Note that it required 0.9 second for the current to rise to practically its full value, after the pressure of 110 volts was thrown across the coil. But once the current reached the value of 10 amperes, which it should have according to Ohm's law, it remained constant. After the voltage had been on for 0.2 second, the current had risen to only 6.6 amperes. There must be some property in a single coil wound in this way which was not appreciably present in a straight wire and which now opposes any rise of the current.

If, as before, we open the switch S, we find that the current has a tendency to keep flowing, as is evidenced by the spark at the breaking points of the switch. There is evidently some property in the circuit in this form which opposes the dying out of the current. This property in a single circuit which opposes both the building up and the dying out of the current is called **self inductance.** A momentary e.m.f. which is set up in the opposite direction to the change that is taking place is called the **induced e.m.f.** It is this e.m.f. induced in the circuit which keeps the current down. The current in any circuit is always proportional to the algebraic sum of the impressed e.m.f. and induced e.m.f. When a circuit has high self inductance, this induced e.m.f. is great if we quickly build up a current, or destroy a current in the circuit.

During the time the current was increasing, the induced e.m.f. was opposing the impressed e.m.f. and the difference between the two voltages determined the current. Thus, at the end of 0.2 second, since the current was only 6.6 amperes and the impressed voltage was 110 volts, there must have been an induced e.m.f which opposed the impressed e.m.f. and cut the current down. For a growing current we might express this in the form of an equation:

$$i = \frac{E - e}{R}$$

where E = impressed voltage
i = momentary value of growing current, shown in table on preceding page
e = momentary value of induced e.m.f.
R = resistance of circuit.

The induced e.m.f. at the end of 0.2 second can then be found as follows:

$$6.6 = \frac{110 - e}{11}$$

$$e = 110 - 72.6 = 37.4 \text{ volts.}$$

At the end of 0.4 second, the induced voltage would be much less. Using the 0.4-second values,

$$8.9 = \frac{110 - e}{11}$$

$$e = 110 - 97.9 = 12.1 \text{ volts.}$$

The curve in Fig. 263 is steeper at the 0.2-second point and the current there is changing more rapidly than at the 0.4-second point. The induced e.m.f. is greatest when the current is **changing** at the greatest rate, and dies out entirely when the current becomes steady.

If the switch is opened and the current is dying out, the self inductance of the circuit tends to keep it flowing, by setting up an induced e.m.f. to cause a current to flow in the same direction. This induced e.m.f. is also proportional to the rate of change of current. And since a current has a tendency to die out very rapidly as the circuit is broken, the induced e.m.f. may be very great. Thus when a circuit of high self inductance is broken, the induced e.m.f. is frequently high enough to break down the air at the switch and an arc is formed.

Thus we have seen that even a single circuit possesses the property of inductance if it has a magnetic field. The inductance sets up an e.m.f. which opposes any change in the existing current of the circuit. This property is called **self inductance,** when the momentary e.m.f. is induced in the **same** circuit in which the current change takes place. It is exactly like the induced e.m.f. which the property of **mutual inductance** sets up in another coil. The induced e.m.f. in each case obeys Lenz's law and tends to set up a current which **opposes any change** in the existing magnetic field.

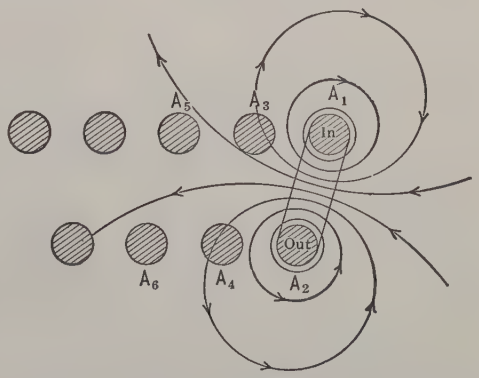


FIG. 264. The expanding field about A_1 and A_2 cuts across A_3 and A_4.

203. Cause of Self Inductance. Make-and-Break Spark Coil. Consider Fig. 264. A current is sent into the coil. A field in growing around A_1 cuts A_3, and by the right-hand rule, induces an e.m.f.

which tends to send a current **out**. But a current of **in** is being sent through A_3. This growing current of **in** in A_3 is thus opposed by the e.m.f. of **out** induced by the growing field around A_1. Similarly, the current in A_5 is opposed by an e.m.f. induced by the growing field around A_3, and the current in A_4 by the growing field of current in A_2, etc.

Also, if we consider the entire field within the coil, we see that an e.m.f. is induced in the coil, which tends to set up a current opposing the building up of a field within it. Thus a growing current in such a coil is said to be "choked" back, and takes some measurable length of time to come up to the full value as expressed by Ohm's law.

In a like manner, when the circuit is broken and the field around the wires and within the coil starts to die out, the wires are again cut by the field, though this time in a direction which sets up an e.m.f. tending to maintain the field and current as it is.

If the magnetic field is sufficiently strong and the current is broken suddenly, the voltage induced by the dying field cutting the wires of the coil may be many times the original impressed voltage. Advantage is taken of this fact in the **make-and-break** spark coil used with marine gas engines.

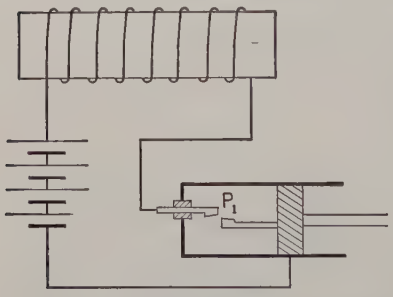


Fig. 265. A make-and-break induction coil for gas-engine ignition.

A single coil of many turns, as in Fig. 265, is wound on a core of soft iron wires. A moving point P_1 makes contact with a stationary point. Both are inside the cylinder of a gas engine. This causes a current to flow through the coil and set up a strong magnetic field. When the point P_1 leaves the stationary point there is a sudden breaking down of this magnetic field which sets up enough induced e.m.f. to cause a spark to jump across the gap now existing between the two points.

It may be stated as a general principle that whenever an inductive circuit (one having a strong magnetic field) is suddenly broken, the induced e.m.f. is likely to set up an arc at the breaking point.

This is also illustrated in the special field switch used in the shunt field of large generators. The inductive effect of the windings is so great that if the circuit were broken too suddenly the high e.m.f.

induced by the dying field might be great enough to puncture the insulation of the field winding. For this reason a special switch is used, which first connects a resistance across the field terminals and then disconnects the field from the line. The energy of the field is then slowly dissipated in heating the resistance and no high e.m.f. is induced.

204. Inductance a Property of the Circuit. The property of inductance is a **property of the circuit,** not of the electric current or voltage. It depends entirely upon the shape and size of the circuit and upon the magnetic permeability of the surrounding medium. If the medium consists of iron, which has a high permeability, and the circuit consists of a coil of wire, the inductance of the circuit is great. On the other hand, if the circuit consists of a straight wire strung in the air, which has low permeability, the inductance of the circuit will be small. The current, voltage and power, ordinarily have nothing to do with the amount of inductance of the circuit, though these are often affected by the inductance.

Inductance, then, is not a **material thing,** but merely a **term** which expresses the result of a certain **arrangement** of wires, iron, air, etc., in the electric and magnetic circuits. It has been likened to **moment of inertia,** which is not a **material thing,** but merely a term which expresses the result of a certain arrangement of weights, etc., about an axis.

205. Computation of Self Inductance. The Henry. The unit of **inductance** is the **henry.** A circuit is said to have **one henry of self inductance** when a change of the current of one ampere will cause 10^8 cuttings of magnetic lines by the circuit. Of course if this change of one ampere takes place in one second there will be induced an average voltage of one volt. In fact, one definition of a henry is the inductance of a circuit in which a change of one ampere per second induces one volt.

If we have a curve (or table) which shows how the current in a coil changes with time, we can make use of this definition to find the inductance of the coil.

In Art. 202 we learned how to find the momentary induced e.m.f. from the equation:

$$i = \frac{E - e}{R}.$$

We used this equation to find the induced e.m.f. in the coil of Fig. 262 at the end of 0.2 second and at the end of 0.4 second, and found

these two values to be 37.4 volts and 12.1 volts respectively. For the period from 0.2 second to 0.4 second the **average** induced voltage is $\frac{37.4 + 12.1}{2} = 24.8$ volts.

From the definition given above, we know that this average induced voltage depends on the value of the inductance of the coil in henries and the average **rate of change** of current in the coil in amperes per second. This relation can be expressed by the equation:

$$e_{av} = L\frac{i_1 - i_2}{t_1 - t_2},$$

in which e_{av} = average induced e.m.f. in volts,
i_1 = current in amperes at time t_1,
i_2 = current in amperes at time t_2,
$t_1 - t_2$ = time in seconds required for current to change from i_1 to i_2,
L = self inductance in henries,
$\frac{i_1 - i_2}{t_1 - t_2}$ = average rate of change of current.

In order to induce an average e.m.f. of 24.8 volts, the current changed from 6.6 amperes at 0.2 second to 8.9 amperes at 0.4 second. Putting these values in the equation for average induced e.m.f.

$$24.8 = L\frac{6.6 - 8.9}{0.2 - 0.4},$$

or

$$L = \frac{0.2 \times 24.8}{2.3} = 2.16 \text{ henries.}$$

When the inductance of the coil is known the equation for e_{av} can be used to find the average voltage induced during a change of current.

Example 2. A circuit has a self inductance of 2.4 henries. What voltage will be induced in the circuit when the current changes from 12 amperes to 4 amperes in 0.10 second?

Solution.

$$e_{av} = 2.4\frac{(12 - 4)}{0.10}$$

$$= 192 \text{ volts.}$$

Example 3. What flux cuttings did the change of current produce in the circuit of Example 2?

Solution.

1 henry produces flux cuttings of 10^8 lines for 1 ampere change.
1 henry produces flux cuttings of $10^8 \times 8$ lines for an 8-ampere change.
2.4 henries produce flux cuttings of $2.4 \times 8 \times 10^8$ or 19.2×10^8 lines for a change of 8 amperes.

If the time interval $t_1 - t_2$ used in finding the rate of change of current in the coil is very large the value of inductance found will be somewhat inaccurate. The smaller the time interval is made, the more accurate the value of inductance becomes. The inductance of a coil can be found exactly from the equation

$$L = \frac{0.4343\, Rt}{\log_{10}\left(\frac{E}{e}\right)}$$

in which

L = inductance of coil in henries,
R = resistance of coil in ohms,
E = voltage applied to terminals of coil,
t = any length of time after the voltage E is applied, in seconds,
e = induced e.m.f. in volts at the time t.

Using this equation, we find that at the time $t = 0.2$ second, the induced e.m.f. $e = 37.4$ volts. The terminal voltage is 110 volts and the resistance of the coil is 11 ohms. The inductance is then

$$L = \frac{0.4343 \times 11 \times 0.2}{\log_{10}\left(\frac{110}{37.4}\right)} = \frac{0.4343 \times 11 \times 0.2}{0.468}$$

$$= 2.04 \text{ henries.}$$

Using the first method, we obtained the value 2.16 henries, which is about 6 per cent higher than the more accurate result. For most practical purposes this error can be tolerated, and the less accurate method has the advantage that one does not need a table of logarithms in order to use it. Care should be taken, however, to work with small time intervals in order to prevent the introduction of serious error.

Prob. 4–12. A coil of 20 ohms resistance is switched across the terminals of a 200-volt line. One tenth second after the switch closes, the current in the coil is 3.68 amperes, and at 0.2 second the current is 6 amperes. What average voltage is induced in the coil during this period?

Prob. 5–12. What is the inductance of the coil of Prob. 4–12 as found from the average induced e.m.f.?

Prob. 6–12. What per cent error is there in the value of inductance found in Prob. 5–12?

Prob. 7–12. A generator field of 8 henries inductance is carrying a current of 3.2 amperes. If the field circuit is opened by a switch what time must the current take to die out if the average induced e.m.f. is not to be greater than 200 volts?

206. The Computation of the Self Inductance of Long Coils. The flux set up by a current in a long coil is given by the equation which was developed in Chapter VI.

$$\phi = \frac{1.26\ NIA\mu}{l}$$

in which

ϕ = flux in lines.
N = number of turns.
I = current in amperes.
A = area of core in square centimeters.
μ = permeability of core.
l = length of core in centimeters.

The flux per ampere =

$$\frac{\phi}{I} = \frac{1.26\ NA\mu}{l}.$$

If all this flux cuts every turn of the coil when a change of one ampere is made in the current, the total average flux cuttings per ampere change would equal

$$\frac{1.26\ (NA\mu)}{l} N$$
$$= \frac{1.26\ N^2A\mu}{l}.$$

Since one henry equals 10^8 flux cuttings per ampere change, the self inductance of coil in henries would equal

$$\frac{1.26\ N^2A\mu}{10^8\ l}.$$

That is,

$$L_{av} = \frac{1.26\ N^2A\mu}{10^8\ l}.$$

Example 4. What is the average inductance of the primary coil of a transformer having 400 turns, if the iron core is 100 centimeters long

and 300 square centimeters in cross-section? Assume the average permeability to be 3000.

Solution.

$$L_{av} = \frac{1.26\ N^2 \mu A}{10^8\ l}$$

$$= \frac{1.26 \times 400 \times 400 \times 3000 \times 300}{10^8 \times 100}$$

$$= 18.1 \text{ henries.}$$

Example 5. Assume the current in Example 4 to change from 36 to 2 amperes in $\frac{1}{3}$ of a second. What average voltage would be induced?

Solution.

$$e_{av} = L\left(\frac{i_1 - i_2}{t_1 - t_2}\right)$$

$$= \frac{18.1\ (36 - 2)}{1/3}$$

$$= 1850 \text{ volts.}$$

Prob. 8–12. If the change in Example 5 had taken place in $\frac{1}{12}$ of a second, what average voltage would have been induced?

Prob. 9–12. If wood had been used for the core in Example 4, what would the inductance of the coil have been?

Prob. 10–12. An air-core solenoid is 100 cm. long and 7.5 cm. in diameter. It is wound with No. 10 wire spaced so that there are 3 turns per cm. What is the inductance of the coil?

Prob. 11–12. If the coil of Prob. 10–12 is doubled in length and the winding is continued at the same spacing, what is the inductance? What would be the diameter of coil required to make the inductance the same as in Prob. 10–12?

Prob. 12–12. A coil of 650 turns is wound on a wrought-iron ring. The mean diameter of the ring is 20 inches and the cross-section area is 15 square inches. Permeability of iron = 1250. What is the average inductance of the coil?

Prob. 13–12. A current of 10 amperes is flowing in the coil of Prob. 12–12. If the current is reduced to half its original value in 0.06 second, what average e.m.f. is induced?

Prob. 14–12. The core of a transformer has two coils wound on it. The primary consists of 150 turns, the secondary of 2000 turns. The core is annealed sheet steel, average permeability of 3600, 200 cm. long, 900 sq. cm. cross-section area. What is the inductance of primary and secondary coils?

207. The Computation of Mutual Inductance. When two circuits are so situated that a change of the current in one circuit

causes a flux to cut the other circuit, the two circuits are said to possess **mutual inductance.** When a change of one ampere in one circuit causes 10^8 flux cuttings in the other circuit, the mutual inductance of the two circuits is said to be **one henry.** This may also be expressed:

When a change of one ampere per second in one circuit induces one volt in the other, the two circuits have a mutual inductance of **one henry.** Both expressions have the same meaning.

If two long coils are wound interlaced on the same core, all the flux set up by one may be considered to cut the other.

Thus the flux per ampere of coil 1 would be

$$\frac{\phi}{I_1} = \frac{1.26\ N_1 \mu A}{l}$$

and the flux cuttings of the second coil by a change of one ampere in the first would equal

$$\frac{1.26\ (N_1 \mu A)}{l} N_2.$$

Since one henry mutual inductance exists when 10^8 flux cuttings are produced by a change of one ampere, the mutual inductance of the two coils can be expressed

$$M = \frac{1.26\ N_1 N_2 A \mu}{10^8\ l}$$

in which

M = mutual inductance of coils in henries.
N_1 = number of turns in first coil.
N_2 = number of turns in second coil.
A = area of core in square centimeters.
l = length of core in centimeters.

Note that the equation is the same as that for the **self-inductance** of the two coils wound as one on a core, **except** that instead of squaring the total number of turns in a single coil, the number of turns on one coil is multiplied by the number of turns on the other.

Prob. 15–12. What is the mutual inductance of two coils wound on a circular wooden core 10 cm. in diameter and 200 cm. long? One coil contains 150 turns, and the other 600 turns. Assume all the flux set up by one coil threads the other.

Prob. 16–12. If the core of Prob. 15–12 were of iron ($\mu = 2000$), what would the mutual inductance be?

Prob. 17–12. (*a*) If the current in coil containing 150 turns, Prob. 15–12, were changed at the rate of 400 amperes per second, what voltage would be induced in the other coil? (*b*) Answer same question for coils in Prob. 16–12.

Prob. 18–12. If the current in coil containing 600 turns, Prob. 16–12, is changed at the rate of 120 amperes per second, what voltage will be induced in the other coil?

208. Coupled Circuits. Circuits which possess mutual inductance are sometimes said, especially in radio work, to be coupled. When all the flux set up by one circuit cuts all turns of the second circuit, as assumed in the last article, the circuits are said to have **unity coefficient of coupling.**

In this case the self inductance of the first coil is

$$L_1 = \frac{1.26\ N_1^2 A\mu}{10^8\ l}.$$

The self inductance of the second coil is

$$L_2 = \frac{1.26\ N_2^2 A\mu}{10^8\ l},$$

and the mutual inductance

$$M = \frac{1.26\ N_1 N_2 \mu A}{10^8\ l}.$$

Note that $$M = \sqrt{L_1 \times L_2}$$

or $$\frac{M}{\sqrt{L_1 \times L_2}} = 1.$$

The expression $\dfrac{M}{\sqrt{L_1 \times L_2}}$ is called the coefficient of coupling and is equal to unity only when, as in above case, all the flux set up by the current in one coil cuts all the turns of the other coil. In most cases some of the flux set up by the current in one coil does not cut all the turns in the other coil and the value of M is less than the value of $\sqrt{L_1 \times L_2}$ and therefore the coefficient of coupling is less than unity.

In power transformers a coefficient of coupling of 0.98 or 0.99 is desirable, and the coils are said to be closely coupled. In radio transformers a coefficient of less than 0.50 is usually desirable for sharp tuning. Such a transformer would be said to couple loosely the two circuits connected to its terminals.

Figure 266 shows a vario-coupler, of which the coefficient of coupling can be varied by changing the position of the inner coil.

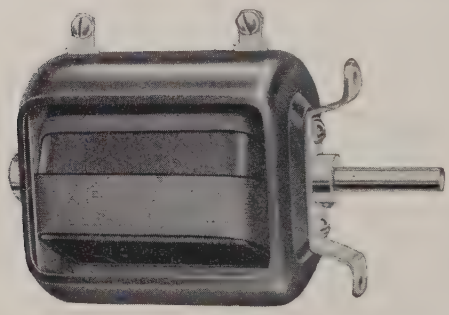


Fig. 266. A vario-coupler, or variometer. *General Radio Co.*

Prob. 19–12. What is the mutual inductance between two coils with self inductances of 0.04 and 0.008 henry respectively, if the coefficient of coupling is 0.64?

Prob. 20–12. Two coils have self inductances of 0.8 henry and 1.2 henry respectively. Their mutual inductance is 0.6 henry. What is the coefficient of coupling?

Prob. 21–12. Two coils, one 6″ long, the other 4″ long, are wound on a fibre tube 1.5 inch in diameter. The mutual inductance is 0.001 henry and the coefficient of coupling is 0.2. If there are 1600 turns in the 6-inch coil how many turns are there in the other?

209. Inductance of Series Circuits. If two coils located a large distance apart are connected in series, the total inductance in the circuit is merely the sum of the two self inductances. When the series coils are located closer together so that there is coupling between them, the inductance of the circuit increases or decreases because when the current changes, each coil has induced in it an e.m.f. due to its self inductance and an additional e.m.f. due to mutual inductance. The mutual induced e.m.f. may add to or subtract from the self induced e.m.f.

Let us designate the coils as A and B. Since they are in series, the same current flows in both. Due to a change in the current, coil A has one e.m.f. induced because of its self inductance and another caused by the mutual inductance between A and B. If the connections are such that the flux in A due to the current in B is in the same direction as the flux in A due to current in itself, the two induced voltages will be in the same direction when the current changes and they therefore add. If the two fluxes are in opposite directions, the induced voltages will oppose each other and the resulting induced voltage is the difference.

The same process is going on in coil B due to its self induced e.m.f. and that set up by mutual inductance with coil A. If the two voltages in A are aiding each other, those in B will also aid

each other, and vice versa. Furthermore, since the mutual inductance is the same for both coils and the rate of change of current is also the same, the mutual induced voltage in A is exactly equal to that in B. Hence the total mutual e.m.f. is twice that of either coil.

The total induced e.m.f. in the series circuit is the sum of the two self induced voltages plus or minus twice the mutual induced voltage of one coil. If the flux in either coil due to its own current is in the same direction as that due to the current in the other coil, the mutual induced voltage adds to the self induced voltages; if the fluxes are opposite, the mutual induced voltage subtracts from the self induced voltages.

This may be expressed by the equations

$$L_T = L_1 + L_2 + 2M \quad \text{(fluxes aiding)}$$

and

$$L_T = L_1 + L_2 - 2M \quad \text{(fluxes opposing)}$$

in which L_T = total inductance of series circuit,
L_1 = self inductance of coil 1,
L_2 = self inductance of coil 2,
M = mutual inductance between coils.

These simple relations are applied to great advantage in many important electrical devices. If we wish to have an inductance which can be adjusted to any value, it can easily be obtained by connecting two coils in series and changing their relative positions (which changes M) until the total inductance has the desired value. Variable inductances of this type frequently look like the variocoupler in Fig. 266. The fixed coil is connected in series with the movable coil and the inductance is changed by turning the shaft. The maximum variation of inductance thus obtainable is four times the mutual inductance.

Another important application is in the construction of noninductive resistors. These are required in many types of electrical measuring instruments and in other devices. By winding the resistance with a single piece of wire which has been looped back on itself, two series coils are formed and they are very closely coupled. The mutual inductance in this case is practically equal to the self inductance of each coil and the fluxes oppose, so the total inductance is very nearly zero.

Prob. 22–12. If the two coils in Prob. 15–12 are connected in series aiding, what is the total inductance of the series circuit?

Prob. 23–12. In the circuit of Prob. 22–12, a certain current change induces an average e.m.f. of 60 volts in the 150-turn coil. What average e.m.f. is induced in the 600-turn coil? What is the total induced e.m.f. in the circuit due to mutual inductance?

Prob. 24–12. Two coils when connected in a series circuit have a total inductance of 8 henries. When the connections to one of the coils are reversed, the inductance of the circuit is found to be 6 henries. What is the mutual inductance between the coils?

210. Inductance of Transmission Lines. It is necessary at times to compute the inductance of straight parallel conductors, especially when used in telephony, telegraphy, or long-distance power transmission. When twin non-magnetic wires are strung in this way in the air, and used as "line and return," the inductance may be computed by means of the following equation:

$$L = \left(1.482 \log_{10} \frac{S}{r} + 0.161\right) l$$

where

L = inductance of line and return in millihenries (thousandth henries);
l = length of line in miles;
S = space between centers of wire;
r = radius of wires (in same units as S).

Prob. 25–12. Compute the inductance in a 100-mile aërial line consisting of two copper wires 20 inches apart, each No. 0 B. & S.

Prob. 26–12. What average voltage would be induced across the terminals of the line in Prob. 25-12 if the current changed from +20 amperes to −20 amperes in 1/120 of a second?

SUMMARY OF CHAPTER XII

The kinetic energy of an electric circuit lies in its magnetic field. The strength of the magnetic field, other things being equal, depends upon the shape of the electric circuit and the medium in which the field lies. It is convenient to express this effect of "shape of circuit and medium of field" by means of an algebraic expression.

INDUCTANCE is the term applied to this expression, as described above. It may be likened to the algebraic expression used in mechanics and known by the term Moment of Inertia. As long as a current is at a constant value in a circuit, the property of Inductance does not make itself known, just as the moment of inertia of a fly wheel does not affect the running of a machine as long as the speed is constant.

LENZ'S LAW. FIRST PART. By means of this property of inductance there is an induced e.m.f. set up whenever the strength of any magnetic field is changed, tending to cause an electric current to flow in such a direction as to oppose any change taking place in the strength of the field.

LENZ'S LAW. SECOND PART. This induced e.m.f. is equal to the product of the coefficient of inductance by the rate of change of the current.

SELF INDUCTANCE is the property which causes this induced e.m.f. to be set up in the same electric circuit in which a change in current, and consequently a change in magnetic field, is taking place.

MUTUAL INDUCTANCE is the property which causes an induced e.m.f. to be set up in a second circuit, by a change of current in the first.

HENRY: UNIT OF INDUCTANCE. When a change of one ampere per second causes 10^8 flux cuttings, i.e., sets up an induced e.m.f. of one volt, the circuit is said to possess an Inductance of one henry. The symbol for Coefficient of Self Inductance is L; for Mutual Inductance M.

EQUATIONS FOR INDUCTANCE FOR LONG COILS.

Self Inductance, $$L = \frac{1.26\ N^2 \mu A}{10^8\ l},$$

Mutual Inductance, $$M = \frac{1.26\ N_1 N_2 \mu A}{10^8\ l}.$$

COILS CONNECTED IN SERIES. When two coils are connected in series, the total inductance is

$$L_T = L_1 + L_2 + 2\ M \text{ (if fluxes aid)}$$

$$L_T = L_1 + L_2 - 2\ M \text{ (if fluxes oppose)}$$

CIRCUITS POSSESSING MUTUAL INDUCTANCE are said to be COUPLED. They are CLOSELY COUPLED when a large fraction of the flux set up by one circuit threads through all the turns of the second circuit, and LOOSELY COUPLED when this fraction is small.

This fraction is called the COEFFICIENT OF COUPLING and equals $\frac{M}{\sqrt{L_1 \times L_2}}$. In radio, most coupled circuits are loosely coupled.

FOR STRAIGHT TRANSMISSION LINES

$$L = l\left(1.482 \log_{10} \frac{S}{r} + 0.161\right)$$

INDUCTION COILS. JUMP SPARK: TWO COILS. By suddenly breaking the current in the primary coil, which is of few turns wound on an iron core, a high e.m.f. is induced by mutual induction across the terminal of the secondary coil which has many turns. If these terminals are separated by a small space, a spark will jump

across this gap when the primary circuit is broken. Accordingly, these secondary terminals are often located within the cylinder of an explosion engine for the purpose of firing the explosive mixture, by means of the spark thus produced.

RUHMKORFF: TWO COILS. An automatic device in an arrangement like the above, opens and closes the primary circuit very rapidly, thus causing a flow of sparks across the terminals of the secondary. In general use with gas engines.

MAKE-AND-BREAK SPARK: SINGLE COIL. When even a small current is broken in a single coil of many turns wound on iron core, the self induction causes a spark to jump across the break. The break may be made to occur in the cylinder of a gas engine and explode the mixture.

TRANSFORMERS consist of primary and secondary coils wound on a core of laminated annealed steel. An alternating current in one coil causes a continual change in the magnetic field. This induces an e.m.f. in the other coil. The ratio of the e.m.f.'s across the two coils approximately equals the ratio of the number of turns in the two coils. Transformers can therefore be used to raise or lower the voltage in an alternating-current circuit. Efficiency, very high.

PROBLEMS ON CHAPTER XII

Prob. 27–12. A coil containing 1000 turns is threaded by 10,000 flux lines at a certain instant. One tenth second later the flux through the coil is 5000 lines. What average voltage is induced by this change of flux?

Prob. 28–12. The flux in Prob. 27–12 was caused by another coil having a self inductance of 0.06 henry and a mutual inductance with the first coil of 0.01 henry. What was the average induced voltage in the coil which produced the flux? The coil in Prob. 27–12 is open and carries no current.

Prob. 29–12. Find the inductance of a coil 70 cm. long, 12 cm. in diameter, having 200 turns of wire, (*a*) with air core; (*b*) with iron core of 2400 permeability.

Prob. 30–12. Find the inductance of a primary coil of a transformer having 1200 turns. Length of magnetic circuit is 125 cm., area 150 sq. cm. Permeability of iron = 4500.

Prob. 31–12. The current in a circuit is changed from 0 to 180 amperes in 2 seconds. The average induced voltage is 1.6 volts. What is the inductance of the circuit?

Prob. 32–12. What average voltage is required to reverse a current of 80 amperes in 0.20 second in circuit of Prob. 31–12?

Prob. 33–12. A "choke" coil is formed by winding 800 turns on a ring consisting of annealed sheet iron, permeability 3800, 150 cm. long and 400 sq. cm. cross-section area. What is the inductance of coil?

Prob. 34–12. When 500 turns of wire are wound on an iron ring, 130 sq. cm. section and average length of 175 cm., the inductance is 1.2 henries. What value does this give for the permeability of iron?

Prob. 35–12. If 1000 turns were wound on the iron ring of Prob. 33–12, what would the inductance be? Assume μ to be constant.

Prob. 36–12. A concentrated coil of 300 turns of small wire (small to cut down the leakage of the flux) has an inductance of 0.026 henry. The number of turns is increased to 600. What will be the approximate change in inductance?

Prob. 37–12. On a steel ring of 12 cm. mean diameter and 5 sq. cm. cross-section are wound two coils, one having 180 turns. The permeability of the steel is 2500. The coefficient of coupling is 0.85 and the mutual inductance is 0.50. How many turns are there on the second coil?

Prob. 38–12. A transformer core of annealed steel is 100 cm. long and 450 sq. cm. cross-section area. The primary coil has 2000 turns of wire, and the secondary, 200 turns. Permeability of the core is 5000. What is the mutual inductance of the coils? Coefficient of coupling = 0.98.

Prob. 39–12. What is the self inductance of each of the two coils in Prob. 38–12?

Prob. 40–12. What is the inductance of a line 200 miles long, consisting of No. 2 (B. & S.) copper supported 3 feet apart in the air?

CHAPTER XIII

CAPACITANCE

Materials are usually divided electrically into two groups, conductors and insulators. To which group any given material belongs is determined by the degree to which it will conduct electricity, — a good insulator naturally being a poor conductor.

211. Conductors and Insulators. According to the most modern theories, all matter is made up of atoms. These atoms in turn are made up of positive and negative charges of electricity. The positive charge consists of a relatively large and heavy part called the **nucleus,** while the negative charge consists of small particles called **electrons.** It is the flow of these electrons that constitutes an electric current. They are so small, however, that 6,300,000,000,000,000,000 or 6.3×10^{18} must flow by a point in one second to produce a current of one ampere.

If in a material there are many free electrons, or some electrons which are so loosely attached to the atoms that a moderate voltage per centimeter will detach the electrons and cause them to flow along, we say the material is a good **conductor.** This is true of all metals, particularly of silver, copper, aluminum, etc., and we accordingly make use of the most economical of these metals as conductors for all of our machinery and transmission lines.

On the other hand if the electrons of a material are so securely attached to the atoms that a pressure or stress of a large voltage per centimeter is necessary to tear them loose and set them flowing, we say that material is a poor **conductor** but a good **insulator.** When a moderate voltage per centimeter is applied to mica, glass, porcelain and the like, practically no electrons are caused to flow. These materials together with rubber, paper, oils and varnishes are therefore good insulators.

212. Condenser Action. Capacitance. The Farad. When an insulator is placed between two conductors a **condenser** is formed. When a voltage is applied to the plates, it cannot cause the electrons to flow through the insulator but it can strain the **bonds** by which the electrons are attached to the atoms in the insulator.

The **bonds** are more or less flexible, and while a moderate pressure

cannot break them, it will flex or stretch them and cause the electrons to move a little in position. In Fig. 267, D represents a plate of dielectric or insulating material between the two metal plates (or electrodes) A, B. When the positive terminal of a battery is connected to metal plate A and the negative terminal is connected to the metal plate B, the electrons (one of which is represented by q) being negative are repelled from the negative plate B and attracted to the positive plate A. The pressure is not strong enough to break the bond, which is represented by the short straight line, but it flexes it and moves the electron a little toward plate A. The insulating plate D is made up of an enormous number of electrons, all of which are moved slightly, from the negative toward the positive plate, and stay in this new position as long as the pressure is maintained between the plates. But note that there is a momentary current, which stops as soon as the electrons have reached their new positions. Thus when the voltage is applied across A and B there is a momentary current as the electrons move slightly, which will soon die out and remain zero as long as the pressure remains constant. When the pressure is removed by short circuiting the battery, the electrons, due to the elasticity of the bonds, will resume their original positions. Thus there will be a momentary current in the reverse direction. In general whenever the electric stress across the dielectric of a condenser is changed, there will be a current in the condenser (called the displacement current) which is due to the slight motion of the electrons. When the voltage is steady there is no current.

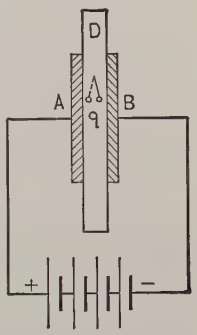


FIG. 267. A simple condenser.

This property which produces a current in a condenser when the voltage across it changes, is called **capacitance** and it is measured in **farads. When a change of one volt per second across it produces a current of one ampere, the condenser is said to have a capacitance of one farad.**

The farad is a much too large unit for practical purposes, so the **microfarad** (one millionth of a farad) is generally used.

This relation between change of voltage and current can be approximately expressed by the equation

$$i_{av} = C\frac{e_1 - e_2}{t_1 - t_2}$$

in which i_{av} = the average current in amperes,
C = capacitance in farads,
e_1 = voltage in volts at time t_1,
e_2 = voltage in volts at time t_2,
$t_1 - t_2$ = time of change from e_1 to e_2 in seconds,
$\frac{e_1 - e_2}{t_1 - t_2}$ = average rate of change of voltage.

Example 1. A condenser has a capacitance of 10 microfarads. What average current will flow when the voltage is raised from 5 to 65 volts in 2 seconds?

Solution.

$$i_{av} = C\frac{e_1 - e_2}{t_1 - t_2}$$

$$\frac{10}{10^6} \times \frac{65 - 5}{2} = 0.0003 \text{ ampere.}$$

Prob. 1–13. A condenser C with a capacitance of 4 microfarads is connected as shown in Fig. 268. The slide wire is 80 feet long and of a total resistance of 100 ohms. If the slide S is being moved to the right at the rate of 60 feet per second, what current flows in the condenser C? Assume that the current taken by the condenser does not change the voltage distribution along the slide wire.

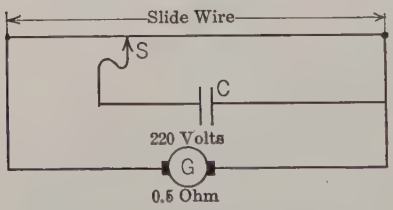


Fig. 268. A method for varying the voltage across a condenser.

Prob. 2–13. What would be the current in the condenser in Prob. 1–13 if a resistance of 50 ohms were in series with the generator?

Prob. 3–13. What is the capacitance of a condenser in which a current of 0.0062 ampere flows when the voltage across its terminals rises at a uniform rate from 100 to 350 volts in 0.2 second?

213. Dielectric Constant and Dielectric Strength. The capacitance of such an arrangement as is shown in Fig. 267 has been found to depend upon the material of the dielectric used. If D is a sheet of glass, the capacitance will be found to be about $1\frac{1}{2}$ times what it is when D is composed of mica. We explain this by saying that the **dielectric constants** of the two materials are different.

The **dielectric constant** of a material is a measure of its effectiveness when used in a condenser, just as the permeability of a material is a measure of the effectiveness of the material for use in a magnetic

circuit. It will be remembered from the study of magnetic circuits that the permeability of iron is a number of thousands, and of other materials a much lower figure, but that the permeability of air is unity. In the same way, when we study condensers we find that while the dielectric constant of most insulating materials is high compared with that of air, yet air or even a vacuum has a dielectric constant; that is, both will give condenser effects between parallel plates. When the dielectric of a condenser is stressed, we can visualize the movement of electrons which constitutes the displacement current. It has not yet been satisfactorily explained, however, why there is still a current when the dielectric is removed and replaced by a vacuum, as experiment shows is probably true. There is by no means as much range in the dielectric constant of various materials, however, as there is in the permeability.*

The dielectric constant of air has been chosen as unity, and all other materials have been referred to air as a standard. The dielectric constant of a material on this basis is called its relative **dielectric constant.** Note in Table V that glass has a relative dielectric constant ranging between six and ten, depending upon the grade of the glass. Mica has about two-thirds of this. Some liquids have high dielectric constants, that of glycerine being about fifty-six.

The dielectric constant of a material must not be confused with its **dielectric strength,** which is the voltage per centimeter that must be applied to tear the electrons from the atoms and rupture the material. The **dielectric strength** is a measure of the insulating property of the material. The **dielectric constant** is the measure of its **condensive** effect.

Prob. 4–13. A certain condenser using acetone as the dielectric has a capacitance of 1.84 microfarads. The pressure across it varies from 8 volts to 34 volts in 0.003 second. What average current will flow in this condenser under this condition, if the acetone is replaced by glycerine?

214. Parallel-Plate Condensers. A parallel-plate condenser may be constructed as shown in Fig. 267 or, in order to increase the amount of capacitance, as shown in Fig. 269. Such condensers may be built up of sheets of metal and plates of glass. They may be

* Experiments on carefully prepared Rochelle salt crystals show that very high dielectric constants do exist. Values as high as 7000 have been reported, but no practical application has been made because of the extreme difficulty of controlling the process. (See paper by H. Mueller, Physical Review, Jan. 1935.)

constructed of paraffined paper and tinfoil. The construction depends, of course, upon the voltage to which they are to be subjected.

The capacitance of such a unit is its current-per-volt change per second. Since this current depends upon the number of electrons in a cross-section which can move in position, it is to be expected that the capacitance will be proportional to the cross-sectional area of the sheet of dielectric between the plates. Also, since the extent to which the electrons will move depends upon the force to which they are subjected, it is to be expected that the capacitance of such a condenser will be inversely proportional to the thickness of the dielectric, and hence proportional to the electric stress applied to the electrons. Such is found experimentally to be the case. The capacitance is also proportional to the dielectric constant, K, of the material used. We may thus write for the capacitance of a parallel-plate condenser

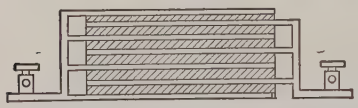


FIG. 269. A condenser of several plates.

$$C = \frac{8.84\ KA}{10^8 \times l}$$

in which C = capacitance in microfarads,
K = relative dielectric constant (air = unity),
A = area (one side) in square centimeters of all the dielectric actually between the plates,
l = thickness of dielectric in centimeters.

Inexpensive condensers up to 8 microfarads are commonly made of alternate sheets of aluminum foil and waxed paper. These are wound into rolls and made very compact. Condensers of large capacitance (10 to 50 microfarads) are made by using an insulating layer formed directly on the metal plates of the condenser, the space between being filled with an electrolyte in liquid or paste form. The thickness of the layer thus formed is very small and a high capacitance is thus obtained.

Figure 270 shows a variable condenser of the type commonly used in radio work and for measurements with high frequency alternating currents. It consists of a number of movable plates arranged on a shaft so that they can be moved between a set of fixed plates. Air is usually the dielectric although occasionally a condenser of this type is immersed in a liquid dielectric to increase the capacitance

obtainable. The condenser shown in Fig. 270 is a precision type built especially for measurements requiring variable condensers of known capacitances.

Example 2. What is the capacitance of a condenser built with 2000 plates, in which the dielectric consists of sheets of paraffined paper, 0.005 cm. thick. The part of each sheet actually between plates has an area 16 × 20 sq. cm.

Solution.

$$C = \frac{8.84\ KA}{10^8 \times l} = \frac{8.84 \times 2.1 \times 16 \times 20 \times 2000}{10^8 \times 0.005}$$
$$= 23.8\ \mu\text{f}.$$

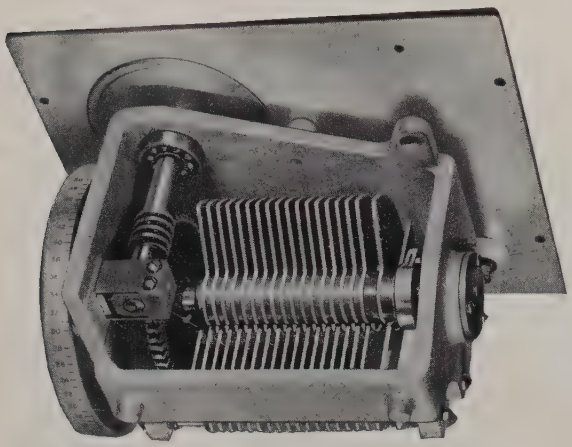


FIG. 270. A variable condenser. *General Radio Co.*

Prob. 5–13. A rolled aluminum foil and paper condenser consists of 20 strips of foil, 2 inches wide and 40 inches long, separated by paraffined paper 0.002 inch thick. What is the capacitance after rolling and connecting alternate sheets of foil together?

Prob. 6–13. What is the capacitance of the condenser of Prob. 5–13 before it is rolled?

215. Charge on a Condenser. Electrons repel one another so powerfully that some are crowded out to the surface, but cannot leave it on account of the attraction of the metal for them, except when the temperature is high. These electrons act like an incompressible fluid in the body of the metal.

But if a sufficiently high electric pressure is exerted on the electrons a few more can be crowded onto a surface. This is especially true

if an oppositely charged plate is very near, for the surface electrons of the first plate are attracted to it.*

When a dielectric is placed between the plates, some of the electrons belonging to the atoms in it are forced over towards one plate and the atoms thus attract other electrons of the metal and crowd more out of the metal to the surface of the plate. The plate thus has a greater charge than it would have if the dielectric were not present. For it must be remembered that a body is charged when it has a greater or smaller number of electrons on it than normally. If it has a greater number, it is negatively charged because the electrons are particles of negative electricity. If it has fewer than the normal number, then the body is positively charged. When just sufficient electrons are present to satisfy or neutralize the positive nuclei, then we say that the body is not charged.

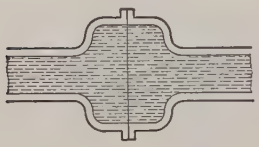


Fig. 271. A hydraulic analogy of a condenser.

An electric condenser has an excellent hydraulic analogy in a rubber diaphragm stretched across a pipe as shown in Fig. 271. If this pipe is connected to a centrifugal pump, as in Fig. 272, and the pump is started running, the diaphragm will be stretched as shown, and will stay stretched as long as the pump operates. During the period when the pump is accelerating and hence increasing the pressure, there will be a flow of water in the pipe which will cease soon after the pressure becomes constant. If the pump is stopped, there will be a flow of water back, and the diaphragm will spring back to its midposition.

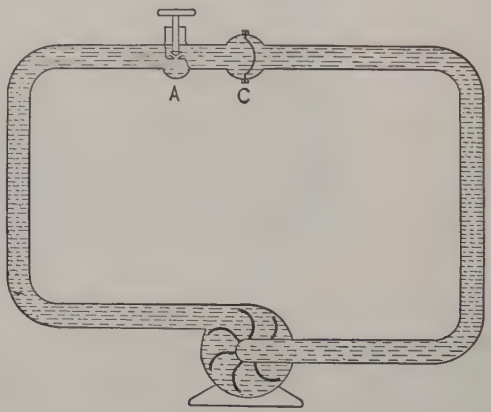


Fig. 272. A hydraulic analogy of the electric circuit in Fig. 273.

Similarly, if an electric condenser is connected in series with a

* This partially explains why a vacuum can act as the dielectric of a condenser, although it does not explain the fact that a current passes in the vacant space between the plates when the voltage across them changes.

generator, as shown in Fig. 273, and the generator started, there will be a flow of current around the circuit after the generator is started if the switch S is closed, but the flow will soon stop if the generator speed and hence the voltage become constant. If the generator is stopped, and the voltage thus removed, there will be a flow back through the circuit for a short period of time.

Suppose that instead of stopping the generator we first open the switch at S. This is equivalent to closing a valve at A in Fig. 272. In such a case the current cannot flow back even though the generator is stopped. We now say that the condenser is charged. The electrons in the dielectric tend to spring back to their positions before displacement, but they cannot do so because the electrons trapped on the plates exert forces which keep the electrons in the dielectric displaced. The electrons on the plate thus act like the volume of water trapped between the valve A and the displaced diaphragm C in Fig. 272. The water cannot escape and since it cannot be compressed very much the diaphragm cannot return to its midposition. The difference in pressure of the water across the valve is that set up by the pump at the time the valve was closed. Likewise when the switch S in Fig. 273 is opened, the voltage across it is equal to the generator voltage at the instant the switch was opened. We say that the condenser is charged to this voltage.

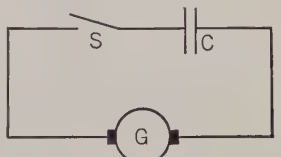


FIG. 273. A circuit containing a condenser C.

The amount of the charge can be found as follows.

If the voltage rises at a uniform rate from 0 to E volts in t seconds, the rate at which voltage changes will equal $\frac{E}{t}$ and the current will equal

$$I = C \times \frac{E}{t}.$$

The quantity of electricity put into the condenser will then be $I \times t$.

$$It = C\frac{E}{t} \times t,$$

$$= CE.$$

But $It = Q,$

therefore $Q = CE,$

in which

Q = quantity of electricity in coulombs or ampere-seconds,
C = capacitance in farads,
E = e.m.f. in volts.

This may also be written as

$$C = \frac{Q}{E}.$$

Thus the capacitance can be expressed as the **charge per volt** across the terminals. We therefore have two equivalent definitions of capacitance:

(1) The capacitance is that factor by which the rate of voltage change is multiplied to determine the current. A condenser has one farad capacitance when a change of one volt per second will cause one ampere to flow in the condenser.

(2) The capacitance is the charge per volt. A condenser has one farad capacitance when one volt will put one coulomb charge on it.

Example 3. How many coulombs of electricity will a condenser of 15 microfarads capacitance hold, when the pressure between its terminals is 200 volts?

Solution.

$$\begin{aligned} Q &= CE \\ &= 0.000015 \times 200 \\ &= 0.003 \text{ coulomb.} \end{aligned}$$

Example 4. If the 0.003 coulomb in Example 3 takes 1/500 of a second to flow into the condenser, what is the average current?

Solution.

$$\begin{aligned} I &= \frac{Q}{t} = \frac{0.003}{0.002} \\ &= 1.5 \text{ amperes.} \end{aligned}$$

Prob. 7–13. What is the capacitance of a condenser that holds 0.0009 coulomb under a pressure of 81 volts?

Prob. 8–13. How many volts would be required to put 0.005 coulomb into the condenser of Prob. 7–13?

Prob. 9–13. What average current flows if it takes 0.005 second to charge the condenser in Prob. 7–13?

Prob. 10–13. The condenser in Prob. 7–13 takes 0.005 second to charge completely on a 250-volt circuit. What average charging current flows?

Prob. 11–13. How many coulombs will a 10-microfarad condenser hold when charged to 1000 volts?

216. A Comparison of Inductance and Capacitance. The inductance of a circuit has been likened to the flywheel on the shaft of an engine. The capacitance is like the elasticity of the shaft or connections. The inductance makes its presence known when we change the current in the circuit; the capacitance, when we change the voltage. The most useful equation dealing with inductance is

$$e_{av} = L\frac{i_1 - i_2}{t_1 - t_2}.$$

The similar equation dealing with capacitance is

$$i_{av} = C\frac{e_1 - e_2}{t_1 - t_2}.$$

When a steady voltage is suddenly applied to an inductance with resistance in series the current gradually rises to its normal value as in Fig. 263.

When a steady voltage is suddenly applied to a capacitance with a resistance in series, the current starts in at its maximum value and gradually falls to zero as in Fig. 274.

Inductance and capacitance produce opposite effects in a circuit, just as a flywheel and a spring on a line of shafting would produce opposite effects. This is well illustrated by the effect of the flywheel of an automobile motor which hinders a quick starting by the starting motor, while the Bendix spring in the starter allows a certain play in the connection between the starting motor and gasoline motor.

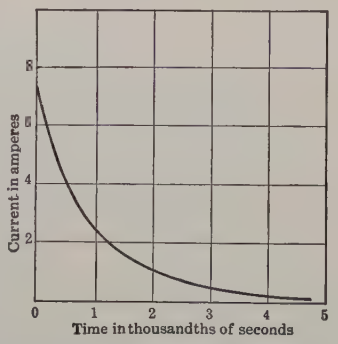


FIG. 274. A curve showing the charging current of a condenser.

217. Measurement of Capacitance. Direct Deflection of Ballistic Galvanometer. In measuring capacitance we make use of the fact, that in order to charge a circuit with electricity, electrons must flow along a conductor. If a ballistic galvanometer is inserted in series in the line, this flow of electrons can be made to give a momentary deflection, or throw, proportional to the number of electrons or the amount of the charge.

Thus in Fig. 275, when the switch S is closed, the electrons flowing along the upper conductor to charge plate A must pass through

galvanometer G. This will cause the galvanometer to "throw." Since the galvanometer is constructed so that the needle swings, or "throws," to a reading proportional to the moving force and immediately returns to zero, it is said to be a **ballistic** galvanometer. This term distinguishes it from one which requires a steady current to cause its deflections to be proportional to the moving force.

A more common way of connecting the galvanometer is shown in Fig. 276. The switch S is thrown down to charge the condenser plates A and B, and then quickly thrown up allowing the condenser to discharge through the galvanometer. A special charge and discharge switch is required for accurate results.

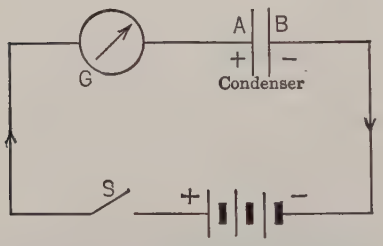


Fig. 275. The galvanometer G measures the charge entering the condenser.

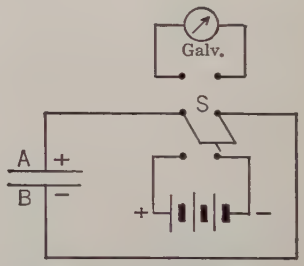


Fig. 276. The condenser AB is discharged through the ballistic galvanometer to measure the charge.

To measure the capacitance of any device, such as a given length of telephone cable, it is first charged for one minute, then suddenly discharged through the galvanometer and the throw noted. A condenser of known capacitance is then inserted in the place of the unknown and charged for a minute, and the throw of the galvanometer noted as the condenser is discharged. The capacitances of the two pieces are then in the same ratio as the galvanometer throws, **provided the throws have not too great a difference.** Since the known capacitance is usually a variable standard condenser, its capacitance can be adjusted until the galvanometer throw on discharge is very nearly equal to the throw caused by the unknown capacitance. In this way a considerable degree of precision can be obtained.

Prob. 12–13. In a circuit arranged as in Fig. 276, a cable is charged to a given potential. On being discharged through a ballistic galvanometer, it causes the instrument to deflect 14.6 scale divisions. A standard condenser of 1.5 microfarads capacitance after being charged to the same potential, on discharge causes the same galvanometer to deflect 11.2 scale divisions. What is capacitance of cable as tested?

218. Capacitance Measurement. Bridge Method. Another method for measuring capacitance is shown in Fig. 277. It resembles a Wheatstone bridge, with the exception that condensers C_1 and C_2, one known and the other unknown, replace two of the resistances. A source of alternating current supply takes the place of the battery and a pair of telephone receivers, T, replaces the galvanometer. The alternating current is usually of a relatively high frequency (1000 cycles per second commonly used) so that an

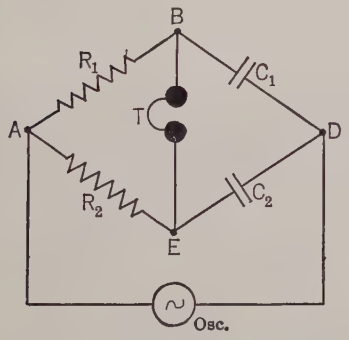


FIG. 277. A Wheatstone bridge arranged to measure capacitance.

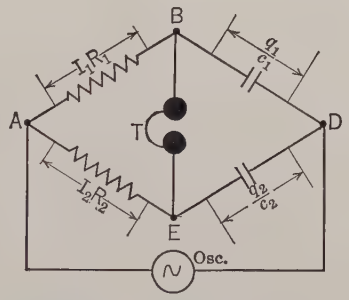


FIG. 278. Voltage drops in the capacitance Wheatstone bridge.

easily audible tone can be heard in the receivers. This current is supplied usually from either a magnetically driven tuning fork oscillator or from a vacuum tube oscillator. A balance is obtained, as in a Wheatstone bridge, by adjusting R_1 and R_2 until no sound is heard in the receiver T.

The following equation is then used to compute the capacity of the unknown, say C_2:

$$\frac{R_1}{R_2} = \frac{C_2}{C_1}.$$

Note that the ratio of the capacitances is the **inverted** ratio of the resistances. In this respect it differs radically from the Wheatstone bridge equation for resistance measurements.

The proof of the above equation is as follows:

When there is no sound in receiver T (Fig. 278), B and E must be at the same potential. The voltage from A to B (I_1R_1) must equal the voltage from A to E (I_2R_2), or

$$I_1R_1 = I_2R_2. \tag{1}$$

Also the voltage from B to D must equal the voltage from E to D.

The voltage across a condenser equals the charge in it divided by its capacitance. (See paragraph 215.)

If q_1 be the charge on C_1, and q_2 the charge on C_2 at any instant, then

$$\text{Voltage } (B \text{ to } D) = \frac{q_1}{C_1};$$

$$\text{Voltage } (E \text{ to } D) = \frac{q_2}{C_2}.$$

Therefore,

$$\frac{q_1}{C_1} = \frac{q_2}{C_2},$$

or,

$$C_2 q_1 = C_1 q_2. \tag{2}$$

But in order that the quantity (q_1) get to the condenser (C_1) it must flow through (R_1) as there is no flow along EB.

In like manner the quantity q_2 must flow through R_2.

Thus we have equation,

$$I_1 \text{ (current in } R_1) = \frac{q_1}{t}; \tag{3a}$$

and

$$I_2 \text{ (current in } R_2) = \frac{q_2}{t}; \tag{3b}$$

where t = the time in seconds in which the charges flow through the resistances. This time must be the same in each resistance since the two resistances are in parallel across the same alternator.

Then, multiplying (3a) by R_1 and (3b) by R_2, we have,

$$I_1 R_1 = \frac{q_1}{t} R_1;$$

and

$$I_2 R_2 = \frac{q_2}{t} R_2;$$

substituting these values in (1),

$$\frac{q_1 R_1}{t} = \frac{q_2 R_2}{t};$$

or

$$q_1 R_1 = q_2 R_2. \tag{4}$$

Dividing equation (4) by (2) we have,

$$\frac{q_1 R_1}{q_1 C_2} = \frac{q_2 R_2}{q_2 C_1};$$

or

$$\frac{R_1}{C_2} = \frac{R_2}{C_1}.$$

This may be transposed to read

$$\frac{R_1}{R_2} = \frac{C_2}{C_1}.$$

Prob. 13–13. In a bridge arranged as in Fig. 277, no sound is heard in the telephone receiver when $R_1 = 3.684$ ohms and $R_2 = 2.972$ ohms. What is the capacitance of C_1 if the capacitance of C_2 is 0.894 microfarads?

Prob. 14–13. In Fig. 277, suppose that the oscillator and T are interchanged in position. What will be the balance equation of the bridge for this condition?

219. Locating a Break in a Telephone Cable. An arrangement similar to the above is often used in locating a break in one of a pair of telephone cables. In Fig. 279, C_1 represents a pair of good wires, C_2 represents a pair in which one of the wires is broken at F.

When bridge is balanced,

$$\frac{R_1}{R_2} = \frac{C_2}{C_1},$$

where C_2 represents the capacity of the faulty pair.

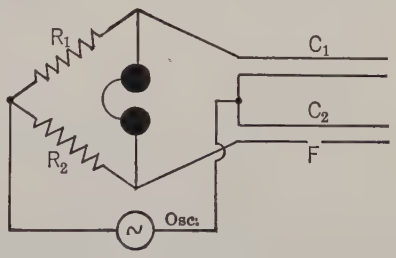


FIG. 279. A method for finding the location of a break in a cable by measuring capacitance.

Now, if the same conditions exist with regard to the wires (except that one wire of one pair is broken) the capacitance of the faulty pair as far as the break will have the same relation to the capacitance of the good pair, that the distance out to the break has to the length of good pair.

Thus:

$$\frac{l_2}{l_1} = \frac{C_2}{C_1};$$

where l_2 = distance to break;
l_1 = length of good pair;
C_2 = capacitance to break;
C_1 = capacitance of good pair.

Therefore

$$\frac{R_1}{R_2} = \frac{l_2}{l_1}.$$

If the length of the good pair is known, the distance l_2 out to the break can easily be computed.

Prob. 15–13. In a cable test for a break, arranged as in Fig. 279, the length of the pair of good wires (C_1) is 5 miles. A balance is obtained when $R_1 = 62$ ohms and $R_2 = 80$ ohms. How far out is the break in the faulty cable (C_2)?

Prob. 16–13. In a testing outfit arranged as in Fig. 279, the telephone receiver gives no sound, when $R_1 = 122$ ohms and $R_2 = 336$ ohms. If the length of the good pair of cables, C_1, is 14,200 feet, how far out is the break in the faulted pair, C_2?

Prob. 17–13. If the capacitance of the good pair in Prob. 16–13 is 1.64 microfarads, what is the capacitance of the faulty pair as far out as the break?

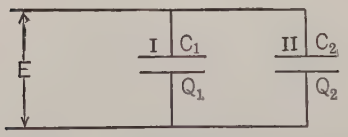


FIG. 280. Condensers I and II are in parallel.

220. Condensers in Parallel. Consider two condensers I and II, Fig. 280, joined in parallel across the mains, the voltage of which is E. The capacitance of condenser I is C_1; of condenser II, C_2. Find the combined capacitance of the two when so joined.

Let C be the combined capacitance;

Let Q_1 = quantity of charge in condenser I;

Let Q_2 = quantity of charge in condenser II.

Then total quantity in both condensers $= Q_1 + Q_2$.

$$Q_1 + Q_2 = CE. \tag{1}$$

(Quantity of total charge equals total voltage times total capacitance.)

But $$Q_1 = C_1E$$

and $$Q_2 = C_2E.$$

Therefore, $$Q_1 + Q_2 = (C_1 + C_2)E. \tag{2}$$

From (1) and (2) we have

$$CE = (C_1 + C_2)E,$$

or

$$C = C_1 + C_2.$$

Thus the capacitance of condensers joined in parallel equals the sum of the capacitances of the separate condensers. Joining condensers in parallel is merely adding the plate area of one to that of the other.

Example 5. What is the capacitance of 4 condensers of 3, 0.2, 7, and 2.5 microfarads respectively, when joined in parallel?

Solution.

$$C = C_1 + C_2 + \cdots.$$

$$C = 3 + 0.2 + 7 + 2.5 = 12.7 \text{ microfarads.}$$

Prob. 18–13. Two condensers of 3 and 5 μf. capacitance respectively are connected in parallel. What voltage is required to force a charge of 0.0035 coulomb into the parallel combination?

Prob. 19–13. What is the charge on each condenser in Prob. 18–13?

221. Condensers in Series. Condensers in series present a peculiar phenomenon. Let condenser I of capacitance C_1, Fig. 281, be joined in series with condenser II of capacitance C_2. Let E be voltage across combination, E_1 across condenser I, and E_2 across condenser II.

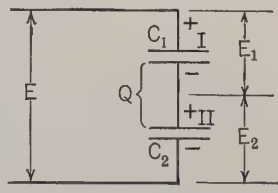


FIG. 281. Condensers I and II are in series.

The charge Q is sent into the condensers under the action of the voltage E. Since the two condensers are in series the same charge must be sent into each, just as the same current is sent through resistances in series. If electricity acts as an incompressible fluid, then there must be as much displacement of electrons in condenser II as in condenser I when they are in series. Thus the charge sent into each is Q, and the charge sent into the combination is also the same, Q.

Let C = combined capacitance of C_1 and C_2.

$$E = \frac{Q}{C}; \tag{1}$$

$$E_1 = \frac{Q}{C_1};$$

$$E_2 = \frac{Q}{C_2}.$$

But
$$E = E_1 + E_2. \tag{2}$$

Therefore, from (1) and (2),

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2};$$

and

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}.$$

Thus the reciprocal of the combined capacitance of condensers in

series equals the sum of the reciprocals of the capacitances of the separate condensers.

Example 6. If the capacitance of condenser I in Fig. 281 is 2 microfarads and that of condenser II, 5 microfarads, what is the capacitance of the two condensers joined in series?

Solution.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2};$$

$$\frac{1}{C} = \frac{1}{2} + \frac{1}{5} = \frac{7}{10};$$

$$C = \frac{10}{7} = 1.43 \text{ microfarads.}$$

Prob. 20–13. What charge is sent into condensers, of above example, if the voltage across the combination is 125 volts?

Prob. 21–13. What is the voltage across condenser I and across condenser II, Fig. 281, if voltage across the two in series is 600 volts? Capacitance of I = 2 microfarads, of II = 8 microfarads.

222. Capacitance of Telephone and Telegraph Cables, etc. One of the greatest obstacles to sending telegraphic messages across the ocean is the large capacitance of the submarine cable. The insulating material around the wire acts as a dielectric between two conductors, the wire and the water. The great length of the cable gives a sufficiently large area to the dielectric to produce a very high capacitance. In transmitting a message, then, it is necessary alternately to fill up and empty a piece of apparatus of great capacitance. The time necessary for these operations puts many difficulties in the way of efficient and rapid transmission.

Telephone cables being laid in pairs, and often in conduits, also possess considerable capacitance which would make it almost impossible to operate long lines, were it not for the possibility of "loading" a line with inductance coils. As has been explained, **capacitance** and **inductance** have practically the opposite effect on a circuit; inductance tending to choke back the flow of electricity, capacitance tending to increase the flow. Under the proper conditions, one may be made to neutralize the other. Thus the inductance of the "loading" coils on a telephone line practically neutralizes the capacitance effect of the long insulated cables. Another striking example is a new submarine cable which has a sheath of the metal Permalloy described in Art. 97. Because of the

presence of the sheath the self inductance of the circuit is increased to the extent of neutralizing most of the undesirable capacitance.

Sometimes condensers are used in a similar way, to counteract the effect of inductance. Examples of this are frequently found in alternating current practice.

223. Capacitance of Cables. The capacitance of submarine cables and of cables laid in metal sheaths is an important factor in telegraphy and telephony.

The capacitance of such lines may be computed by means of the same equation as for a plate condenser. A more precise equation, however, for the capacitance of a cable is the following:

$$C = \frac{0.0388\ Kl}{\log_{10}\frac{D}{d}},$$

where C = capacitance of cable in **microfarads;**
l = length of cable in **miles;**
D = outside diameter of dielectric (insulation);
d = inside diameter of dielectric in same units as D;
K = relative dielectric constant of insulation.

Prob. 22–13. A No. 6 (B. & S.) copper wire covered with 0.30 cm. of gutta percha insulation is held in a lead sheath. Find capacitance per mile of this cable and grounded sheath.

Prob. 23–13. A submarine telegraph cable is of copper 0.42 inch in diameter, and covered with 0.6 inch of gutta percha insulation. What is capacitance between cable and water of 100 miles of such cable?

224. Capacitance of an Aërial Line. The capacitance of twin wires may be found by means of the following equation:

$$C = \frac{0.0194\ l}{\log_{10}\frac{S}{r}},$$

where C = capacity in **microfarads;**
l = length of **one wire** in **miles;**
S = space between wires;
r = radius of each wire in same units as S.

Prob. 24–13. What is the capacitance per mile of a line consisting of two No. 2 B. & S. wires separated by a space of 30 inches?

Prob. 25–13. What is capacitance of a 60-mile line consisting of two No. 4 (B. & S.) copper wires hung 40 inches apart?

225. Mechanical Force on a Condenser. Electrostatic Voltmeters. When two bodies are charged electrically, there is always a small mechanical force acting between them, called an electrostatic force. This force tends to pull the bodies together if the charges are opposite in sign, and to push them apart if the charges are alike in sign. Electrostatic attraction and repulsion are similar to magnetic attraction and repulsion, although the electrostatic forces are very small compared with the magnetic forces. The force between two charged plates of a condenser is given by the equation:

$$f = \frac{1.59\ KAE^2}{l^2 \times 10^{11}} \text{ ounces,}$$

in which

f = force between two similar plates, in ounces;
K = relative dielectric constant;
A = area of one side of one plate in square inches;
l = distance between plates in inches;
E = electromotive force between plates in volts.

Fig. 282. An electrostatic voltmeter. *General Electric Co.*

Note by Example 7, how exceedingly small this force ordinarily is. Yet when the voltage, E, between the plates is very high the force becomes large enough to be used to measure the voltage. Thus in Fig. 282 the electrostatic force is great enough on high voltages to actuate the delicately suspended vane and cause it to indicate the voltage between the movable and stationary vanes.

Example 7. What is the force between two plates 64 inches square separated 0.64 inch in air and subjected to an e.m.f. of 100 volts?

Solution.

$$f = \frac{1.59\ KAE^2}{l^2 \times 10^{11}}$$

$$= \frac{1.59 \times 1 \times 64^2 \times 100^2}{0.64^2 \times 10^{11}} = 0.00159 \text{ ounce.}$$

Prob. 26–13. In an electrostatic voltmeter similar to that shown in Fig. 282, the effective area of each plate is 18 square inches. The distance between plates is 3 inches. What force acts on the plates when a potential difference of 40,000 volts is impressed between them?

226. The Insulation of Aërial Lines. For lines operating below 30,000 volts the pin insulator, described in Art. 305, has been in common use, but for higher voltages the increased size, weight and cost of this type has made its use prohibitive and insulators of the strain type, shown in Fig. 283, have been developed.

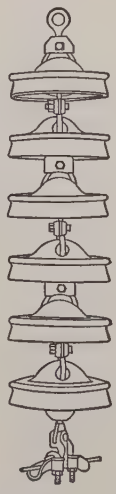


Fig. 283. A string of strain insulators.

The strain type has the advantage over the pin type in that a number of insulators are strung together in series and thus divide the total voltage of the line among the several units. This is illustrated in Fig. 284 and 285. Note in Fig. 284 that although all the units are exactly alike, the voltage does not divide equally among them, but is greater across the end units, being especially large across the unit connected to the conductor. The reason for this is as follows.*

The insulating material of each insulator being between two metal pieces forms a condenser. In addition the air between each piece of metal and the ground forms other condensers.

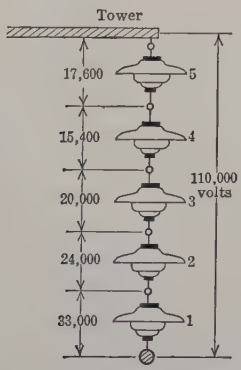


Fig. 284. The voltage distribution in a string of strain insulators.

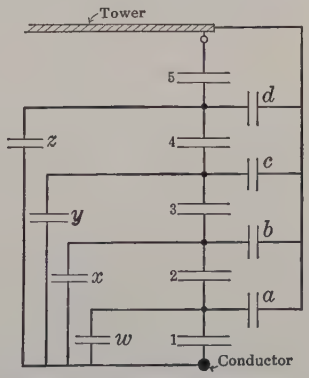


Fig. 285. The combinations of condensers formed by a string of strain insulators.

* For a more complete treatment of this problem see "Electric Power Transmission and Distribution" by L. F. Woodruff. (John Wiley and Sons.)

And likewise the air between the plates of the units and the metal of conductor forms still other condensers. Thus in Fig. 285 the metal between the insulators 1 and 2 forms condenser (*a*) with the air as dielectric and the tower or ground as the other plate. But the same metal forms condenser (*w*) with the air as dielectric and the conductor as the other plate.

The charge on condenser (*a*) is fairly large because of the large area of one plate (the ground) and also because of the high voltage between the metal parts and the ground. On the other hand, the charge on condenser (*w*) will be lower because of the small area of both plates and the lower voltage between the metal parts and the conductor. Condensers (1) and (*w*) are in parallel, and condensers (2) and (*a*) are in parallel. The parallel combination of (1) and (*w*) is in series with the parallel combination of (2) and (*a*).

The sum of the charges on (*w*) and (1) must be the same as the sum of the charges on (2) and (*a*). That is,

$$Q_1 + Q_w = Q_2 + Q_a$$

or

$$Q_2 = Q_1 + Q_w - Q_a.$$

But we have seen that Q_a is larger than Q_w. Therefore $Q_1 + Q_w - Q_a$ is smaller than Q_1. Therefore Q_2 is smaller than Q_1.

The capacitances of all the units are the same; let us call it C. Then the voltage across condenser (1) is

$$E_1 = \frac{Q_1}{C}$$

and the voltage across condenser (2) is

$$E_2 = \frac{Q_2}{C}.$$

But we have seen that Q_2 is smaller than Q_1, therefore E_2 is smaller than E_1, as shown in Fig. 284.

Similarly the voltage across each succeeding unit will be found to be smaller than that of the one below it, until a point is reached where the charge on the air condenser formed by the metal parts and the conductor is larger than the charge on the condenser formed by the metal parts and the ground. This occurs when we compare the condensers (4) and (5).

The sum of the charges on condensers (4) and (*z*) is the same as

the sum of the charges on condensers (5) and (*d*). This may be written

$$Q_4 + Q_z = Q_5 + Q_d$$

or

$$Q_5 = Q_4 + Q_z - Q_d.$$

Now although the capacitance of condenser (*d*) is larger than that of condenser (*z*), because of the large area of one plate (the ground), still the voltage across (*z*) is much larger than the voltage across (*d*). Thus the charge on (*z*) is larger than the charge on (*d*). Therefore

$$Q_4 + Q_z - Q_d \text{ is larger than } Q_4.$$

But

$$Q_4 + Q_z - Q_d = Q_5,$$

therefore Q_5 is larger than Q_4.

The voltage across Q_5 is

$$E_5 = \frac{Q_5}{C}.$$

The voltage across Q_4 is

$$E_4 = \frac{Q_4}{C}.$$

Thus E_5 must be larger than E_4.

This shows that the voltage across either end unit is greater than that across the unit next to the end, although the voltage across the unit on the ground end of the string is usually but slightly greater than the voltage across the unit next to it. The voltage across the conductor end, however, is usually about 30 per cent of the entire voltage, regardless of the number of units in the string. This shows the futility of trying to insulate against greater and greater voltage by increasing the number of units in a string. The added units have almost no effect toward decreasing the voltage across the unit on the conductor end of the string. This unequal distribution can be somewhat corrected by making the capacitance of unit No. 2 sufficiently smaller than that of unit No. 1 to make up for the loss of charge through the parallel connection of condensers formed by the air and ground. The capacitance of unit No. 3 is similarly made smaller than that of No. 2. This is done by merely decreasing the size of the metal caps sufficiently to give less area of dielectric between the metal parts which form the plates of the condensers.

SUMMARY OF CHAPTER XIII

In an **INSULATOR** the electrons are so closely bound to the atoms that they cannot be torn away by ordinary voltages. In a solid **CONDUCTOR** there are free electrons.

A **CONDENSER** consists of an insulator between two conductors.

While ordinary stresses will not tear the electrons from the atoms of an insulator, they may strain the bonds and cause the electrons to move slightly, as the voltage changes. This property in a condenser is called its **CAPACITANCE.** When a voltage change of one volt per second produces sufficient movements of electrons to constitute a momentary current of one ampere in a condenser, the condenser is said to have a **CAPACITANCE** of one **FARAD.** A microfarad is one-millionth of a farad.

The **DIELECTRIC CONSTANT** of a material is a measure of its effectiveness when used as the dielectric in a condenser. See Table V for values.

PLATE CONDENSERS consist of thin sheets of lead, tin, or aluminum foil, separated by thin sheets of insulating material. The capacitance of a plate condenser is

$$C = \frac{8.84\,KA}{10^8 \times l} \text{ microfarads.}$$

The **QUANTITY OF ELECTRICITY** moved in a condenser is called its **CHARGE** and is computed by the equation

$$Q = EC.$$

CAPACITANCE produces effects which are opposite to the effects produced by **INDUCTANCE.**

Capacitance may be measured: (1) By direct deflection method, using standard condenser. Deflections of ballistic galvanometer on discharge of standard and unknown condensers, charged to same potential, are proportional to the respective capacitances. (2) By bridge, using source of A-C power, standard variable resistances, standard condenser, and telephone receiver. Equation of balanced bridge is

$$\frac{R_1}{R_2} = \frac{C_2}{C_1},$$

which is not the Wheatstone bridge equation for resistances.

LOCATING BREAK IN CABLE. Apparatus arranged as in bridge measurement of capacitance; using good pair of twin wires of known length exactly like the faulty pair. Equation is

$$\frac{l_1}{l_2} = \frac{R_2}{R_1}.$$

CAPACITANCE OF CONDENSERS IN PARALLEL. Equals sum of their separate capacitances.

CAPACITANCE OF CONDENSERS IN SERIES. Equals the reciprocal of the sum of the reciprocals of their separate capacitances.

CAPACITANCE OF CABLES. All cables have some capacitance. Submarine cables have a large capacitance due to water forming one plate, separated but a small distance by the insulation from the conductor which forms the other plate.

The usual equation for capacitance of a SUBMARINE CABLE is:

$$C = \frac{0.0388\ K l}{\log_{10} \frac{D}{d}} \text{ microfarads.}$$

The capacitance of an AËRIAL line is

$$C = \frac{0.0194\ l}{\log_{10} \frac{S}{r}} \text{ microfarads.}$$

The mechanical force exerted between the plates of a condenser is

$$f = \frac{1.59\ KAE^2}{l^2 \times 10^{11}} \text{ ounces.}$$

The ELECTRIC STRESS ON A STRING OF STRAIN INSULATORS is greatest across the unit next the conductor, the voltage across it being usually nearly 30% of the voltage across the total string. Adding more units will not materially affect this.

PROBLEMS ON CHAPTER XIII

Prob. 27–13. What is the capacitance of a condenser made up of 800 plates of aluminum foil 25 × 20 cm. with a dielectric of paraffined paper 0.015 cm. thick?

Prob. 28–13. What is the capacitance of a condenser having 800 plates of aluminum foil 25 × 20 cm., if the dielectric consists of mica sheets 0.015 cm. thick?

Prob. 29–13. If condensers of Prob. 27–13 and Prob. 28–13 were joined in parallel, what would be their joint capacitance?

Prob. 30–13. If condensers of Prob. 27–13 and 28–13 were joined in series what would their joint capacitance be?

Prob. 31–13. What charge would condensers joined as in Prob. 29–13 hold, if a voltage of 400 volts were placed across the terminals?

Prob. 32–13. If the voltage across the condensers connected as in Prob. 30–13 were changed from 300 to 800 volts, in 0.005 second, what average current would flow from the condensers?

Prob. 33–13. A condenser of 6 μf. capacitance in series with a condenser of unknown size is placed across a generator. When the

unknown capacitance contains 0.004 coulomb, the potential difference across both condensers is 800 volts. What is the capacitance of the unknown condenser?

Prob. 34–13. What would be the size of a plate condenser of one farad capacitance, if it could be made of the following materials? Mica sheets 0.002 inch thick and lead foil 0.0025 inch thick with an area of 50,000 square inches.

Prob. 35–13. A 6-microfarad condenser is charged to a potential of 300 volts and a 12-microfarad condenser to a potential of 480 volts. They are then joined together, positive to positive and negative to negative. What is voltage across the parallel combination?

Prob. 36–13. If the condensers in Prob. 35–13 had been joined in series after being charged, what would voltage across the combination become?

Prob. 37–13. What charge would there be on each condenser in Prob. 35–13 after being joined in parallel?

Prob. 38–13. What charge would there be on condensers in Prob. 36–13 after being joined in series?

Prob. 39–13. It is desired to build up a capacitance of 6 microfarads. There are at hand 3 condensers of 4, 5, and 3 microfarads respectively. Show by diagram what arrangement of these condensers will produce a capacitance nearest to 6 microfarads. Compute exact capacity of such an arrangement.

Prob. 40–13. How would the condensers of Prob. 39–13 be arranged to produce minimum capacitance? What would be the value of this minimum capacitance?

Prob. 41–13. What is the capacitance of a 100-mile two-wire transmission line, if the total inductance is 150 millihenries?

Prob. 42–13. What is the capacitance between two wires of the 154-mile Big Bend-Oakland transmission line? The wires, No. 000 B. & S., are spaced 10 feet apart.

Prob. 43–13. What is the capacitance to sheath per mile of a rubber-covered single-conductor cable having a wire of No. 0000 B. & S. and insulation $\frac{7}{64}$ inch thick? $K = 3.42$.

Prob. 44–13. If impregnated paper is substituted for the rubber in the cable of Prob. 43–13, what will be the capacitance per mile? The relative dielectric constant of impregnated paper may be taken as 2.8.

Prob. 45–13. The Sierra and San Francisco Power Co. transmits power from Stanislaus to San Francisco, a distance of 138 miles. The wires are No. 00 B. & S. and are spaced 96 inches apart. Compute the capacitance of two of these wires if used as line and return.

Prob. 46–13. One method of obtaining a very high d-c voltage is to charge a number of condensers in parallel and then connect them in series so that the individual voltages add. A device of this type

is called an impulse generator. An impulse generator consists of 50 condensers each of 0.1 mfd. capacitance which are charged in parallel to 20,000 volts and are then connected in series. If the series condenser group is discharged in 0.00002 second, what average current does it deliver during discharge?

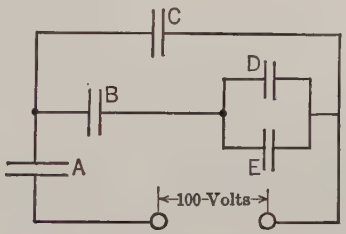


Fig. 286. The condensers are connected in series-parallel across a 100-volt supply.

Prob. 47–13. The group of condensers in Fig. 286 are connected across a 100-volt line. The capacitances are: $A = 2$ μf., $B = 4$ μf., $C = 1$ μf., $D = 3$ μf., $E = 5$ μf.

(*a*) What is the capacitance of the series-parallel arrangement?
(*b*) What is the charge on each condenser?
(*c*) What is the voltage across each condenser?

CHAPTER XIV

ALTERNATING CURRENTS

The last fifty years have seen the invention of the alternating-current system of power generation and distribution and a remarkable growth in the use of alternating-current equipment. Wherever electric power is to be transmitted more than a few miles, alternating current is generally used. Modern radio is based almost entirely on the application of alternating-current principles. In order to understand most modern electrical appliances it is necessary to be familiar with the general laws of alternating-current electricity.

227. Cycle: Frequency. An electric current which flows back and forth at regular intervals in a circuit is called an **alternating current.** When the current rises from zero to a maximum, returns to zero, and increases to a maximum in the opposite direction and finally returns to zero again, it is said to have completed a **cycle.** For convenience a cycle is divided into 360 degrees. This cycle of changes is repeated over and over again. Figure 295 is a graphical representation of the cycle of such a current. The maximum value in one direction is reached at the 90-degree point, and in the other direction at the 270-degree point. The number of times this cycle is repeated in one second, is called the **frequency** of the current. Thus a current which rises to a maximum in each direction 60 times a second is said to make 60 cycles per second, or to have a frequency of 60 cycles per second. The standard frequencies for electrical power in the United States are 25 and 60 cycles per second. The frequency of radio waves is in the order of one million cycles per second.

228. Water Analogy. If we liken the flow of a direct current to the flow of water in a river, we may liken the flow of an alternating current to the ebb and flow of the tide in a narrow channel. Of course the frequency of such a tidal flow would be extremely small. Some idea of the nature of direct currents and alternating currents can be derived from the study of Fig. 287 and 288.

Figure 287 represents a pipe circuit containing a pump with valves so arranged that the water in the pipe flows in one direction only,

independent of the direction of the piston motion. This is the water analog to a direct current system. The pipe takes the place of the wires, the pump of the generator. The valves of the pump represent the commutator of the generator. Both valves and commutator allow the current to flow in but one direction.

The pump in Fig. 288 has no valves. The direction of the current in the pipe then depends upon the direction of the piston motion. The pump without valves represents a generator without a commutator. Such a generator would deliver an alternating current to the line, as explained in Chapter VII.

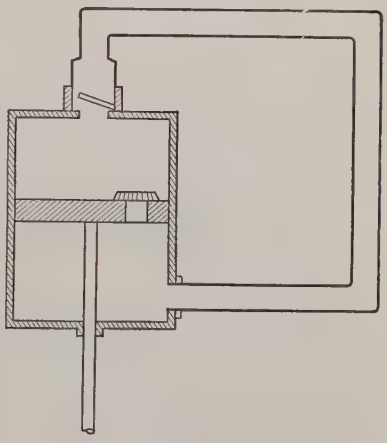


FIG. 287. A hydraulic analogy to a direct-current generator and circuit.

We will now consider, in greater detail, the e.m.f. and current set up in a single-coil armature of such an alternating-current generator.

229. Alternating-Current Generator. Single Coil. Figure 289 is a simple diagram in perspective of a single coil armature. An alternating e.m.f. will be set up in the coil if it is revolved, as explained in Chapter VII.

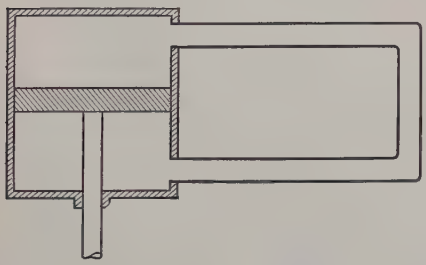


FIG. 288. A hydraulic analogy to an alternating-current generator and circuit.

The following details in connection with the e.m.f. induced should be studied carefully.

When the coil is in the position shown in Fig. 290, it is cutting no lines of force. There is therefore no voltage induced in it. This is called the neutral or zero position.

When the coil has moved 30° from the zero, as shown in Fig. 291, it is cutting lines of force, and there is an e.m.f. induced of **out** at *A*, and **in** at *B*.

Suppose we plot the e.m.f. as ordinates and the position of the coil in degrees from zero as abscissae, Fig. 295.

The line e_1 represents the instantaneous voltage at the 30° point, or when coil is at 30° as shown in Fig. 291.*

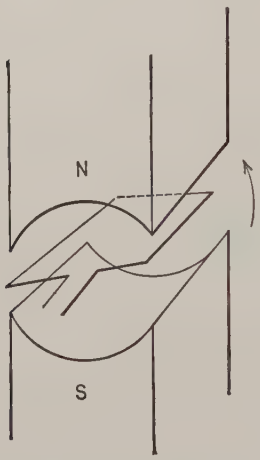


Fig. 289. A two-pole alternating-current generator with a single coil armature.

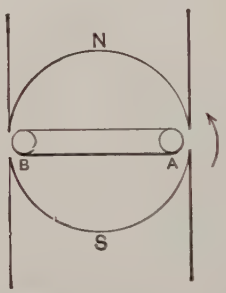


Fig. 290. The coil of the generator of Fig. 289 passing through the zero position where it does not cut any magnetic lines. Fig. 299 is the vector diagram for the voltage at this instant.

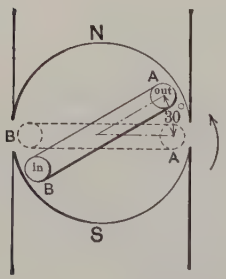


Fig. 291. The coil is passing through the 30-degree position. Fig. 300 is the vector diagram of the voltage at this instant.

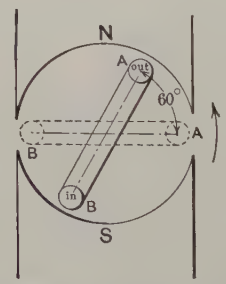


Fig. 292. The coil is passing through the 60-degree position. Fig. 301 is the vector diagram of the voltage at this instant.

* The angular position of the coil and the electrical angle are the same in a two-pole machine, but not in others. When the electrical angle differs from the angular position of the coil, it is customary to speak of the position of the coil as so many "mechanical degrees" and of the electrical angle as so many "electrical degrees." This causes no confusion in actual practice.

When the coil has moved 60° from the zero position, as in Fig. 292, the instantaneous e.m.f. has increased to a value represented by the line e_2, Fig. 295.

When the coil has moved 90° from the zero position, it is then cutting lines at the fastest rate as it is moving across them at right

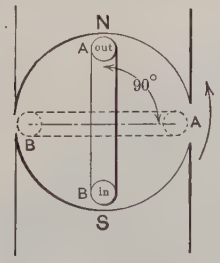


FIG. 293. The coil is passing through the 90-degree position and cutting lines at the maximum rate.

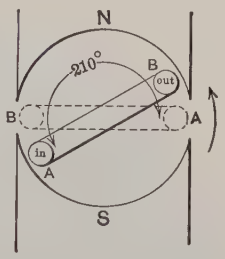


FIG. 294. The coil is passing through the 210-degree position and is now cutting lines in the opposite direction.

angles, as shown in Fig. 293. The line E_m, Fig. 295, represents the instantaneous induced voltage at this instant.

As the coil moves on from this point, the induced voltage across it begins to decrease, and at length becomes zero again, as the coil reaches a point 180° from the zero position. Lines e_3 and e_4, Fig.

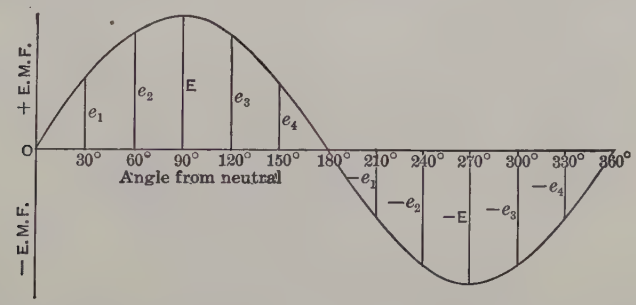


FIG. 295. A sine curve showing the value of the e.m.f. in the coil of Fig. 289 at each instant during one complete cycle.

295, show the instantaneous value of the e.m.f. at 120° and 150° respectively. At 180° the instantaneous e.m.f. is zero, but as soon as it passes this point the coil begins to cut lines in the opposite direction, and an e.m.f. is now induced which is in at A, and out at B, as shown by Fig. 294. The instantaneous value of this induced

e.m.f. is represented by the line $-e_1$ in Fig. 295. The value continues to increase in this reverse direction, until a position 270° from zero is reached. Here the voltage becomes a maximum and is represented by the line $-E_m$, Fig. 295. From here on, it gradually decreases until it becomes zero again as it reaches 360° (which is really the zero point at which we started to follow the movement of

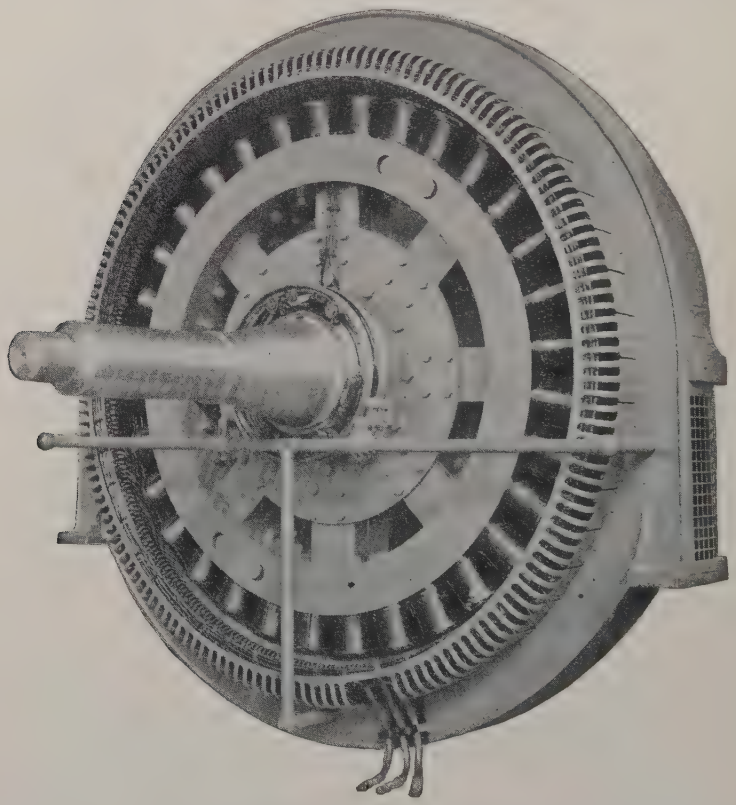


Fig. 296. A modern alternating-current generator of the revolving-field type. *General Electric Co.*

the coil). The lines $-e_3$ and $-e_4$ represent the values of the e.m.f. at 300° and 330° respectively. The curve, Fig. 295, plotted from these values, represents the various instantaneous values of the e.m.f. at all times, during one complete cycle. Note that the e.m.f. is continually changing, and if we wish to indicate its value we must state it as for a given instant only. At any other instant it may be greater or less than this value.

As the armature turns around it merely repeats this cycle of values, shown by the vertical lines in Fig. 295.

A curve like Fig. 295 is known as a **sine curve** and approximates more or less closely the curves of the e.m.f. values of most alternating-

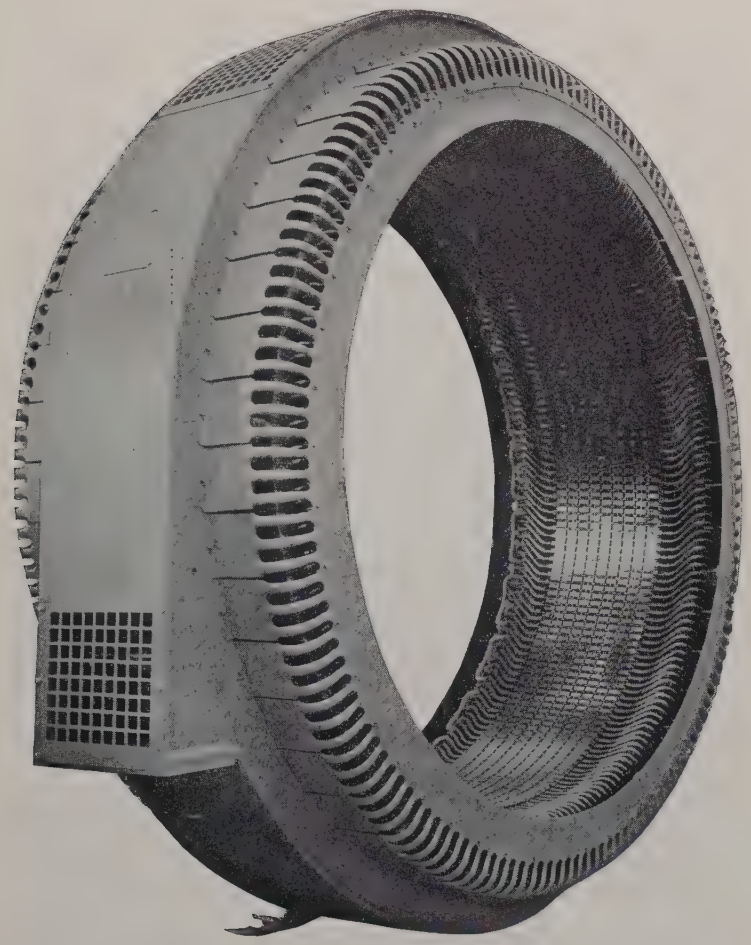


Fig. 297. The armature construction of an alternating-current generator of the revolving-field type. *General Electric Co.*

current generators. The sine wave is the simplest form of alternating curve and is nature's standard. Electric generators do not inherently deliver a sine wave of voltage but they are designed in such a way as to cause them to. In order to simplify computation

we shall consider all wave forms of voltage and current to have the sine form.

230. Alternating-Current Generator. Revolving-Field Type.* Modern alternators are usually constructed with a revolving field

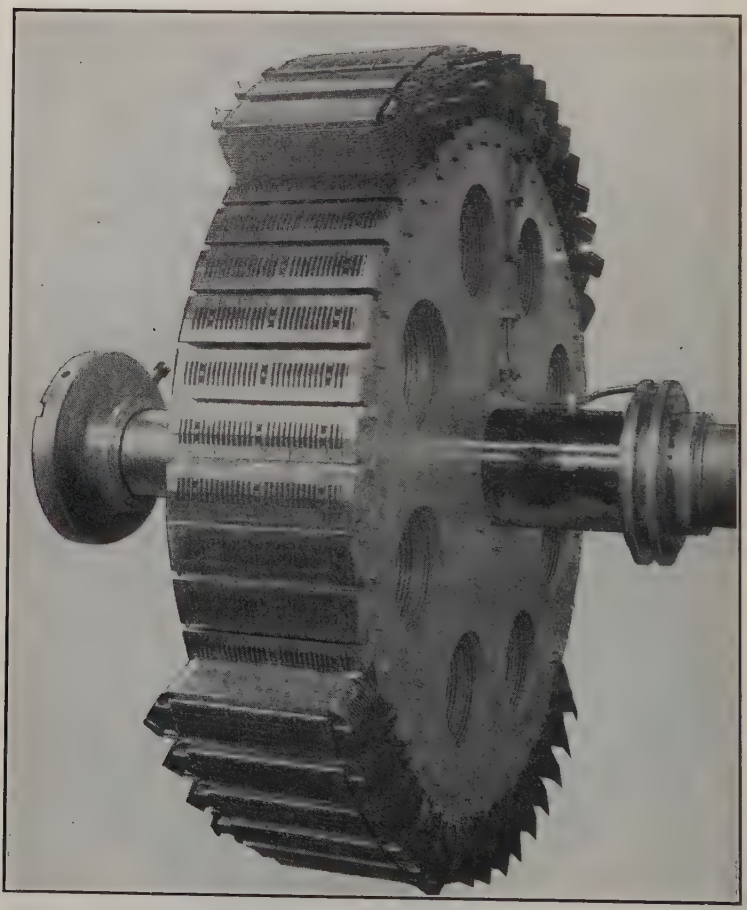


Fig. 298. The partly assembled field structure of an alternating current generator. *General Electric Co.*

rather than with a revolving armature. Note in Fig. 296 that it is the field poles which revolve. The armature coils are imbedded in the frame of the machine as shown in Fig. 297, and are cut by the

* For a more detailed description of modern alternating-current generators, see "Alternating-Current Electricity," by Timbie and Higbie.

revolving magnetic flux. Of course it makes no difference whether the coils cut the flux or the flux cuts the coils; the result is an alternating e.m.f. set up in the coils. The revolving-field type has the advantage that the high alternating voltages and large alternating currents are confined to stationary wires and are not taken off from moving contacts. Figure 298 shows the appearance of a partly assembled revolving field element. Note that the only moving contacts are at the two small collecting rings, which take the small direct current from the brushes and conduct it to the field coils. Of course the field coils carry direct current and must be separately excited from some source of constant e.m.f.

231. Vector Diagram. Figures 299–308 show another and a simpler way of representing the relation of the e.m.f. induced in an armature coil to the electrical angle, when the e.m.f. follows a sine curve.

The greatest value which the e.m.f. attains is represented by the line E_m, called a **vector.** This line is supposed to be rotating in a counter-clockwise direction about the end O. By drawing this vector to some scale, and at an angle to an axis, equal to phase angle, we determine the instantaneous value at that position, or phase, by measuring the vertical component of the vector E_m. The X axis has been used for convenience in Fig. 299–308 to represent the zero position of the vector.

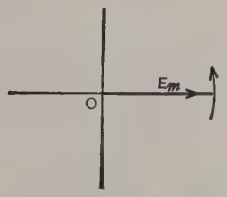


Fig. 299. Vector diagram of voltage when the coil is passing through the zero position as in Fig. 290.

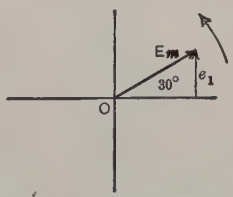


Fig. 300. Vector diagram of voltage when the coil is passing through the 30-degree position as in Fig. 291.

Thus in Fig. 299, which is the vector diagram for coil in position shown in Fig. 290, the vector E_m has not moved from the zero position and has no vertical component. Thus the value of the e.m.f. in this position is 0. This is the value of the e.m.f., as we have seen, when the coil is in the position of Fig. 290.

In Fig. 300, the vector diagram represents the instantaneous e.m.f., when the position of the coil is as shown in Fig. 291. The

coil has moved 30° from the zero position. The vector E_m is therefore at an angle of 30° to the horizontal. The vertical component of E_m is now e_1, which equals the instantaneous value of the induced e.m.f., when the coil is 30° from the zero position. The vertical component e_2 at 60° is shown in Fig. 301.

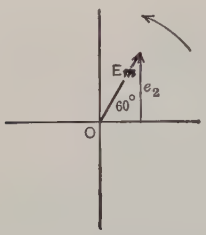


Fig. 301. Vector diagram of voltage when the coil is passing through the 60-degree position as in Fig. 292.

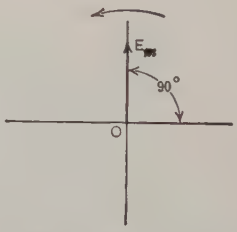


Fig. 302. Vector diagram of voltage when the coil is passing through the 90-degree position as in Fig. 293.

When the vector reaches the 90° position, the vertical component is equal to the vector itself, which is its maximum value. Figure 293 represents the coil in this position and Fig. 302 represents the vector diagram of the e.m.f. at this time.

Figures 303, 304, and 305 are the vector diagrams for the coil in

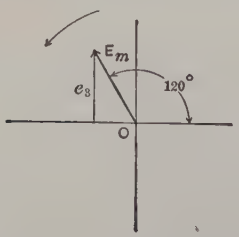


Fig. 303. Vector diagram of voltage when the coil is passing through the 120-degree position.

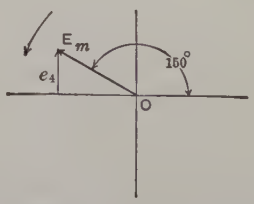


Fig. 304. Vector diagram of voltage when the coil is passing through the 150-degree position.

positions 120°, 150°, and 180° respectively. Note that e_4 is the instantaneous value for the e.m.f. when the coil is at the 150° point and its value corresponds to e_4 of Fig. 295. At 180° the instantaneous value of the e.m.f. is zero. Accordingly, in the vector diagram, Fig. 305, there is no vertical component of E_m, showing that the instantaneous value of the e.m.f. is zero.

When the coil passes the 180° point, the e.m.f. reverses. This is

shown in Fig. 306, by the fact that the line $-e_1$, which is the vertical component of E_m, is below the X axis. At 270°, the instantaneous value of the e.m.f. becomes a maximum in this reversed direction. This appears in Fig. 307, where the vertical component of E_m equals $-E_m$, and is below the line, thus being a minus quantity.

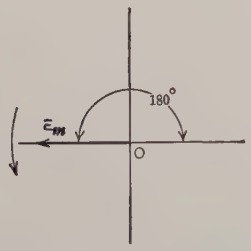


FIG. 305. Vector diagram of voltage when the coil is passing through the 180-degree position.

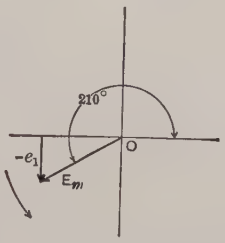


FIG. 306. Vector diagram of voltage when the coil is passing through the 210-degree position as in Fig. 294.

In the same manner, $-e_4$, in Fig. 308, corresponds to $-e_4$ in Fig. 295, and represents the instantaneous value of the e.m.f., when the coil is at the 330° point.

Note carefully:

(*a*) That the lines E_m, e_1, e_2, e_3, e_4, and $-e_1$, $-e_2$, $-e_3$, $-e_4$ repre-

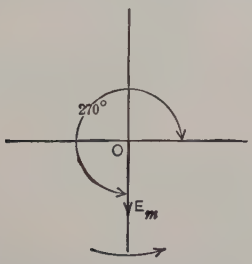


FIG. 307. Vector diagram of voltage when the coil is passing through the 270-degree position.

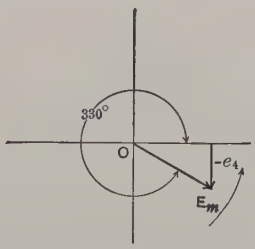


FIG. 308. Vector diagram of voltage when the coil is passing through the 330-degree position.

sent merely instantaneous values of the e.m.f. at the various positions of the coil, or at various phases in a cycle.

(*b*) That E_m represents the maximum value that these instantaneous values ever reach.

(*c*) That the maximum is reached twice during each cycle.

It follows from this that e_1, e_2, and $-e_1$, $-e_2$, etc., can be regarded as merely certain fractions of the maximum value E_m. Now we have stated that the curve, Fig. 295, was called a sine curve. It is so called because the ratio between the instantaneous values e_1, e_2, etc., and the maximum value E_m equals the **sine of the electrical angle at that instant.**

Thus in Fig. 300, e_1, the instantaneous value at the 30° phase, is a certain fraction of E_m. The value of this fraction is the sine of the angle 30°, since the phase is 30°. By consulting a table of sines, we find that sin 30° = 0.5. Thus e_1 is $\frac{1}{2}$ of E_m. So if E_m = 150 volts, the instantaneous value of the voltage when only 30° of the cycle had been completed would be $\frac{1}{2}$ of 150 or 75 volts.

In Fig. 301, e_2 is that fraction of E_m which is equal to the sine of the angle 60°. Sin 60° = 0.866. Therefore, $e_2 = 0.866 \times E_m$.

If $E_m = 150$ volts,

$e_2 = 0.866 \times 150 = 130$ volts.

The vector diagram then is a sort of snapshot taken at a given instant and shows only what is going on in the armature or circuit at that instant. A sine curve on the other hand is a sort of moving picture film on which can be read the happenings throughout an entire cycle.

232. Equation for Instantaneous Value of E.m.f. The rule then for finding any instantaneous value of the voltage in an alternating current circuit, when the voltage follows the sine wave, is:*

Multiply the maximum value of the voltage by the sine of the angle passed through by the voltage. This is generally stated in the form of an equation:

$$e = E_m \sin \phi;$$

where

e = instantaneous voltage;
E_m = maximum voltage;
ϕ = angle (in degrees).

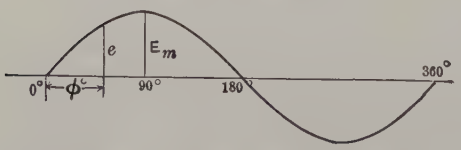


Fig. 309. A sine curve of voltage.

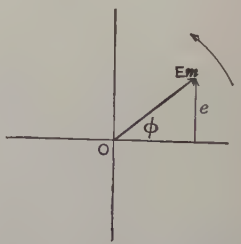


Fig. 310. A vector diagram of voltage.

* Unless otherwise stated, the wave form of an alternator is assumed to be that of a sine curve.

There are, then, **three** general ways of representing an alternating e.m.f. and of determining its value at various instants.

(1) By the graphical representation of the **sine curve,** as in Fig. 309.

(2) By the vector diagram, as in Fig. 310.

(3) By an equation, as

$$e = E_m \sin \phi.$$

It is best for one starting the subject of Alternating Currents, to do the problems by all three methods, using (1) to get a general idea of the different phase relations, etc., (2) and (3) to obtain correct numerical results.

Example 1. What is the instantaneous value of an alternating e.m.f. when it has reached 45° of its cycle? The maximum value is 600 volts.

Solution.

Method (1)

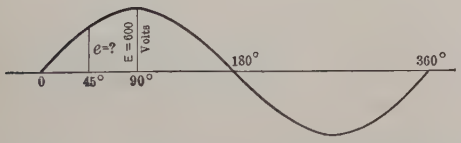


Fig. 311. By Method (1) the length of the ordinate e (to scale) is the value at $\phi = 45°$.

Method (2)

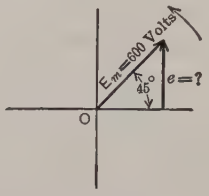


Fig. 312. By Method (2) the length of the vertical projection (to scale) is the value of e at $\phi = 45°$.

Method (3)

$$e = E_m \sin \phi,$$
$$e = 600 \sin 45°$$
$$= 600 \times 0.707 = 424 \text{ volts.}$$

Solve by three methods as in Example 1.

Prob. 1–14. What is the instantaneous value at the 70° phase of an alternating e.m.f. of maximum value 3250 volts?

Prob. 2–14. When the e.m.f. of Prob. 1–14 has reached 230° what is its instantaneous value? Note from methods (1) and (2) that e has the same numerical value as at 230°–180° = 50° phase, but that the sign is negative.

Prob. 3–14. The instantaneous value of an alternating e.m.f. is 320 volts at 125°. What is its maximum value?

Prob. 4–14. If an alternating e.m.f. has the value 280 volts at 50°, what is its value at 150°?

Prob. 5–14. The maximum value of an alternating e.m.f. is 500 volts. What are the instantaneous values at the following phases: 25°, 50°, 75°, 100°, 125°, 150°, 250°, 325°?

Prob. 6–14. Plot the sine curve, to a convenient scale, for one complete cycle of the e.m.f. of Prob. 5–14.

233. Average Value of an Alternating E.m.f. Since half of the instantaneous values of an alternating e.m.f. are negative and half are positive, and since the negative values are exactly equal to the positive values, the average of a complete cycle of values must be zero. But the actual average value is not zero. It would be as reasonable to say that the actual average value of the pressure exerted by the piston of the water pump of Fig. 288 is zero, just because the pressure alternates equally in each direction. The actual average value in both cases is the average of all instantaneous values **regardless of signs,** which only indicate direction.

The average value of an alternating e.m.f. can be found easily by plotting a number of the instantaneous values throughout the cycle and finding their average. Or, the average may be computed from the equation by calculus. The results of both of these methods show that the average value of an alternating e.m.f. is 0.637 times the maximum value. In the form of an equation, this may be written,

$$E_{av} = 0.637\ E_m.$$

The average of an alternating e.m.f. is little used except to compute the maximum value.

In finding the average voltage induced in a single loop revolving between the poles of a generator we have merely to find the average number of lines cut per second and divide by 10^8, as explained in Chapter VII. Of course if this loop is composed of many turns of wire so closely looped together that the same voltage is induced in all the wires at the same instant we also have to multiply the average voltage induced in one conductor by the number of conductors, in order to obtain the average voltage induced in the whole coil. Such a coil is said to have a **concentrated** winding.

Example 2. What is the maximum voltage generated in a drum armature with concentrated windings consisting of 300 series conductors in each path? The armature has a speed of 1200 r.p.m. Each conductor cuts twice through a field of 1,500,000 lines of force during each revolution.

Solution.

$$E_{av} = \frac{\text{lines cut per sec.}}{10^8}$$

$$= \frac{1{,}500{,}000 \times 2 \times 1200 \times 300}{10^8 \times 60}$$

$$= 180 \text{ volts.}$$

But $E_{av} = 0.637\ E_m$.

Then

$$E_m = \frac{E_{av}}{0.637}$$

$$= \frac{180}{0.637} = 282 \text{ volts.}$$

Prob. 7–14. Let the maximum value of an alternating e.m.f. for convenience be taken as 100 volts. Compute the instantaneous e.m.f. for every 2° during the first quarter of a cycle. Start with 1°, 3°, 5°... to 89°. This will give 45 instantaneous values. The sum of these values divided by 45 will give the average value of the e.m.f. for the first quarter, which of course equals the average value for the whole cycle. What fraction is this average value of the maximum value?

Prob. 8–14. In an alternator, one set of coils containing 350 concentrated conductors is being cut 8 times each second by 5×10^7 lines. What maximum voltage is induced in the coils?

Prob. 9–14. What is the voltage in coils of Prob. 8 when 0.85 of the cycle is completed?

Prob. 10–14. How much of the cycle in Prob. 8–14 is complete when the instantaneous e.m.f. is 350 volts positive and increasing?

Prob. 11–14. It is desired to generate a maximum alternating e.m.f. of 1500 volts. The number of concentrated conductors in series in each path of the armature is 500. There are two poles and the machine is run at a speed of 3600 r.p.m. How many lines of force must there be in the field?

Prob. 12–14. A 6-pole a-c generator has 600 concentrated conductors in series on the armature. Flux from each pole is 6×10^6 lines. It is desired to generate a voltage of 450 volts, maximum. At what speed must machine run?

Prob. 13–14. What would be the instantaneous voltage when the e.m.f. of machine in Prob. 11 had completed 115° of its cycle?

Prob. 14–14. What would be the average voltage of the generator in Prob. 12–14 if the speed were reduced one-half and the flux doubled?

Prob. 15–14. (*a*) What is the frequency of the generator in Prob. 11–14? (*b*) Of the generator in Prob. 12–14?

234. Computation of Current in A-C System. The diagrams, Fig. 299–310, which have been drawn to show the relation between

instantaneous values of alternating e.m.f. and the maximum value, serve just as well to show the relation between instantaneous values of alternating current and the maximum value. We have merely to put i_1, i_2, I_m, i_3, i_4, $-i_1$, etc., in the place of e_1, e_2, etc., and all diagrams show current instead of voltage values.

The same three methods can be used to solve problems in current values that we used for e.m.f. values.

(1) Thus in Fig. 313, which is similar to Fig. 295 except for the lettering, i_1 is the instantaneous current at the phase 30° from the zero, i_2 the value of the current at 60°, and I_m at 90°, etc. The current thus follows a curve of the same shape as that of the e.m.f.

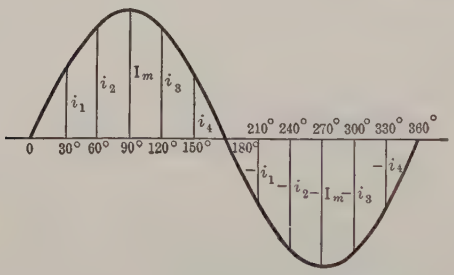


Fig. 313. A sine wave of alternating current.

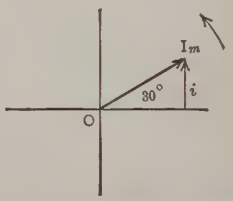


Fig. 314. Vector diagram of a current at the 30° phase.

(2) The instantaneous values can also be represented by a vector or clock diagram as in Fig. 314, which is identical with Fig. 300 except for the lettering. (3) I_m represents the maximum value of the current, and i represents the instantaneous value of the current when it is at the 30° phase; i is equal to $I_m \sin 30°$, and can be found from the general equation:

$$i = I_m \sin \phi;$$

where i = instantaneous value of an alternating current;
I_m = maximum value;
ϕ = phase angle in degrees.

As in the case of the alternating e.m.f. it is best to use all three methods for solving each problem, until the ideas of each are thoroughly mastered.

Prob. 16–14. An alternating current has a maximum value of 150 amperes. What is its instantaneous value at 20°?

Prob. 17–14. What is the maximum value of an alternating current if the value at 310° is −80 amperes?

Prob. 18–14. For the current of Prob. 17–14, what are the instantaneous values at 70° phase; 160°; 220°?

235. Average Value and Effective Value of Alternating Currents. The average value of an alternating e.m.f. has been found to be 0.637 times the maximum value. The average value of an alternating current is found in the same manner, from the equation

$$I_{av} = 0.637\ I_m.$$

Thus if the maximum value of an alternating current is 50 amperes, the average value is $50 \times 0.637 = 31.8$ amperes.

The average value is of little use, however, as we rate an alternating current on its **effective value.**

An alternating current has **no unit** of its own, but is measured in terms of the unit of direct current, the **ampere.** Now an ampere is defined as that **steady rate of flow** which will deposit a standard amount of silver from a standard solution in one hour. But an alternating current is **not a steady current,** neither will it deposit any definite amount of silver from a solution, since practically all it deposits during one-half a cycle it takes off the next half, when it is flowing in the reverse direction.

Accordingly, in order to compare the alternating with the direct current, we must use some other property which both kinds of current possess. The most useful is the heating effect of each.

Therefore an alternating current is said to be equivalent to a direct current when it produces the same average **heating** effect, under exactly similar conditions. This value is called the **effective value** of an alternating current, and is measured in amperes. It is somewhat greater than the **average value,** being equal to 0.707 of the maximum value when the current has a sine wave form. This is because the heating effect of a current depends upon the **average of the squares** of each instantaneous value of the current. Note that the heating effect of a negative current is the same as that of a positive current. The heating depends on the square of the current, which is always positive.

The effective value of an alternating current is thus often called the "Square root of the mean square," because it can be found by squaring a number of the instantaneous values in a cycle, finding the average of these squares, and then extracting the square root. For brevity, this name is shortened to "root-mean-square current," or simply "r.m.s. current." These terms both mean **effective value, based on the heating effect.**

For all practical purposes the effective value of an alternating current may be found from the following equation:

$$I = 0.707\ I_m \quad \text{or} \quad I = \frac{I_m}{\sqrt{2}};$$

where I = effective value of an alternating current;

I_m = maximum value.

236. Effective Value of Alternating E.m.f. In the same way, the value of alternating e.m.f., by which the voltage is rated, is the **effective** value, and is found by the equation:

$$E = 0.707\ E_m,$$

where E = effective value of alternating e.m.f.,

E_m = maximum value.

When no special value is designated, the **effective** value is always understood, for both current and voltage.

Proof: that the **effective** value of an alternating current equals the "square root of the average square."

Suppose two calorimeters are so arranged that one measures the heat generated per hour by an alternating current flowing through a resistance R. The other measures the heat generated per hour by a direct current flowing through an equal resistance R.

Let the two currents be so regulated that the amount of heat generated per hour by each current be equal.

Then by definition:

$$I_d = I;$$

where I_d = direct current in amperes;

I = effective value of alternating current.

The heat generated by the direct current equals,

$$H = 0.24\ I_d^2 Rt;$$

The average heat generated by an alternating current equals,

$$H = 0.24\ (\text{av. } i^2)Rt.$$

where i = instantaneous value of the current.

But the heat generated by the direct current equals the heat generated by the alternating current.

Therefore,

$$0.24\ I_d^2 Rt = 0.24\ (\text{av. } i^2)Rt;$$

$$I_d^2 = \text{av. } i^2;$$

$$I_d = \sqrt{\text{av. } i^2}.$$

But $$I_d = I.$$

Thus,

$$I = \sqrt{\text{av. } i^2}.$$

Effective value = $\sqrt{\text{Average of squares of instantaneous values}}$.

Since the effective value of voltage and current is always a definite fractional part of the maximum values, the **effective** value may be used in **vector diagrams.** To use the effective value this way, is merely equivalent to reducing the scale of the diagram.

Example 3. The maximum value of an alternating current is 65 amperes. What is the effective value?

Solution.

$$I = 0.707 \times 65 = 45.9 \text{ amperes.}$$

Prob. 19–14. The maximum value of an alternating current is 10 amperes. Find the instantaneous values of a complete cycle for every 2°. Square these instantaneous values and find the average of the squares. Extract the square root of the average to find the effective current. What fraction of the maximum current is this effective current?

Prob. 20–14. Using the instantaneous values of current found in Prob. 19–14, find the average current. What fraction of the maximum current is this average current?

Prob. 21–14. What is the maximum value of an alternating current of 80 amperes?

Prob. 22–14. What is the instantaneous value at the 210° phase of an alternating voltage of 2300 volts?

Prob. 23–14. What is the effective value of an alternating current if its instantaneous value at 240° is −36 amperes?

Prob. 24–14. An electric iron draws an alternating current of 3.8 amperes. What maximum current flows through the iron?

Prob. 25–14. At what speed must the generator of Prob. 11–14 run in order to generate 550 volts?

Prob. 26–14. It requires 120 amperes direct current to thaw out a frozen water pipe in a given time. What value of alternating current would be required to thaw the pipe in the same time? What is the maximum value of this current?

237. Phase Relations of Current and Voltage. Whenever an alternating e.m.f. sends a current through a circuit, of course this current is an alternating current. In the case of pure sine curves, the curve and equation representing the current will have the same form as the curve and equation for the e.m.f., as explained above. The vector diagrams will be similar also. Accordingly, the curves

for the alternating voltage and current in a circuit can both be drawn on the same pair of axes, as in Fig. 315. Their vector diagrams also can both be drawn on one pair of axes as in Fig. 316.

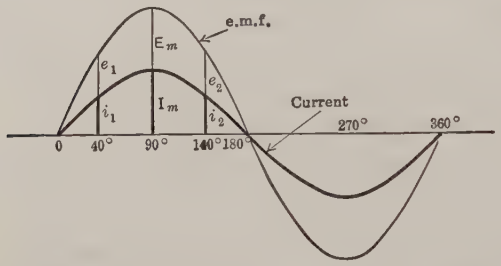


Fig. 315. Sine curves of current and voltage in phase.

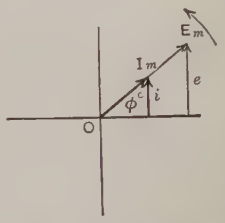


Fig. 316. Vector diagram of current and voltage in phase.

There are **three** phase relations possible between the current and e.m.f.:

(1) Current and voltage curves may be "**in phase.**" Figures 315 and 316.

(2) Current curve may "**lag**" behind voltage curve. Figures 318 and 319.

(3) Current curve may "**lead**" voltage curve. Figures 321 and 322.

238. (I) E.m.f. and Current in Phase. The current and voltage may be "**in phase**" as in Fig. 315 and 316. That is, they both are zero at the same instant, both pass through their maximum values at the same instant, and, in fact, are in the same phase throughout their entire cycles. This is the case when the **inductance** and **capacitance** effects in the circuit are balanced, or when the circuit contains **resistance** only.

Figure 315 represents the relations of the current and voltage when they are "**in phase.**" The heavy line represents the current. Note that both the curves are zero at the same instant, and that they reach their maximum values E_m and I_m at the same instant of time. The value e_1, at the 40° point, corresponds to the value i_1 at the 40° point and is the same fractional part of E_m that i_1 is of I_m.

Figure 316 also shows the vector relation of e.m.f. to current when they are "in phase." The lines I_m and E_m coincide in direction at all times, and thus the angle ϕ is always the same for both. The instantaneous value e is the same fractional part of E_m that the

instantaneous value i is of I_m. This fraction is represented by the expression "sin ϕ." That is,

$$i = I_m \sin \phi$$

and

$$e = E_m \sin \phi.$$

239. Ohm's Law in A-C Circuits. There is always likely to be some confusion about the use of Ohm's law, when the student reaches alternating current work. This confusion disappears when one understands that Ohm's law deals with **voltage, current** and **resistance** only, and always holds true as far as the relative values of these three alone are concerned. The law says nothing about **inductance** or **capacitance** and is not concerned with the effect of these. Accordingly the student may be sure that as far as the **resistance** alone is concerned, the voltage required to force a certain current through a certain **resistance** is always the product of the **current times the resistance,** whether the voltage is direct or alternating. This statement is true for both effective values and instantaneous values of voltage and current. It also holds for average values, but these are rarely applied in practice.

If some other factor like a counter e.m.f. due to inductance, etc., is in the circuit, allowance must be made for it according to some other law, not Ohm's law. Therefore do not have any doubts about the validity of Ohm's law. It **always applies** in so far as **resistance** affects the results.

Example 4. A circuit containing resistance only has an alternating e.m.f. impressed upon it, of which the effective value is 110 volts. If the resistance is a lamp of 55 ohms, what is the instantaneous current through the lamp, when the voltage is at the 30° instant? What effective current is drawn by the lamp?

Solution.

$$E = 0.707\ E_m$$

$$E_m = \frac{110}{0.707} = 156 \text{ volts.}$$

Since there is resistance only in the circuit, the current will be in phase with the voltage and will be at a maximum value when the voltage is at a maximum.

Thus by Ohm's law,

$$I_m = \frac{E_m}{R}$$

$$= \frac{156}{55} = 2.83 \text{ amperes.}$$

The instantaneous value at the 30° instant, drawn to scale, as in Fig. 317, = 1.4 amperes.

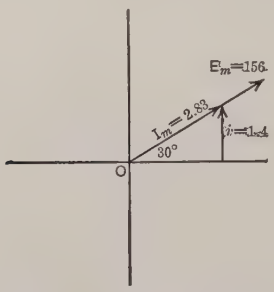


Fig. 317. Vector diagram of voltage and current in Example 4.

Or by the equation:

$$I = I_m \sin \phi$$
$$= 2.83 \sin 30°$$
$$= 2.83 \times 0.500 = 1.42 \text{ amperes.}$$

We can also find the current directly from the value of instantaneous e.m.f. At 30°

$$e = E_m \sin 30° = 156 \times \tfrac{1}{2} = 78 \text{ volts}$$

$$i = \frac{e}{R} = \frac{78}{55} = 1.42 \text{ amperes.}$$

The effective value of current is also found by Ohm's law.

$$I = \frac{E}{R} = \frac{110}{55} = 2 \text{ amperes.}$$

Prob. 27–14. The r.m.s. voltage in an alternating-current circuit containing 60 ohms resistance (no inductance or capacitance) is 185 volts. What is the effective current?

Prob. 28–14. At what phase angle is the current when 1.0 ampere flows in the circuit of Prob. 27–14?

Prob. 29–14. What average current flows in the circuit of Prob. 27–14?

Prob. 30–14. An alternating voltage of 250 volts maximum is impressed on a resistance of 50 ohms. Draw a vector diagram showing the voltage and current at 230°. What are the instantaneous values of voltage and current at this time?

Prob. 31–14. A 10-ohm resistance and a 12-ohm resistance are in series across an alternating voltage of 110 volts. What current flows in the circuit? What is the voltage across each resistance?

Prob. 32–14. At the instant the voltage across the 10-ohm resistance in Prob. 31–14 is 5 volts, what is the voltage across the 12-ohm resistance?

240. (II) Lagging Current. The current may "**lag**" behind the voltage as in Fig. 318 and 319. Note in Fig. 318 that the current curve is 50° behind the voltage curve; that is, the voltage curve has a value e_1 at 50° while the current has a value of zero at 50°. Also, the voltage curve reaches its maximum at 90° point while the current curve reaches it at the 140° point, 50° later. So when the voltage curve has become zero again at 180°, the current curve

has just gone 40° beyond the maximum value and still has a value of i_2.

Figure 319 shows how this same phase difference is represented by a vector diagram.

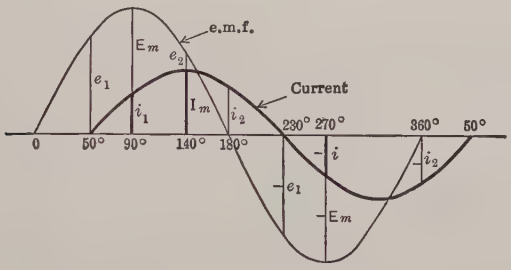


FIG. 318. Sine curves in which the current lags 50° behind the voltage.

Since the current reaches its maximum value 50° later than the voltage, the vector I_m lags 50° behind the vector E_m. Thus when the vector E_m has reached 65°, the vector I_m has reached only 15°. The instantaneous values at this instant are e and i.

$$e = E_m \sin 65°,$$

and

$$i = I_m \sin 15°$$

or

$$i = I_m \sin (65° - 50°).$$

The general equations for a "lagging" current are

$$e = E_m \sin \phi,$$
$$i = I_m \sin (\phi - \theta);$$

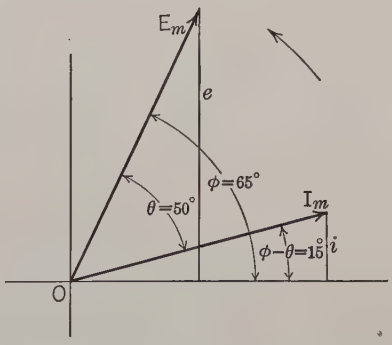


FIG. 319. The vector diagram of the voltage and current of Fig. 318.

where ϕ = phase angle of the voltage in degrees;

θ = difference in phase between E_m and I_m.

Example 5. In an inductive a-c circuit the current lags 20° behind the voltage. The maximum value of the current is 45 amperes. What is the instantaneous value of the current, when the voltage is at its 85° point?

Solution. Draw a vector diagram as in Fig. 320. The value of i when voltage is at 85° equals 41 amperes.

Or by the equation:

$$i = I_m \sin(\phi - \theta);$$
$$= 45 \sin(85° - 20°)$$
$$= 45 \sin 65°$$
$$= 40.8 \text{ amperes.}$$

Prob. 33–14. In Example 5 what instantaneous values will the current and voltage have when the voltage is at the 135° point? Maximum voltage is 600 volts.

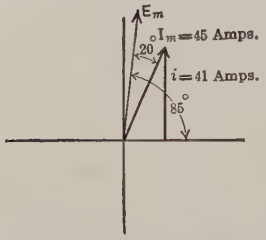


Fig. 320. Current lags 20° behind voltage. Voltage is at 85° point.

Prob. 34–14. The phase difference in an inductive a-c circuit is 45°. The current has an instantaneous value of 65 amperes when the voltage is at its maximum. (*a*) What is the maximum value of the current? (*b*) What is the effective value of the current?

Prob. 35–14. (*a*) What is the maximum value of the voltage (Prob. 34–14), if the instantaneous value is 750 volts, when the current is at its maximum? (*b*) What is the r.m.s. value of the voltage?

Prob. 36–14. The maximum value of an alternating voltage is 2300 volts, the maximum value of the current is 60 amperes. If the instantaneous value of the current is 20.5 amperes, when the instantaneous value of the voltage is 1762 volts, what is the " phase difference " between current and voltage?

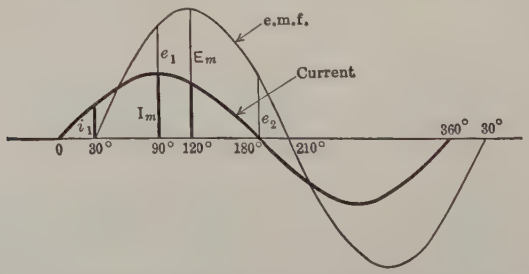


Fig. 321. Sine curves in which the current leads the voltage by 30°.

241. (III) Leading Current. The current curve may " **lead** " the voltage curve as in Fig. 321 and 322.

Notice that the current, Fig. 321, has reached a value i_1, at 30° while the voltage is at zero. The current thus " leads " the voltage by a " phase difference " of 30°. Also, while the current reaches its maximum at 90° the voltage does not reach its maximum until 120°; again a difference of 30°. And so on throughout all parts of the cycle.

In the vector diagram Fig. 322, the current vector I_m is 30° ahead of the voltage vector E_m so that when vector I_m has moved 55° from the X axis, vector E_m has gone but 25°. The equations for the instantaneous values at this instant would be:

$$e = E_m \sin 25°;$$
$$i = I_m \sin 55°$$

or

$$i = I_m \sin (25° + 30°).$$

The general equations for a "leading" current are:

$$e = E_m \sin \phi,$$
$$i = I_m \sin (\phi + \theta),$$

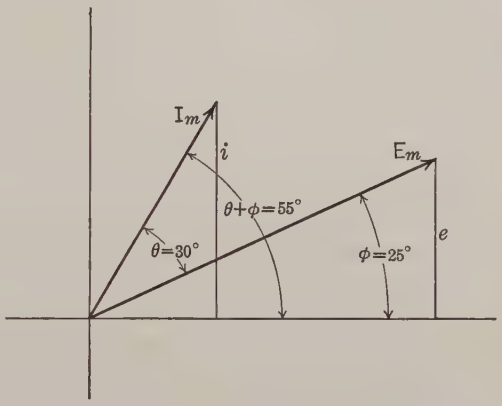


FIG. 322. The vector diagram of the voltage and current of Fig. 321.

where θ = phase difference between E_m and I_m.

Example 6. In a certain a-c circuit the current leads the voltage by 40°. The effective current is 100 amperes. What is the instantaneous current when the voltage is 0?

Solution. Find maximum value of current from equation:

$$I = 0.707\ I_m.$$

$$I_m = \frac{100}{0.707} = 141 \text{ amperes.}$$

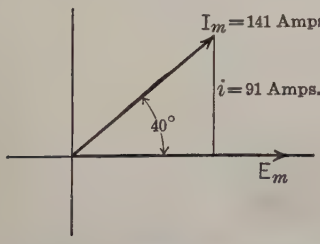


FIG. 323. Current leads voltage by 40°. Instantaneous voltage is zero.

Draw the vector diagram to scale as in Fig. 323, and note that i measures 91 amperes.

Or by the equation:

$$i = I_m \sin (\phi + \theta)$$
$$= 141 \sin (0° + 40°) = 90.6 \text{ amperes.}$$

Prob. 37–14. If the voltage in Example 6 is at a maximum value, what is the instantaneous value of the current?

Prob. 38–14. A current of 25 amperes leads a voltage of 300 volts by 70°. When the instantaneous voltage is 100 volts what is the value of the current?

Prob. 39–14. What is the instantaneous voltage in Prob. 38–14 when the current is zero?

242. Inductive Reactance, a Cause of a Lagging Current. We have seen in Chapter XII that, if an electric circuit contains inductance, an e.m.f. is induced in it whenever there is a **change** in the current flowing in the circuit. This **induced** e.m.f. always opposes the **change** in the **current.**

Now in an alternating current circuit we have a current which is **always** changing, so, if there is any inductance in the circuit we must always have an e.m.f. induced opposing this **change.** When the current is rising in value, the induced e.m.f. opposes this rise and the value of the current flowing is less than the " impressed voltage divided by the resistance," just as the current in a motor which is running is less than the impressed voltage divided by the resistance of the motor, on account of the opposing e.m.f. generated in the rotating armature. Accordingly, the rise of current in the a-c line takes place **later than the rise in the impressed voltage.** This causes the current curve to lag behind the voltage curve on the rising side of the curve.

But when the current is decreasing, the induced e.m.f. opposes the decrease and tends to keep the value of the current greater than the impressed e.m.f. alone would do. Thus the current falls off later

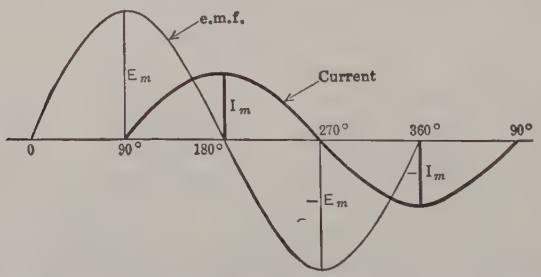


FIG. 324. Sine curves of current and voltage in a circuit containing inductance only. The current lags 90° behind the voltage.

than the impressed voltage. This causes the current curve to lag behind the voltage curve on the descending side. Thus the current curve " lags " behind the voltage curve throughout the entire cycle. If a circuit contains nothing but inductance (that is, if the resistance is negligible and the capacitance has less effect than the inductance) the current lags 90° behind the voltage.

This is evident, if we consider that the only voltage required would be that necessary to overcome the induced e.m.f. This induced e.m.f. would be greatest, not when the current was greatest,

but when the current was **changing at the fastest rate.** By inspecting Fig. 324, it is seen that the current is **changing in value most rapidly as it passes through the zero value.** Thus the voltage is greatest when the current is zero. But the voltage is always greatest when it is at its own 90° point. The current then must be zero when the voltage reaches its 90° point. The current therefore lags 90° behind the voltage.

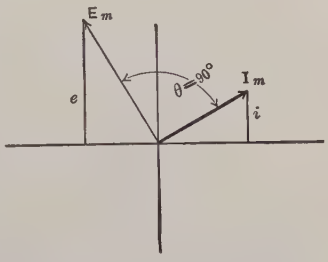


FIG. 325. Vector diagram of current and voltage in a circuit containing inductance only. The current lags 90° behind the voltage.

Again, the voltage must be least when the current is changing at its lowest rate. The current changes at zero rate when it has a maximum value. The voltage, then, is zero when the current is a maximum.

Thus the current and voltage always differ in phase by 90°, which is the difference in phase between the maximum and minimum values. Whenever the voltage is zero, the current is maximum, and whenever the voltage is maximum the current is zero. Figure 325 shows the vector diagram for voltage and current in a circuit containing inductance only.

Since simple resistance in a circuit does not cause a current to "lag" or "lead," we can say that inductance has an effect which is at 90° or at right angles to the effect of resistance. The result of this will be seen later when the combined effect of inductance and resistance is explained. If there is also resistance in series with the inductance, the angle of lag will be less than 90°.

FIG. 326. Conventional diagram of a circuit containing inductance only.

Figure 326 is the conventional way of representing a circuit containing inductance only. The current equation for this circuit is

$$i = I_m \sin(\phi - 90°).$$

This opposition of self inductance to the flow of the current is called **inductive reactance,** and is measured in **ohms** just as **resistance** is. Also, just as the current flowing through a resistance is found by the equation

$$\text{Current} = \frac{\text{Voltage}}{\text{Resistance}},$$

so the current flowing through an **inductive reactance** is found by the equation

$$\text{Current} = \frac{\text{Voltage}}{\text{Reactance}};$$

or

$$I_m = \frac{E_m}{X_L}, \qquad I = \frac{E}{X_L},$$

where I = current in **amperes**;

X_L = inductive reactance in **ohms**;

E = voltage necessary to force current through inductive reactance.

It must be remembered that if we use a maximum value of voltage we get a maximum value of current; if we use the effective value of voltage, we get the effective value of the current; if we use the average value of the voltage, we get the average value of the current.

The current value found in this way is always 90° behind the voltage producing it.

Example 7. In an a-c circuit containing 40 ohms inductive reactance (no resistance), the effective voltage is 220 volts. (*a*) What is the effective current? (*b*) What is the maximum current? (*c*) What are the values of current and voltage when the voltage is at 140° point?

Solution. (*a*) Effective current $= \dfrac{\text{Effective voltage}}{\text{Reactance}}$,

or

$$I = \frac{E}{X_L}.$$

$$I = \frac{220}{40} = 5.5 \text{ amperes.}$$

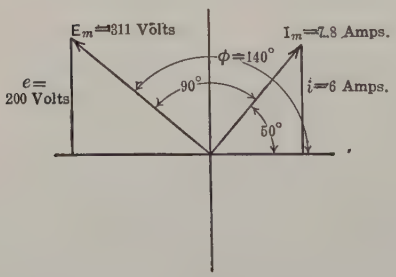


FIG. 327. Voltage is at 140°, current lags 90°.

(*b*) $I = 0.707\, I_m$;

$$I_m = \frac{I}{0.707}$$

$$= \frac{5.5}{0.707} = 7.8 \text{ amperes.}$$

(*c*) Draw a vector diagram, Fig. 327.

$$E = 0.707\, E_m;$$

$$E_m = \frac{220}{0.707} = 311 \text{ volts.}$$

When e.m.f. is at 140°, by equation

$$e = E_m \sin \phi = 311 \sin 140°$$
$$= 311 \times 0.643 = 200 \text{ volts}$$
$$i = I_m \sin (\phi - 90°) = 7.8 \sin (140° - 90°)$$
$$= 7.8 \sin 50° = 5.97 \text{ amperes.}$$

Prob. 40–14. An effective voltage of 220 volts impressed on a circuit containing inductance only produces a maximum current of 55 amperes. What is the inductive reactance?

Prob. 41–14. What is the phase angle of the voltage in Prob. 40–14 when the instantaneous current is 10 amperes?

Prob. 42–14. An inductive reactance of 50 ohms is connected across an a-c voltage having a maximum value of 1000 volts. Find: (*a*) Effective current; (*b*) instantaneous value of voltage when current is 12 amperes.

Prob. 43–14. What r.m.s. voltage is required to force an average current of 15 amperes through a 25-ohm inductive reactance?

Prob. 44–14. What instantaneous value will voltage of Prob. 43–14 have when the current is at its maximum value?

Prob. 45–14. An inductive reactance of 30 ohms carries a current of 60 amperes. At what instantaneous values of voltage will the instantaneous current be 40 amperes in the positive direction?

243. Capacitive Reactance, a Cause of Leading Currents. Capacitance in a circuit acts like an air chamber in a pump circuit; it tends to oppose any change in pressure, and to keep the pressure

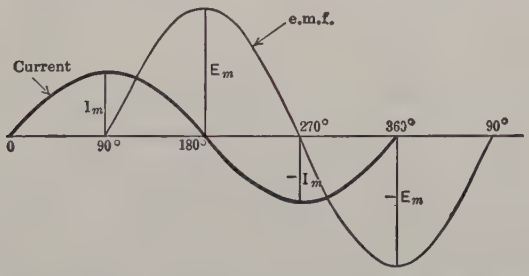


FIG. 328. Sine curves of voltage and current in a circuit containing capacitance only. The current leads the voltage by 90°.

constant. Thus condensers in a circuit might be thought of as reservoirs, in which electricity is being stored during the time when the pressure is rising. These reservoirs or condensers then use the pressure of the stored charge, to maintain the current during the time when the pressure is dying out. A current will thus be flowing

into a condenser, until it is fully charged and then cease. As long as the voltage across the condenser is rising, current will be flowing into the condenser, and the **greatest current will be flowing, when the voltage is rising most rapidly.** The voltage is rising most rapidly when it is at zero value as shown in Fig. 328. Thus the current must be greatest, when the voltage is zero, that is, the phase of the current must be 90° when the voltage phase is zero. The current thus "leads" the voltage by 90°. In the same way the current flows out from the condensers at the greatest rate when the voltage is falling at the fastest rate. The voltage falls fastest at 270°, where it is again zero. Thus the current is at a negative maximum at this point, and is still 90° ahead of the voltage.

Thus if an a-c circuit contains capacitance only, the current "leads" the voltage by 90°. Figure 329 shows the vector diagram of a current leading the voltage by 90°.

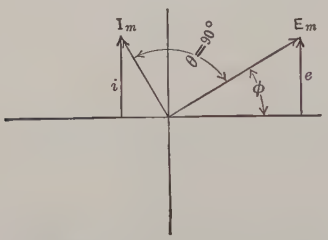


FIG. 329. Vector diagram of current leading voltage by 90°.

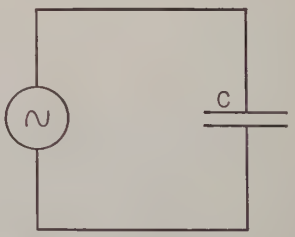


FIG. 330. Conventional diagram of a circuit containing capacitance only.

Figure 330 is the conventional way of representing an a-c circuit containing capacitance only, the current equation for which is

$$i = I_m \sin (\phi + 90°).$$

Of course, if there is resistance in series with the capacitance, the "angle of lead" will not be 90°.

The opposition which capacitance offers to the flow of the current is called **capacitive reactance,** and is measured in **ohms,** just as resistance is. The symbol for capacitive reactance is X_C, and the equation for voltage and current relations in a circuit containing a capacitive reactance of X_C, is similar to Ohm's law for the current-voltage relations in a circuit containing a resistance R. Thus we have the equation:

$$\text{Current} = \frac{\text{Voltage}}{\text{Capacitive Reactance}},$$

or

$$I_m = \frac{E_m}{X_C}, \qquad I = \frac{E}{X_C}, \text{ etc.}$$

where I = current through capacitive reactance;
X_C = capacitive reactance in ohms;
E = voltage necessary to force current through capacitive reactance.

In using the equation, the **maximum** value of the current must be used with the **maximum** value of the voltage; the **effective** current with the **effective** voltage, etc.

It must be remembered that **the current value in this equation is always 90° ahead of the voltage value.**

Example 8. The voltage across a capacitive reactance of 4 ohms is 120 volts. What is the current through the reactance?

Solution.

$$I = \frac{E}{X_C}$$
$$= \frac{120}{4} = 30 \text{ amperes.}$$

Example 9. What is the maximum value of the current in above example?

Solution.

$$I = 0.707\, I_m;$$
$$I_m = \frac{I}{0.707}$$
$$= \frac{30}{0.707} = 42.4 \text{ amperes.}$$

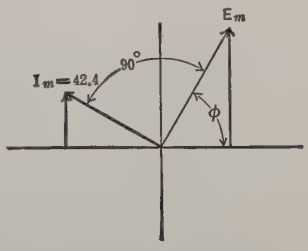


FIG. 331. Vector diagram for Examples 8 and 9. The current leads the voltage by 90°.

A vector diagram for these examples is shown in Fig. 331. Note that the maximum value of the current I_m is 90° ahead of the maximum value of the voltage E_m.

Prob. 46–14. Draw vector diagram for Example 8 with voltage at 120° phase angle and find instantaneous values of current and voltage.

Prob. 47–14. A 240-volt a-c circuit contains a condenser only. If the current is 25 amperes, what is the capacitive reactance of the condenser?

Prob. 48–14. What is the instantaneous value of current in Prob. 47–14 when the instantaneous voltage is 250 volts?

Prob. 49–14. What voltage will force a maximum current of 12 amperes through a circuit containing 65 ohms of capacitive reactance?

Prob. 50–14. (*a*) Draw vector diagram and determine instantaneous value of voltage when instantaneous current in Prob. 49–14 is 5 amperes and decreasing. (*b*) In what phase angle will voltage be at that instant?

244. Computation of Inductive Reactance. In taking up the computation of Inductive Reactance, that is, the opposition offered to the flow of an alternating current by an inductance, it is necessary to have well in mind the meaning of the term **inductance** and its measurement.

Inductance has been defined in Chapter XII as the **inertia reaction** to any **change** in the current flowing in an electric circuit.

A circuit has one unit of **inductance,** that is, one **henry,** when a **change** of one **ampere per sec.** induces an e.m.f. of one **volt.**

This is expressed by the equation:

$$E_{av} = L\frac{I}{t};$$

where E_{av} = average **voltage** induced in circuit;
L = inductance of circuit in **henries**;
I = change of current in **amperes**;
t = time of change in **seconds.**

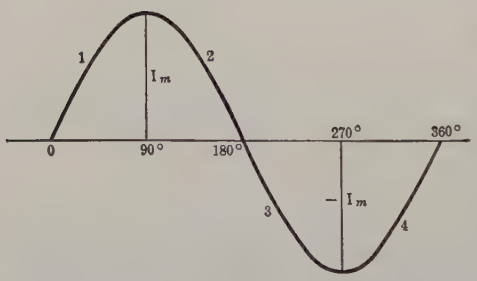


FIG. 332. One cycle of a sine wave of alternating current.

This equation may be stated in words as follows:

The voltage induced in any circuit by change in the current is equal to the product of the inductance in henries times the rate of change of current in amperes per second.

Now in an alternating current, **during one cycle,** the current makes the following **four** changes, as shown in Fig. 332.

(1) The current rises from zero to maximum.
(2) " falls from maximum to zero.
(3) " rises from zero to maximum in opposite direction.
(4) " falls from this maximum to zero.

In each of the four changes, the current makes a complete change from zero to the maximum value I_m, or vice versa, in one direction or the other. The time of each complete change can be found as follows:

The number of cycles per sec. $= f$.

The time for one cycle $= \frac{1}{f}$ sec.

" each change $= \frac{1}{4}$ time for each cycle.

" " $= \frac{1}{4f}$ sec.

The average rate of change, then, of an alternating current, must be I_m amperes in $\frac{1}{4f}$ sec., or $(4fI_m)$ amperes per sec.

Thus the expression,

$\frac{I_m}{t}$ (rate of change of current) must equal $4fI_m$.

The expression $$E_{av} = L\frac{I_m}{t}$$

becomes $$E_{av} = 4fLI_m.$$

That is, the **average voltage** induced in an inductive circuit equals **four** times the product of the **frequency** by the **inductance** by the **maximum current.**

If a circuit contains nothing but inductance, the impressed voltage merely has to overcome the induced voltage, and is therefore equal to it; just as in starting an unloaded machine which has no friction, an impressed force has to overcome merely the inertia reaction and must be equal to it.

Thus the average impressed voltage must equal the average induced voltage. The maximum voltage E_m is generally used in the equation, and this is found as follows:

$$E_{av} = 4fLI_m,$$

but $$E_{av} = 0.637\,E_m,$$

so $$0.637\,E_m = 4fLI_m,$$

and $$E_m = 6.28fLI_m.$$

Since $$6.28 = 2\pi$$

the equation is generally written

$$E_m = 2\pi fLI_m,$$

or $$E = 2\pi fLI.$$

This is the general equation for the voltage necessary to send an alternating current through an inductance, that is, through an inductive reactance.

But we have seen that the voltage necessary to send an alternating current through an inductive reactance was found by the following equation:

$$I = \frac{E}{X_L};$$

or $$E = IX_L.$$

But $$E = 2\pi fLI,$$

so $$IX_L = 2\pi fLI,$$

and $$X_L = 2\pi fL.$$

Therefore the value of an **inductive reactance** in **ohms** is equal to **2 π** times the **frequency** times the **inductance.**

Example 10. What is the inductive reactance of an a-c circuit of 0.08 henry inductance, if the frequency is 60 cycles?

Solution.

$$X_L = 2\pi fL$$
$$= 2 \times 3.14 \times 60 \times 0.08$$
$$= 30.2 \text{ ohms.}$$

Prob. 51–14. What current will flow in circuit of Example 10 if the voltage is 280 volts?

Prob. 52–14. Draw vector diagram for Prob. 51–14, when instantaneous value of the current is 5 amperes.

Prob. 53–14. The inductance of an a-c circuit is 0.08 henry, the voltage is 150 volts. At what frequency will the current be 3 amperes?

Prob. 54–14. What will be the reactance of circuit in Prob. 53–14?

Prob. 55–14. A 2300-volt a-c circuit has a frequency of 60 cycles. What inductance must be placed in the circuit to limit the current to 15 amperes?

Prob. 56–14. What instantaneous value will current in Prob. 55–14 have when the voltage is at the 70° phase angle?

Prob. 57–14. A circuit containing inductance only draws a current of 18 amperes from a 220-volt, 60-cycle line. What is the inductance of the circuit?

Prob. 58–14. What current would be drawn by the circuit of Prob. 57–14 if placed across a 110-volt, 25-cycle line?

245. Computation of Capacitive Reactance. In a circuit containing capacitance, such as a condenser, the current charges or

discharges the condenser **four** times in each cycle. This action may be seen from Fig. 332, which represents the current curve in such a circuit:

(1) At 0° the condenser is fully charged, so it **discharges** until the 90° angle is reached. The current is a maximum at the instant the condenser is fully discharged because the corresponding voltage is zero at this instant and changing at its greatest rate (Fig. 328).

(2) From 90° to 180° the condenser **charges** with opposite sign so the current is in the same direction but decreases as the voltage across the condenser builds up. Note that a condenser plate **discharging** a positive charge causes a current flow in the same direction as a plate **charging** with a negative charge, and vice versa.

(3) The condenser becomes fully charged in the opposite direction at 180° and again **discharges** from 180° to 270°.

(4) From 270° to 360° the condenser again **charges,** and at 360° is in the same condition as at 0° and is ready to repeat the cycle.

The time consumed for each complete charge or discharge is $\frac{1}{4f}$ sec. The quantity of charge in a condenser can be found by the equation (Chapter XIII)

$$Q = EC;$$

where Q = charge in **coulombs**;
E = voltage across terminals of condenser;
C = capacitance in **farads** (or number of **coulombs** per volt).

But the quantity of water or electricity or anything that flows, must equal the average rate of flow times the time of flow; thus

$$Q_m = I_{av} \times t.$$

Since

$$t = \frac{1}{4f},$$

$$Q_m = I_{av} \times \frac{1}{4f} = \frac{I_{av}}{4f}.$$

Since also

$$Q_m = E_m C,$$

then

$$E_m C = \frac{I_{av}}{4f};$$

or

$$E_m = \frac{I_{av}}{4fC}.$$

But

$$I_{av} = 0.637\, I_m,$$

so

$$E_m = \frac{0.637\, I_m}{4fC} = \frac{I_m}{6.28\, fC} = \frac{I_m}{2\,\pi fC} = \left(\frac{1}{2\,\pi fC}\right) I_m.$$

The usual form of the equation is

$$E_m = \left(\frac{1}{2\,\pi fC}\right) I_m$$

or

$$E = \frac{I}{2\,\pi fC}.$$

The law may be stated in words as follows: The **voltage** necessary to cause an alternating current to flow through a line containing capacitance only, equals the product of the **current** times the **reciprocal** of $2\,\pi$ times the **frequency** by the **capacitance.**

But we have seen that the voltage to force a current to flow in a circuit containing capacitance only, also equals:

$$E = X_C I$$

where X_C = the capacitive reactance.

Therefore

$$X_C I = \left(\frac{1}{2\,\pi fC}\right) I;$$

and

$$X_C = \frac{1}{2\,\pi fC}.$$

Thus the value of **capacitive reactance in ohms** equals the **reciprocal** of $2\,\pi$ times the **frequency** times the **capacitance.**

Example 11. What is the capacitive reactance of an a-c circuit containing 35 microfarads capacitance if the frequency is 60 cycles?

Solution.

$$X_C = \frac{1}{2\,\pi fC}$$

$$= \frac{1}{6.28 \times 60 \times 35 \times 10^{-6}}$$

$$= \frac{1}{0.0132}$$

$$= 75.8 \text{ ohms.}$$

Prob. 59–14. What current will 800 volts force through the circuit of Example 11?

Prob. 60–14. What instantaneous value will voltage in Prob. 59–14 have when the instantaneous current is 8.5 amperes and decreasing?

Prob. 61–14. Find the average value of the voltage across a condenser of 22 μf. capacitance, if a current of 2.8 amperes flows, when the frequency is 180 cycles.

Prob. 62–14. What will the instantaneous current value be in Prob. 61–14, when the voltage is at 40° phase angle?

Prob. 63–14. The voltage in a 25-cycle a-c circuit is 220 volts. If a condenser of the following dimensions be placed directly across the line, what current will flow in circuit? 2600 plates of lead foil 80 cm. by 165 cm. Dielectric of paraffined paper 0.008 cm. thick. See Chapter XIII for equation for capacitance of condenser.

Prob. 64–14. Three condensers of 25, 70, and 45 μf. capacitance, respectively, are placed in parallel across a 90-cycle a-c circuit of 220 volts. What is maximum current in circuit?

246. Reactance, a Combination of Inductive and Capacitive Reactance. Inductive reactance and capacitive reactance have exactly opposite effects upon the phase relation of the current curve to the voltage curve.

Inductive reactance causes the current to "lag."
Capacitive reactance causes the current to "lead."

Thus when inductive and capacitive reactances are combined in series, they tend to neutralize each other. The combined effect then is the difference between them, and is called the **reactance.** The **reactance** of such a series circuit is found by subtracting the capacitive reactance from the inductive reactance. This is expressed by the equation:

$$X = X_L - X_C;$$

where X = reactance in **ohms**;
X_L = inductive reactance in **ohms**;
X_C = capacitive reactance in **ohms.**

When the result is positive, the reactance is due to a larger inductive reactance; when negative, to a larger capacitive reactance.

Example 12. What is the reactance of a circuit containing 14 ohms inductive reactance in series with 5 ohms capacitive reactance?

Solution.

$14 - 5 = 9$ ohms reactance (inductive).

Example 13. What is the reactance of a 60-cycle a-c circuit containing an inductance of 1.5 henries in series with a capacitance of 40 microfarads?

Solution.

$$X_L = 2\pi fL$$
$$= 6.28 \times 60 \times 1.5$$
$$= 565 \text{ ohms.}$$

$$X_C = \frac{1}{2\pi fC}$$
$$= \frac{1}{6.28 \times 60 \times 40 \times 10^{-6}}$$
$$= \frac{1}{0.0151}$$
$$= 66.3 \text{ ohms.}$$

$$X = X_L - X_C$$
$$= 565 - 66$$
$$= 499 \text{ ohms reactance (inductive).}$$

Prob. 65–14. A 35-μf. capacitance is in series with a 0.5-henry inductance. What is the reactance of this circuit at 133 cycles?

Prob. 66–14. What current flows in the circuit of Prob. 65–14 if the 133-cycle voltage across it is 1100 volts?

Prob. 67–14. If the circuit of Prob. 65–14 is connected across a 2300-volt, 60-cycle line, what current will flow?

Prob. 68–14. With the voltage in each case at the 50° phase, draw vector diagrams for the conditions of Probs. 66–14 and 67–14. What are the instantaneous currents in the two cases?

247. Impedance, a Combination of Resistance and Reactance. In a circuit containing **resistance** only, the current is in phase with the voltage. In a circuit containing **reactance** only, the current either "leads" or "lags" the voltage by 90°. The effect of reactance on the phase of the current is thus at an angle of 90° to the effect of resistance. The resultant effect, therefore, of a series combination of the two **cannot be found by simple addition or subtraction.** It can easily be found by the graphical method as follows:

Suppose it were required to find the result of a series combination of 4 ohms **resistance** and 3 ohms inductive **reactance.** Draw horizontal line R to scale to represent 4 ohms resistance (Fig. 333). Then, since the effect of reactance is at right angles, 90°, to resistance, draw the line X_L to scale at right angles from end of R, to

represent 3 ohms reactance. The line Z then represents the **impedance,** or resultant of R and X_L. In this case, when scaled off, $Z = 5$ ohms, which is neither the sum of 4 and 3, nor the difference.

Since the lines R, X_L and Z always form a right triangle, of which Z is the hypothenuse, an equation can be formed, from which the value of Z can be found directly, without drawing the lines to scale.

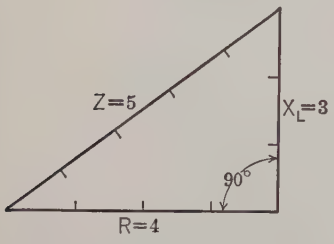


FIG. 333. Graphical method of combining inductive reactance and resistance to find the impedance.

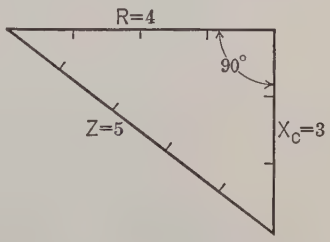


FIG. 334. Graphical method of combining capacitive reactance and resistance to find the impedance.

In every right triangle "the square of the hypothenuse equals the sum of the squares of the other two sides."

Thus

$$Z^2 = R^2 + X_L^2;$$

and

$$Z = \sqrt{R^2 + X_L^2}.$$

In above problem,

$$\begin{aligned} Z &= \sqrt{4^2 + 3^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \\ &= 5 \text{ ohms.} \end{aligned}$$

If the reactance is a capacitive reactance, it is customary to draw the diagram as in Fig. 334, with the line representing the capacitive reactance, X_C, drawn downward to show that it acts in the opposite direction to inductive reactance, which is drawn upward. The mathematical result and the equation are the same as in the case of inductive reactance.

$$Z = \sqrt{R^2 + X_C^2}.$$

If the circuit contains both inductance and capacitance in series with the resistance, then the impedance can be found by first subtracting the capacitive reactance X_C from the inductive reactance

X_L, and combining the result, X, with the resistance, R, as in Fig. 333 and 334. The equation would then become:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{R^2 + X^2}.$$

Figure 335 shows the graphical method of obtaining the impedance in a series circuit of resistance, inductance and capacitance.

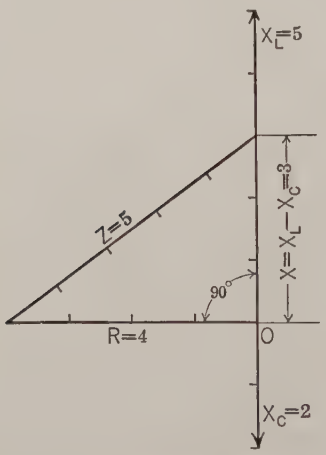


FIG. 335. Graphical method of finding the impedance when the circuit contains resistance, inductive reactance and capacitive reactance.

Suppose 5 ohms inductive reactance and 2 ohms capacitive reactance are in series with 4 ohms resistance.

The resistance R is represented by the horizontal line R, Fig. 335; the inductive reactance X_L by the line X_L drawn up from end of R, at right angles to R. The capacitive reactance is represented by the line X_C drawn down from end of R, at right angles to R. The resultant of X_L and X_C is found by subtracting X_C from X_L and is equal to X. That is:

$$X = X_L - X_C$$
$$= 5 - 2$$
$$= 3 \text{ ohms.}$$

The line Z is then drawn as the resultant of R and X. Z by scale = 5 ohms.

By the equation

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{4^2 + (5 - 2)^2}$$
$$= \sqrt{16 + 9}$$
$$= \sqrt{25}$$
$$= 5.$$

When inductance and resistance exist in the same piece of machinery as in an induction coil, the same current must be forced through both reactance and resistance. The resistance and reactance are therefore considered to be in series.

248. The Induction Motor as a Cause of Lagging Line Current. When a system has a lagging current it is frequently because the load is composed of a number of induction motors. Figure 336

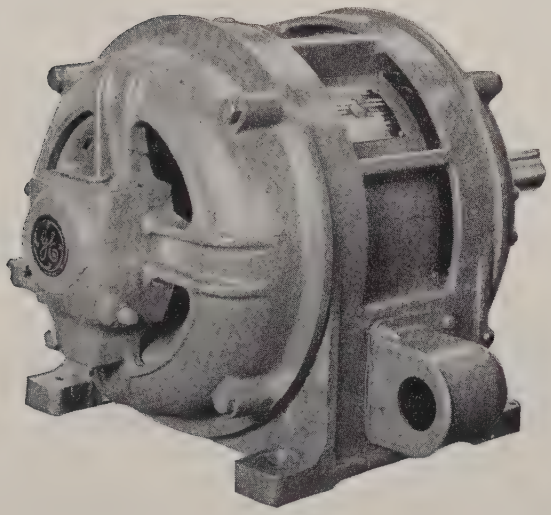


FIG. 336. An induction motor. *General Electric Co.*

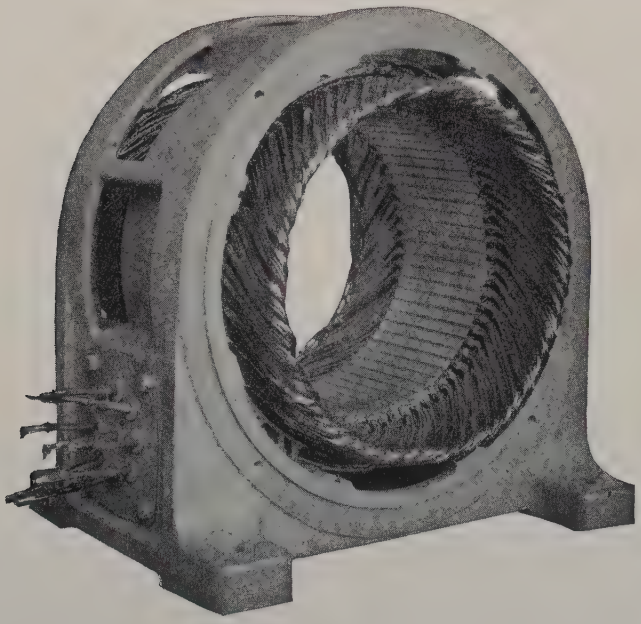


FIG. 337. The stator of an induction motor. *General Electric Co.*

represents a motor of this type. It has the advantage of being extremely rugged and uses no commutator or collecting rings.*

The alternating current is lead directly to the coils in the motor frame which is called the stator as shown in Fig. 337. The rotor is usually composed of a number of short-circuited copper bars laid around axially on the laminated core. This form of rotor, shown in Fig. 338, is called a squirrel-cage rotor and has no electrical connection to the stator or outside line. As is explained in Art. 270 the

FIG. 338. The cast squirrel-cage type rotor of an induction motor. *General Electric Co.*

flux produced by the alternating current in the stator coils revolves around the frame, and induces currents in the short-circuited bars of the rotor. These rotor currents react on the revolving flux and produce a torque, tending to turn the rotor in the direction of the flux rotation.

By using rotors composed of special windings instead of bars, and by adding special windings to the stator, an induction motor can be made to take current in phase with or even leading the voltage.

The current of a squirrel-cage motor lags particularly far behind the voltage on starting or when running under light load. At full load the angle of lag is usually between 25° and 40°. The induction motor has a drooping speed-load characteristic which is similar to that of a direct-current shunt motor.

249. General Law for A-C Circuits. The impedance of a circuit represents the total opposition to flow of the current. Thus the relation between voltage and current can be stated simply as follows:

The **current** flowing in an alternating current circuit equals the **quotient** of the **voltage** divided by the **impedance.** It is repre-

* For full description of Induction Motors, see "Alternating-Current Electricity" by Timbie and Higbie.

sented by the equation

$$I = \frac{E}{Z},$$

where I = effective current in amperes;
E = effective voltage in volts;
Z = impedance in ohms.

Example 14. What current flows in an a-c circuit if 110 volts is applied to 4 ohms impedance?

Solution.

$$I = \frac{E}{Z},$$
$$= \frac{110}{4} = 27.5 \text{ amperes.}$$

Example 15. A series circuit contains an induction coil of 10 ohms resistance and of 6 ohms reactance, and a condenser of 25 ohms capacitive reactance, negligible resistance. If the voltage is 220 volts, what current flows?

Solution.

The reactance of the induction coil must be inductive, thus:

$$X = X_L - X_C$$
$$= 6 - 25$$
$$= -19 \text{ ohms reactance.}$$
$$Z = \sqrt{R^2 + X^2}$$
$$= \sqrt{10^2 + (-19)^2}$$
$$= \sqrt{100 + 361}$$
$$= \sqrt{461}$$
$$= 21.5 \text{ ohms impedance.}$$
$$I = \frac{E}{Z}$$
$$= \frac{220}{21.5}$$
$$= 10.23 \text{ amperes.}$$

Prob. 69–14. What is the impedance of a circuit containing 100 ohms inductive reactance, in series with 75 ohms capacitive reactance and 40 ohms resistance?

Prob. 70–14. What current will flow in circuit of Prob. 69–14 if the voltage is 440 volts?

Prob. 71–14. A 60-cycle 110-volt circuit has 35 ohms resistance in series with 0.2 henry inductance and 20-μf. capacity. What is impedance of the circuit?

Prob. 72–14. What is the maximum current in Prob. 71–14?

Prob. 73–14. What would be the impedance of the circuit in Prob. 71–14 if the frequency were changed to 25 cycles?

Prob. 74–14. What would be the current in circuit of Prob. 73–14?

250. Voltage and Current Relations in Series and Parallel Circuits. The relations existing between voltage and current in series and parallel **direct current** circuits have been explained in Chapter I.

The same method of computing these relations will apply to alternating current circuits. Care must be used, however, to take into account the effect of any difference in phase between the voltage and current.

251. Series Circuits. Voltage and Current Relations. The current flowing in a series circuit is of course the same throughout, whether it be direct current or alternating current.

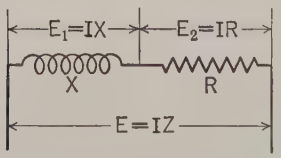


FIG. 339. A series circuit containing resistance R and inductive reactance X.

The **voltage** across a series combination equals the **vector sum** of the voltages across the separate parts.

This is exactly analogous to the rule for direct currents; the sum of the voltages in direct current being the **algebraic sum,** since the voltage and current are always in phase.

The meaning and application of the expression **vector sum** will be made clear by a study of the following illustrations.

Assume a series circuit (Fig. 339) to be made up of a reactance X, and a resistance R.

Let $E = IZ$ = voltage across the combination.
$E_1 = IX$ = voltage across the reactance.
$E_2 = IR$ = voltage across the resistance.

Electromotive force E does **not equal the algebraic sum of** E_1 and E_2, because of the effect of the reactance on the phase relations between current and voltage.

Yet the electromotive force E equals the **vector sum** of the two electromotive forces E_2 and E_1, in exactly the same manner that a single mechanical force equals the **vector sum** of any two components, which are at an angle to each other. Thus the force F (Fig. 340) is equal to the **vector sum** of its two components F_1 and F_2.

If, therefore, we represent the two electromotive forces E_1 and E_2 by lines drawn at the proper phase angle to each other, the electromotive force E will be represented by their **vector sum.**

In Fig. 341 OA is drawn to scale equal to $IR = E_2$. OB is drawn at right angles to OA equal to $IX = E_1$. The resultant or vector sum of OA and OB is OC, equal to $IZ = E$.

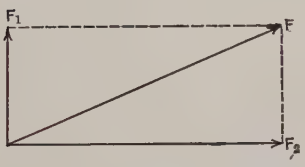


FIG. 340. The force F is the vector sum of the forces F_1 and F_2.

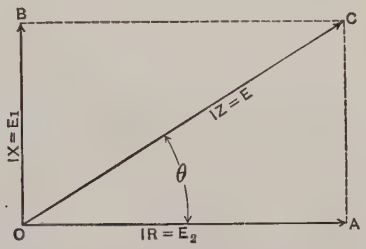


FIG. 341. The voltage E is the vector sum of the voltages E_1 and E_2.

OB is drawn at an angle of 90° to OA, because the effect of reactance is always at a 90° angle to the effect of resistance. In Fig. 334 the line X_C is drawn at right angles to the line R for a similar reason.

By scaling off the line OC, or E, we can find the value of the voltage across the series combination X and R.

Or the value of E may be computed from the equation for the right triangle

$$E^2 = E_1^2 + E_2^2 \quad \text{(since line } OB = \text{line } AC\text{)}.$$

$$E = \sqrt{E_1^2 + E_2^2}.$$

Furthermore, the angle θ is equal to the angle of phase difference between voltage and current in the circuit. This will be apparent on consideration of the following points:

Resistance alone does not throw the current out of phase with the voltage. Therefore the current must have the same phase as the component IR or E_2, which is merely that component of the voltage across the resistance R.

Since the direction of E represents the direction of the voltage across the combination, and the direction of E_2 represents the direction of the current, the angle between E and E_2 must equal the angle of phase difference between voltage across the combination and the current through the combination.

If X is inductive reactance, the angle θ represents an angle of "lag," or the number of degrees the current is behind the voltage.

If X is capacitive reactance, the angle θ represents an angle of "lead," or the number of degrees the current is ahead of the voltage.

This angle may either be measured or be ascertained by means of the following equation:

$$\frac{AC}{OA} = \tan\theta;$$

but $$AC = E_1 \quad \text{and} \quad OA = E_2.$$

Thus $$\frac{E_1}{E_2} = \tan\theta;$$

or since

$$E_1 = IX \quad \text{and} \quad E_2 = IR,$$

then $$\frac{E_1}{E_2} = \frac{IX}{IR} = \frac{X}{R} = \tan\theta.$$

Knowing the value of X and R, the tangent of the angle θ can be computed, and the angle θ found by reference to a table of tangents.

Example 16. In a series a-c circuit consisting of 8 ohms inductive reactance and 10 ohms resistance, a current of 6 amperes is flowing. (*a*) What is the voltage across each part of the circuit? (*b*) What is the voltage across the combination? (*c*) What is the difference in phase between the current and voltage in the combination?

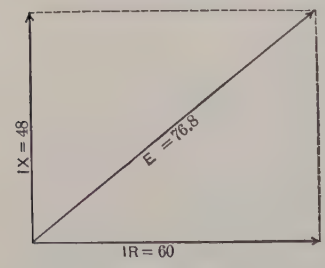


FIG. 342. The voltage E is the vector sum of voltages IX and IR.

Solution. (*a*) $IX = 6 \times 8 = 48$ volts across inductive reactance X. $IR = 6 \times 10 = 60$ volts across resistance R. (*b*) Draw IR to scale horizontally (Fig. 342). Draw IX to scale vertically.

Then E (the resultant voltage of IR and IX) = 76.8 volts.

Angle θ, the angle of difference in phase between voltage and current in circuit, equals 38.59°.

$$\begin{aligned} Z &= \sqrt{R^2 + X^2} \\ &= \sqrt{100 + 64} \\ &= \sqrt{164} \\ &= 12.8 \text{ ohms.} \end{aligned}$$

$$\begin{aligned} E &= IZ \\ &= 6 \times 12.8 \\ &= 76.8 \text{ volts.} \end{aligned}$$

(c) $$\tan\theta = \frac{8}{10} = 0.8;$$

$$0.8 = \tan 38.59°;$$

$$\theta = 38.59°.$$

The current lags 38.59° behind the voltage, since X is **inductive** reactance.

The vector diagram for current and voltage would therefore be like that of Fig. 343.

The diagram of Fig. 344 shows the phase relation of current and voltage by means of sine curves for current and voltage.

When voltage has reached the instantaneous value e, about the 39° point, the current has a value of zero. The current reaches its maximum value I_m about 39° later than the voltage reaches its maximum value E_m. The current curve, then, is everywhere approximately 39° behind the voltage curve.

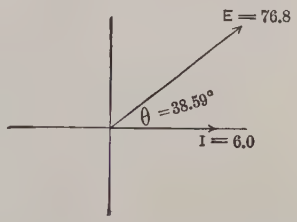


FIG. 343. Vector diagram for current and voltage relations of Example 16.

Note the fact, surprising to students who have worked on direct current circuits only, that the algebraic sum of the voltages across the separate parts (48 + 60 = 108) is greater than the actual voltage across the combination (76.8 volts).

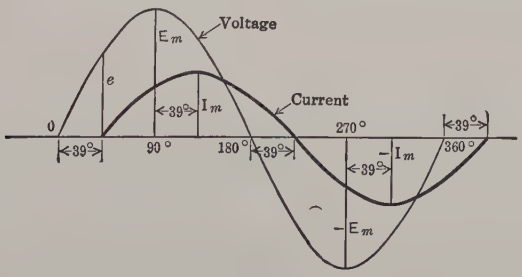


FIG. 344. Sine waves showing the relation between current and voltage in Example 16.

The combined resistance of a number of separate resistances in series equals the algebraic sum of the separate resistances.

The combined reactance of a number of reactances in series equals the algebraic sum of the separate reactances. (Inductive reactances are positive and capacitive reactances are negative.)

The impedance, or combined effect of series resistance and reactance, has been dealt with earlier in the chapter.

The combined impedance of a number of impedances in series equals the **vector sum** of the separate impedances.

Prob. 75–14. A series circuit containing 18 ohms resistance and 10 ohms capacitive reactance is across a 115-volt, 60-cycle line. What current flows in this circuit?

Prob. 76–14. Find the voltage across each part of circuit in Prob. 75–14.

Prob. 77–14. Find the phase relation between the current and voltage in Prob. 75–14.

Prob. 78–14. A 550-volt, 60-cycle line is connected across a series circuit composed of the following elements: (1) A resistance of 20 ohms. (2) A condenser of 80 μf. (3) A coil of 0.15 henry inductance and 30 ohms resistance. (4) A condenser of 50 μf. (5) A 10-ohm resistance. Find: (*a*) Impedance of the series circuit. (*b*) Current through the circuit. (*c*) Voltage across each part. (*d*) Phase angle between current and voltage across each part. (*e*) Phase angle between line voltage and current.

Prob. 79–14. When an inductive reactance of 16 ohms, a resistance of 8 ohms, and a capacitive reactance of 8 ohms are connected in series across a certain line, the voltage across the condenser is found to be 48 volts. (*a*) What is the line voltage? (*b*) What is the phase angle between line voltage and current?

Prob. 80–14. A coil containing 100 ohms resistance and 0.50 henry inductance is placed in series with a 60-μf. condenser and a 30-ohm resistance, across a 60-cycle a-c circuit of 550 volts. What current flows in circuit?

Prob. 81–14. What is phase relation between current and line voltage in Prob. 80–14?

Prob. 82–14. What is the voltage across the coil in Prob. 80–14? What is the phase angle between this voltage and the current?

Prob. 83–14. What is the voltage across the portion of the circuit of Prob. 80–14 which includes the condenser and the 30-ohm resistance? What is the phase relation between this voltage and the line voltage?

252. Resonance in Series Circuits. A series circuit is said to be in **resonance** when the capacitive reactance equals the inductive reactance; that is, when the resulting reactance is zero. When a series circuit is resonant it will allow the maximum current to flow. The process of tuning a radio receiving set consists largely of balancing the inductive and the capacitive reactance for the particular

frequency being received, thus allowing the voltage to send the maximum current through the coils of the receiver. Unlikely as it may seem at first, we can often decrease the impedance of a series circuit by adding the proper kind of reactance in series to it.

Again since capacitive reactance is decreased and inductive reactance is increased by an increase in the frequency, combinations of inductive and capacitive reactances offer but small impedance (that is, are resonant) to voltages of one frequency and offer high impedance to voltages of other frequencies.

Example 17. A series circuit has a resistance of 2 ohms, an inductance of 0.0191 henry and a capacitance of 2 μf. (*a*) What will be the impedance of the circuit to a voltage of 500 cycles per second? (*b*) What capacitance must be added in parallel with the 2-μf. condenser to produce resonance at this frequency? (*c*) What will be the impedance of the circuit at resonance?

Solution.

(*a*)
$$X_L = 2\pi fL = 6.28 \times 500 \times 0.0191$$
$$= 60 \text{ ohms.}$$
$$X_C = \frac{1}{2\pi fC} = \frac{10^6}{6.28 \times 500 \times 2}$$
$$= 159 \text{ ohms.}$$
$$X = X_L - X_C = 60 - 159$$
$$= -99 \text{ ohms.}$$
$$Z = \sqrt{R^2 + X^2} = \sqrt{2^2 + 99^2}$$
$$= 99 \text{ ohms.}$$

(*b*) For resonance
$$X_C = X_L = 60 \text{ ohms.}$$
Therefore
$$60 = \frac{10^6}{6.28 \times 500 \times C}$$
$$C = 5.31.$$

To produce resonance, there must be added $5.31 - 2 = 3.31$ μf.

(*c*) When $X_C = X_L$, then $X = 0$.

The impedance therefore is given by:
$$Z = \sqrt{R^2 + X^2} = \sqrt{2^2 + 0}$$
$$= 2 \text{ ohms.}$$

Note that by adding a capacitance of 3.31 μf. we have reduced the impedance of the series circuit from 99 ohms to 2 ohms, or to $\frac{1}{50}$ of its former value.

Prob. 84–14. For what frequency will the circuit of Example 17 be in resonance, if the 3.31 μf. are not added?

Prob. 85–14. How much inductance would have to be added or taken out of the circuit in Example 17 in order to produce resonance at 500 cycles, if the capacitance remained 2 μf.?

Prob. 86–14. A series circuit has an inductance of 0.035 henry and a resistance of 3.2 ohms. How much capacitance must be added to produce resonance at 1000 cycles per second?

Prob. 87–14. After the capacitance has been added to the circuit of Prob. 86–14, how much more impedance will the circuit offer to impulses of a frequency of 100 cycles per second than to impulses of 1000 cycles per second?

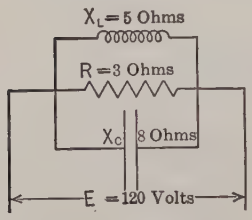


FIG. 345. A parallel combination of resistance, inductive reactance, and capacitive reactance.

253. Parallel Circuits. Voltage and Current Relations. The **voltage** across a parallel combination is the same as the voltage across each branch, as in a direct-current circuit.

The **current** through a parallel combination is the **vector sum** or resultant of the currents through each branch.

Example 18. In the parallel combination, Fig. 345, it is desired to find the current flowing through each branch, and the total current through the combination.

Solution.

$$I_C = \frac{120}{8} = 15 \text{ amperes (current through } X_C),$$

$$I_R = \frac{120}{3} = 40 \text{ amperes (current through } R),$$

$$I_L = \frac{120}{5} = 24 \text{ amperes (current through } X_L).$$

$$I = I_C \oplus I_R \oplus I_L = \text{current through combination.}$$

Note. The sign $\oplus$ is to stand for the **vector sum** of the two quantities between which it is placed.

Draw I_R, Fig. 346, to represent the current through the resistance R. At right angles to I_R (ahead of I_R), draw I_C to represent the current through X_C, leading the current in I_R by 90°. At right angles to I_R, (behind I_R), draw I_L to represent the current through I_L, lagging 90° behind current in I_R. Construct the parallelogram

$OACG$ and draw the diagonal OC. This represents the resultant of I_L and I_R.

That is, $OC = I_R \oplus I_L$.

Construct a parallelogram $ODBC$, and draw the diagonal OB or I. This diagonal I represents the resultant of I_C and OC. That is, $I = OC \oplus I_C = I_R \oplus I_L \oplus I_C$.

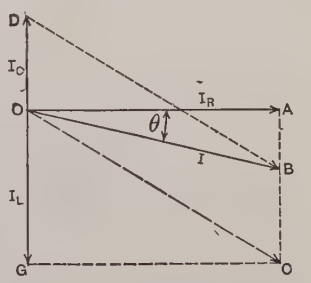


FIG. 346. Graphical method of finding the current through the parallel combination of Fig. 345.

Therefore I represents the amount of the current in the parallel combination of R, X_C and X_L. The voltage must be in phase with I_R, the current through R. Thus the angle θ between I and I_R is a measure of the phase difference between voltage and the current of the combination. In this case θ represents an angle of lag, since the inductive reactance X_L is not entirely counteracted by the capacity reactance X_C.

$I = 41$ amperes;

$\theta = 12.7°$.

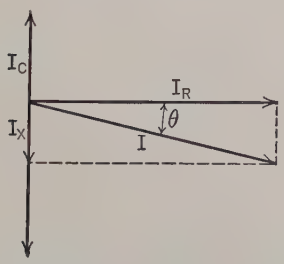


FIG. 347. A simpler graphical method for finding current in Fig. 345.

Instead of Fig. 346, the simpler Fig. 347 may be drawn. The line I_X represents the current through the combined reactances, X_L, and X_C. The resultant current, I, through the whole combination, can then be found by the construction of but one parallelogram. Of course the results obtained are the same. The values may be obtained by computation as follows:

$$\begin{aligned} I &= I_L \oplus I_C \oplus I_R; \\ &= \sqrt{(I_L - I_C)^2 + (I_R)^2} \\ &= \sqrt{(24 - 15)^2 + 40^2} \\ &= \sqrt{9^2 + 40^2} \\ &= \sqrt{1681} \\ &= 41 \text{ amps. (current through combination).} \end{aligned}$$

$$\tan \theta = \frac{I_L - I_C}{I_R} = \frac{9}{40} = 0.225.$$

From tables of tangents

$$0.225 = \tan 12.7°.$$

254. Adding Alternating Current Curves. Currents in the different branches of a parallel circuit may be either

(*a*) in phase,

or

(*b*) out of phase.

(*a*) **In Phase.**

We will consider first the case where the currents are in phase.

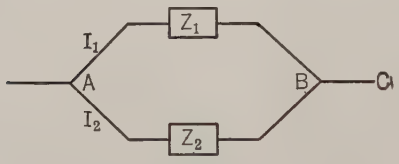


Fig. 348. A simple parallel circuit.

Assume that the currents I_1 and I_2 in the parallel circuits (Fig. 348) are in phase.

Then the equation

$$I = I_1 \oplus I_2$$

becomes

$$I = I_1 + I_2.$$

The curves for this condition are represented by Fig. 349. The current flowing in the part BC of the circuit is represented by the curve I. The current flowing in Z_1 is represented by the curve I_1; the current through Z_2, by the curve I_2. Note that the curve I is a combination of curves I_1 and I_2. Both currents I_1 and I_2 do not flow at once in BC, but merely one current, their resultant I.

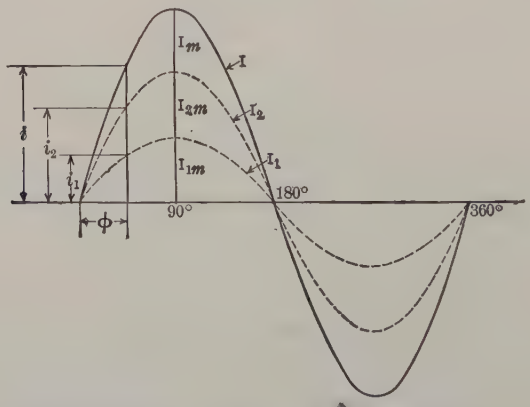


Fig. 349. The sine curve I is the sum of the sine curves I_1 and I_2. The curves are in phase.

Any instantaneous value of I is the sum of the corresponding values of I_1 and I_2. For instance, $i = i_1 + i_2$.

Or,

$$I_m \sin \phi = I_{1m} \sin \phi + I_{2m} \sin \phi.$$

(*b*) **Out of Phase.**

We will consider the following cases:

(1) When the current phases differ by 180°
(2) When the current phases differ by 90°
(3) When the current phases differ by $\theta°$ (general case).

(1) Currents 180° Out of Phase. If the current I_1 lags 180° behind I_2, Fig. 350 will represent the curves.

The equation

$$I = I_1 \oplus I_2$$

becomes

$$I = I_2 - I_1.$$

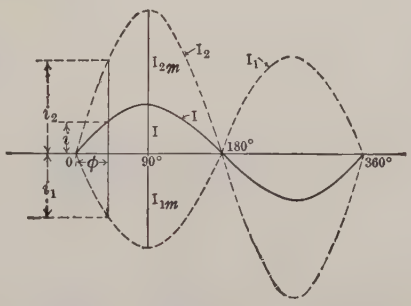


FIG. 350. Current curve I equals the sum of current curves I_1 and I_2 which differ in phase by 180°.

Any instantaneous value, such as i, equals $i_2 - i_1$, but i_2 and i_1 are not in the same phase. The equation for instantaneous value is

$$i = I_m \sin \phi = I_{2m} \sin \phi + I_{1m} \sin (\phi + 180°).$$

Or

$$i = I_{2m} \sin \phi - I_{1m} \sin \phi.$$

(2) Currents 90° Out of Phase. If the current I_2 lags 90° behind I_1, Fig. 351 represents the curves of the current in the different parts of the circuit. The equation for the resultant current I in part BC is

$$I = I_2 \oplus I_1.$$

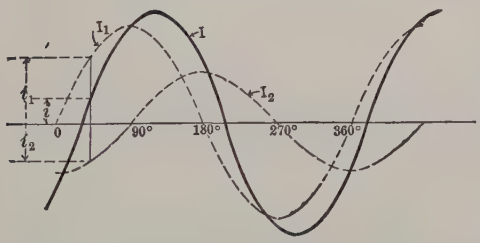


FIG. 351. Current curve I equals the sum of current curves I_1 and I_2 which are 90° out of phase.

Any instantaneous value can be found by adding algebraically the values of I_1 and I_2 at the same instant.

Thus in Fig. 351

$$i = i_1 + i_2.$$

Thus, if two currents which differ in phase by 90° are added together, the resulting current will be the sum of the instantaneous values of the two currents, and will not be in phase with either current. It may be said to differ in phase from one of the currents by an angle of $\theta°$.

The instantaneous value can be found from the equation:

$$i = I_m \sin (\phi - \theta) = I_{1m} \sin \phi + I_{2m} \sin (\phi_2 - 90°).$$

(3) **Currents $\theta°$ Out of Phase.** Let currents I_1 and I_2 differ in phase by $\theta°$.

Then I, Fig. 352, represents the vector sum of these two currents according to the equation,

$$I = I_1 \oplus I_2.$$

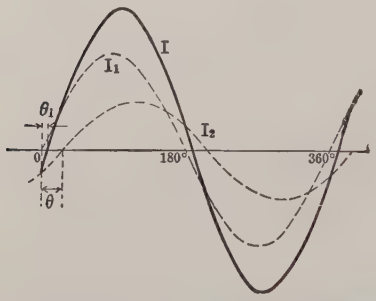


FIG. 352. Current curve I equals the sum of current curves I_1 and I_2 which are $\theta°$ out of phase.

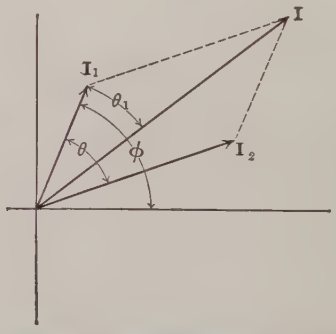


FIG. 353. Vector diagram in which I equals the vector sum of I_1 and I_2 which are $\theta°$ out of phase.

Since I_2 lags $\theta°$ behind I_1, the resultant of the two will be in phase with neither, but may be considered to lag $\theta_1°$ behind I_1. The equation for any instantaneous value then becomes

$$i = I_m \sin(\phi - \theta_1) = I_{1m} \sin \phi + I_{2m} \sin(\phi - \theta).$$

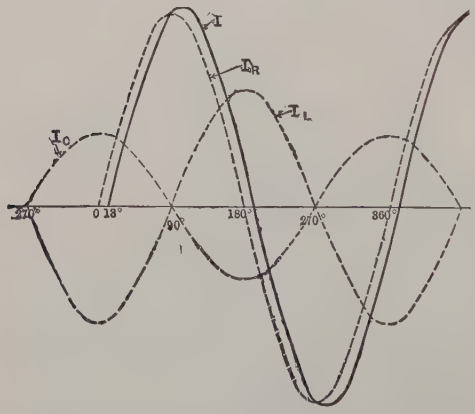


FIG. 354. Curves showing the currents in Example 18, Art. 253. I equals the sum of I_R, I_C and I_L.

Figure 353 represents vector diagram of this case corresponding to curves in Fig. 352.

Thus the current flowing in the circuit of Example 18 would be represented by a single curve I, Fig. 354, made up of curves I_R, I_L and I_C.

Any instantaneous value on this curve would be made up of corresponding instantaneous values on the other three curves.

The curve I also lags approximately 13° behind the voltage, which of course must be in phase with curve I_R. Any instantaneous value may be found from the equation

$$i = I_m \sin(\phi - 13°)$$
$$= I_{Rm} \sin \phi + I_{Lm} \sin(\phi - 90°) + I_{Cm} \sin(\phi + 90°).$$

255. The Impedance of a Parallel Combination. This can always be found without any special rule. Merely divide the **voltage** across the combination (found as above) by the **current** through the combination. If the voltage is not given, **assume** a voltage and proceed as above.

Example 19. What is the impedance of the parallel circuit of Example 18, in which the voltage across the combination is 120 volts and the current through the combination is 41 amp.?

Solution.

$$Z \text{ (of combination)} = \frac{E \text{ (across combination)}}{I \text{ (through combination)}},$$

$$Z = \frac{120}{41} = 2.93 \text{ ohms.}$$

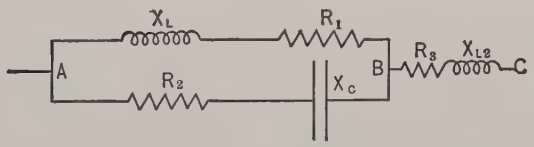


FIG. 355. A series-parallel arrangement of resistances and reactances.

Example 20. A circuit arranged as in Fig. 355 presents all fundamental combinations. (*a*) Find the value of the impedance when the following values are known. (*b*) What is the difference in phase between current and voltage?

$X_L = 6$ ohms. $R_2 = 8$ ohms. $X_{L_2} = 3$ ohms.
$R_1 = 4$ ohms. $X_C = 10$ ohms. $R_3 = 5$ ohms.

Solution.

To find impedance (Z_1) and phase angle of branch composed of X_L and R_1 in series, draw Fig. 356.

By computation:

$$Z_1 = \sqrt{R_1^2 + X_L^2}$$
$$= \sqrt{16 + 36}$$
$$= \sqrt{52}$$
$$= 7.21 \text{ ohms.}$$

$$\tan \theta_1 = \tfrac{6}{4} = 1.5,$$
$$\theta_1 = 56.31°.$$

The impedance of branch composed of X_L and R_1 in series is 7.21 ohms.

The current through this branch of the circuit lags 56.31° behind the voltage.

To find impedance Z_2 and phase relations of branch composed of R_2 and X_C draw Fig. 357.

By computation:

$$Z_2 = \sqrt{R_2^2 + X_C^2}$$
$$= \sqrt{64 + 100}$$
$$= 12.8 \text{ ohms.}$$
$$\tan \theta_2 = \tfrac{10}{8} = 1.25.$$

From table, $\theta_2 = 51.34°$.

Impedance of branch composed of R_2 and $X_C = 12.8$ ohms. The current through this branch leads the voltage by 51.34°.

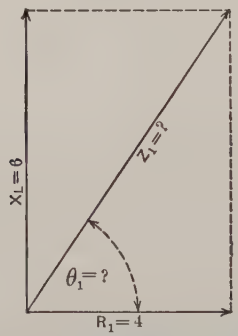


Fig. 356. Graphical solution of the impedance of the series combinations of X_L and R_1 of Fig. 355.

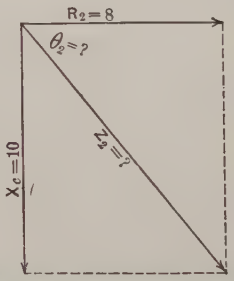


Fig. 357. Graphical solution of the impedance of the series combination of X_C and R_2, Fig. 355.

To find the combined impedance of above branches in parallel draw Fig. 358. By assuming a voltage of 110 volts across AB, the following current must flow:

Through Z_1,

$$I_1 = \frac{E}{Z_1} = \frac{110}{7.21} = 15.3 \text{ amperes.}$$

Through Z_2,

$$I_2 = \frac{E}{Z_2} = \frac{110}{12.8} = 8.6 \text{ amperes.}$$

In Fig. 358, E represents the phase of the voltage E. Current through Z_1 lags 56.3° behind E; then the line, I_1 of 15.3 amperes, at an angle of 56.3° lag to E, represents the current through Z_1. Similarly line I_2 represents a current of 8.6 amperes, 51.3° ahead of E in phase. The line I, resultant of I_1 and I_2, represents the current through the parallel combination; the angle θ represents the difference in phase between the current through combination and voltage across combination.

By computation (referring to Fig. 359),

$$I = I_1 \oplus I_2$$
$$= \sqrt{(I_2 \cos 51° + I_1 \cos 56)^2 + (I_1 \sin 56° - I_2 \sin 51°)^2}$$
$$= \sqrt{(5.41 + 8.56)^2 + (12.7 - 6.68)^2}$$
$$= \sqrt{(13.97)^2 + (6.0)^2}$$
$$= \sqrt{195 + 36}$$
$$= \sqrt{231}$$
$$= 15.2 \text{ amperes.}$$

$$\tan \theta = \frac{6.0}{13.97} = 0.429,$$
$$\theta = 23.2°.$$

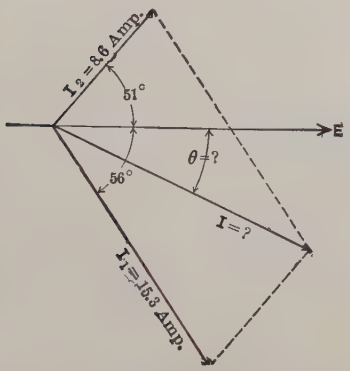


FIG. 358. Graphical method of finding the current in circuit of Fig. 355.

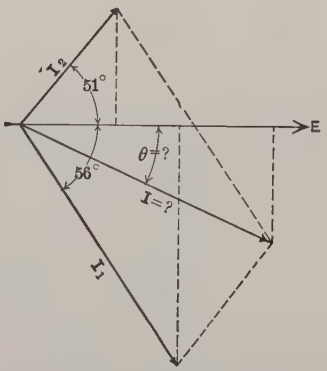


FIG. 359. Vector diagram showing components used in computing current in the combination of Fig. 355.

The current through the parallel combination thus equals 15.2 amperes and lags 23.2° behind the voltage.

The impedance of the parallel combination AB can now be found by the equation:

$$Z = \frac{E}{I},$$
$$= \frac{110}{15.2} = 7.24 \text{ ohms.}$$

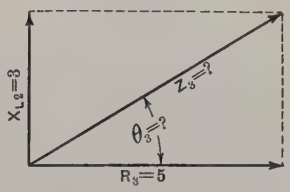


FIG. 360. Graphical method for finding the impedance of R_5 and X_{L_2} in series (Fig. 355).

The impedance Z of the series combination BC can be found from Fig. 360.

By computation:

$$Z_3 = \sqrt{R_3^2 + X_{L2}^2}$$
$$= \sqrt{25 + 9}$$
$$= 5.83 \text{ ohms.}$$
$$\tan \theta = \tfrac{3}{5} = 0.6,$$
$$\theta = 31°.$$

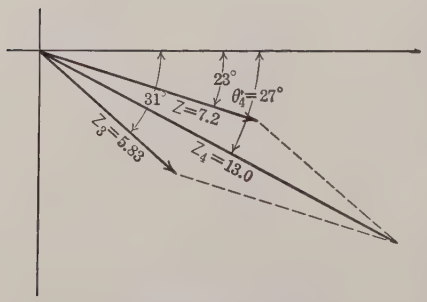


FIG. 361. Graphical method of finding the impedance of the complete circuit of Fig. 355.

The impedance (Z_3) of the series circuit BC is 5.83 ohms, and its effect is to cause the current through it to lag 31° behind the voltage.

But in series with above impedance Z_3, is the impedance Z of the parallel circuit AB, which causes the current to lag 23° behind the voltage across BC. The combined impedance of these two (Z and Z_3) can be found from Fig. 361.

By computation:

$$Z_4 = \sqrt{(Z \sin 23° + Z_3 \sin 31°)^2 + (Z \cos 23° + Z_3 \cos 31°)^2}$$
$$= \sqrt{(2.83 + 3)^2 + (6.66 + 5)^2}$$
$$= \sqrt{5.83^2 + 11.66^2}$$
$$= \sqrt{170}$$
$$= 13.0 \text{ ohms.}$$
$$\tan \theta_4 = \frac{5.83}{11.66} = 0.50.$$

From table, $\theta_4 = 26.6°$.

Thus the impedance of the series-parallel combination is 13.0 ohms, and the phase difference between current and voltage is 26.6° lag.

256. Summary of Voltage, Current and Impedance Relations in Series and in Parallel Combinations.

(1) Series.

Voltage across combination equals vector sum of separate voltages.

Current is same throughout series combination.

Impedance equals vector sum of separate impedances.

(2) **Parallel.**

Voltage is the same across each branch of combination.

Current equals vector sum of separate currents.

Impedance equals (assumed) voltage divided by the current which (assumed) voltage forces through combination.

Prob. 88–14. What current flows through a circuit connected as in Fig. 362 under following conditions?

R_1 has a resistance of 85 ohms.
C_1 has a capacitance of 20 μf.
R_2 has a resistance of 150 ohms.
C_2 has a capacitance of 50 μf.
Frequency = 60 cycles per sec.

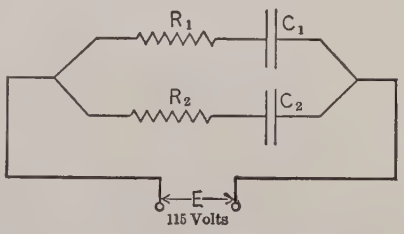


FIG. 362. A series-parallel combination of resistances and condensers.

Prob. 89–14. (*a*) What is difference in phase between E and I of circuit in Prob. 88–14? (*b*) What is the impedance of the circuit?

Prob. 90–14. What is the 25-cycle impedance of a parallel circuit of two branches, each branch of which consists of an inductance coil? The inductance coil in branch A has a resistance of 25 ohms and an inductance of 0.24 henry. The coil in branch B has a resistance of 25 ohms and an inductance of 0.48 henry.

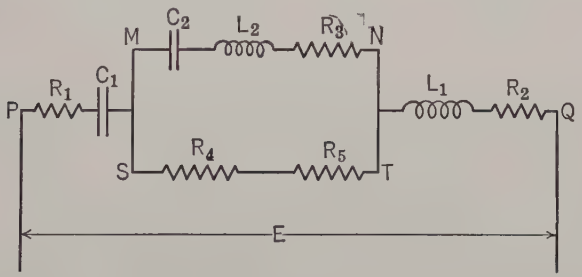


FIG. 363. A series-parallel arrangement of resistances, condensers, and inductance coils.

Prob. 91–14. What current will flow through the parallel combination of Prob. 90–14, if it is put on a 110-volt 60-cycle line?

Prob. 92–14. In Fig. 363,

R_1 = 15 ohms.	R_4 = 50 ohms.	L_2 = 0.06 henry.
R_2 = 5 ohms.	R_5 = 30 ohms.	C_1 = 50 μf.
R_3 = 5 ohms.	L_1 = 0.7 henry	C_2 = 50 μf.

E = 240 volts, 60 cycles.

Find:

(*a*) Current through MN.
(*b*) Current through ST.
(*c*) Current through PQ.
(*d*) Phase relation between line current and line voltage.
(*e*) Impedance of the combination.

257. Resonance in Parallel Circuits. In a **series circuit** it has been stated that resonance exists when the inductive reactance equals the capacitive reactance. Another way of stating it is that resonance exists in a series circuit when the **lagging** component of the **voltage** across the combination equals the **leading** component.

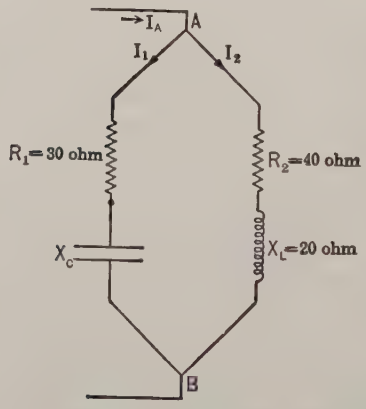


FIG. 364. A series-parallel arrangement which is adjusted for parallel resonance.

In a **parallel** circuit resonance exists when the **lagging** component of **current** through the combination equals the **leading** component.

When a **series** circuit of given resistance is resonant the impedance is a **minimum** and the current for a given voltage is a **maximum.**

In a **parallel** circuit containing inductive reactance in one branch and capacitive reactance in the other branch, resonance exists when the impedance of the combination is a **maximum** and the current through the combination is a **minimum.** This occurs when the lagging current taken by the branch containing inductive reactance equals the leading current taken by the branch containing capacitive reactance.

Example 21. In Fig. 364, R_1 is 30 ohms, R_2, 40 ohms and X_L has a value of 20 ohms at a certain frequency. What value must X_C have at this frequency, in order to produce resonance in the parallel circuit?

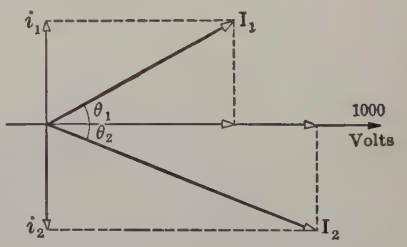


FIG. 365. For resonance, i_1, the reactive component of I_1, must equal i_2, the reactive component of I_2.

Solution.

Assume an alternating voltage of 1000 volts across AB and determine the value of X_C which will make i_1 (Fig. 365), the leading com-

ponent of current in this capacitive branch, equal to i_2, the lagging component of current in the inductive branch.

In Fig. 364 and 365, I_2 represents the current through the inductive branch lagging $\theta_2°$ behind the voltage, and its value can be found as follows:

$$Z_2 = \text{(impedance of inductive branch)} = \sqrt{40^2 + 20^2} = 44.7 \text{ ohms.}$$

$$I_2 = \frac{1000}{44.7} = 22.4 \text{ amperes.}$$

I_2 lags behind the voltage by an angle θ_2, the sine of which is $\frac{20}{44.7} = 0.447$.

From Table IV this angle $\theta_2 = 26.6°$.

The lagging reactive component of I_2,

$$i_2 = I_2 \sin \theta_2$$
$$i_2 = 22.4 \sin 26.6°$$
$$= 10 \text{ amperes.}$$

The leading reactive component of I_1 must also equal 10 amperes to produce resonance.

$$Z_1 \text{ (impedance of capacitive branch)} = \sqrt{30^2 + X_C{}^2}$$

$$I_1 \text{ (current through capacitive branch)} = \frac{1000}{\sqrt{30^2 + X_C{}^2}}$$

I_1 leads the voltage by an angle θ_1 the sine of which is

$$\frac{X_C}{\sqrt{30^2 + X_C{}^2}}$$

$$i_1 = I_1 \sin \theta_1$$
$$= \frac{1000}{\sqrt{30^2 + X_C{}^2}} \times \frac{X_C}{\sqrt{30^2 + X_C{}^2}}$$
$$= \frac{1000\, X_C}{900 + X_C{}^2}$$

But to produce resonance

$$i_1 = i_2 = 10 \text{ amperes.}$$

Therefore

$$\frac{1000\, X_C}{900 + X_C{}^2} = 10.$$
$$100\, X_C = 900 + X_C{}^2$$
$$X_C{}^2 - 100\, X_C = -900.$$
$$X_C{}^2 - 100\, X_C + 2500 = 1600$$
$$X_C - 50 = \pm 40$$
$$X_C = 10 \text{ ohms or } 90 \text{ ohms.}$$

The circuit will be resonant if X_C is made equal to either 10 ohms or to 90 ohms, and there will be no leading or lagging component of current through the combination.

Example 22. If the 30-ohm resistance is left out of the capacitive branch of Fig. 364, what value must X_C have if the current through the combination is to be a minimum when 1000 volts are applied?

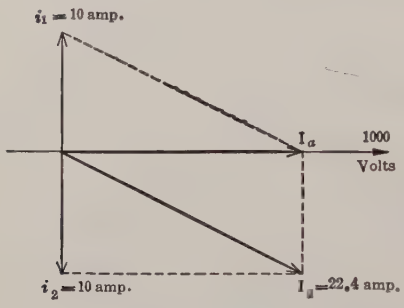


FIG. 366. When i_1 equals i_2, I_a is a minimum.

Solution.

For the current through the combination to be a minimum the circuit must be made resonant.

Draw Fig. 366, making the values of the vectors as found in the previous example.

$$I_2 = 22.4 \text{ amperes.}$$

$$i_2 \text{ (reactive component of } I_2) = 10 \text{ amperes.}$$

For resonance

$$i_1 = 10 \text{ amperes.}$$

Since there is no resistance in the path, I_1 will have no active component. Therefore $I_1 = i_1$.

$$X_C = \frac{1000}{I_1} = \frac{1000}{i_1} = \frac{1000}{10}$$
$$= 100 \text{ ohms.}$$

Note that the current through the combination equals the resultant of i_1 and I_2, or I_a. If i_1 is made any greater or any smaller, the resultant I_a will always be greater. This is made plain in Fig. 367, when i_1 is reduced one-half to i_b the resultant becomes I_b and when i_1 is doubled to i_c, the resultant becomes I_c. Both I_b and I_c are larger than I_a, which is the resultant current when $i_1 = i_2$.

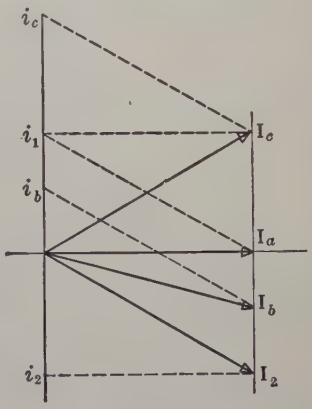


FIG. 367. When i_1 does not equal i_2, I_a is not a minimum.

Parallel resonance is a desirable condition under which to operate constant potential lines. For a given amount of power delivered the line current and generator current will be a minimum. This is demonstrated in Art. 262.

Prob. 93–14. A condenser is in series with a 20-ohm resistance, and this combination is in parallel with a coil of 0.12 henry inductance and 15 ohms resistance. What is the capacitance of the condenser if this circuit is resonant at 1500 cycles?

Prob. 94–14. If the resistance in the capacitive branch of Prob. 93–14 is removed, at what frequency will the parallel circuit be in resonance?

Prob. 95–14. What current is taken by the combination in Prob. 93–14 when connected across 100 volts?

258. Filters. Resonance in Series-Parallel Circuits. We have seen that when a series circuit is in resonance, its impedance is at a minimum value equal to the resistance in the circuit. Parallel resonance produces a maximum impedance which is very high if the resistances in the two branches are kept small. These properties of series and parallel resonant circuits are very useful in radio and wire telephony. By properly combining series and parallel resonant combinations we can design circuits which have minimum impedances at frequencies we desire to receive, and maximum impedances at frequencies which are to be excluded. These combinations of resonant circuits are called **filters.**

A simple series-parallel circuit will act as a filter if its parallel part is made resonant to the frequency to be excluded, and its series part is made resonant to the frequency to be received.

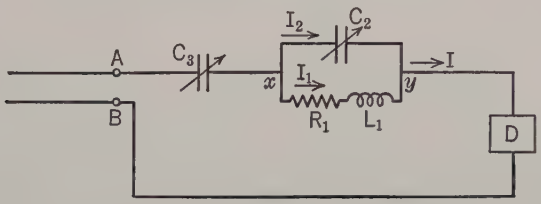


FIG. 368. A filter for suppressing a high-frequency current.

Thus in Fig. 368, a series-parallel filter circuit is connected in series with a detector D which is receiving signals from the line AB. The condenser C_2 is adjusted to produce resonance in the parallel branches at the frequency to be suppressed. This keeps the undesired waves at a minimum. Then the condenser C_3 is adjusted to produce series resonance at the desired frequency. The series circuit consists of C_3, and the parallel group R_1, L_1 and C_2. This reduces the impedance in series with the detector to a minimum at the desired frequency, and thus allows a maximum current to flow.

Example 23. In Fig. 368, the detector has a negligible resistance, L_1 is 0.002 henry, and R_1 is 100 ohms. Adjust the condensers C_2 and C_3 so that current of a frequency of 10,000 cycles per second will be a maximum in the circuit and currents of 40,000 cycles per second will be suppressed.

Solution.

In order to reduce to a minimum a current of 40,000 cycles we make the parallel circuit resonant at 40,000 cycles.

Assume 1000 volts across xy.

$$X_L = 2\pi f L_1$$
$$= 6.28 \times 40{,}000 \times 0.002$$
$$= 502.4 \text{ ohms.}$$

$$I_1 \text{ through } L_1 \text{ and } R_1 = \frac{1000}{\sqrt{100^2 + 502.4^2}}$$
$$= 1.952 \text{ amperes.}$$

I_1 lags $\theta_1°$ behind the 1000 volts (Fig. 369)

$$\tan \theta_1 = \frac{502.4}{100}$$
$$= 5.02$$
$$\theta_1 = 78.7°.$$

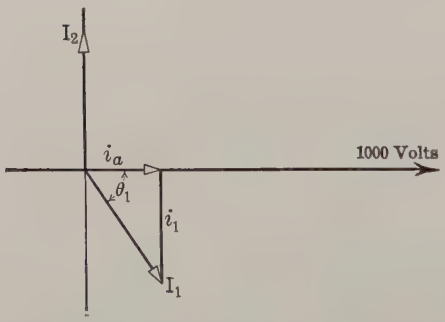


Fig. 369. Vector diagram of current and voltage relations in the parallel branches of Fig. 368.

Lagging reactive component of I_1,

$$i_1 = 1.952 \sin 78.7°$$
$$= 1.914 \text{ amperes.}$$

For parallel resonance I_2, the current through C_2 must equal the lagging component of current i_1 through L_1 and R_1 or 1.914 amperes.

Let X_{C2} equal the reactance of the condenser C_2.

$$I_2 = \frac{1000}{X_{C2}}$$

$$1.914 = \frac{1000}{X_{C2}}$$

$$X_{C2} = \frac{1000}{1.914}$$
$$= 523 \text{ ohms}$$

But $$X_{C2} = \frac{10^6}{2\pi f C_2} \text{ (in microfarads)}$$

$$C_2 = \frac{10^6}{2\pi f X_{C2}}$$

$$= \frac{10^6}{6.28 \times 40{,}000 \times 523} = 0.00761 \ \mu\text{f.}$$

In order to have the maximum current at 10,000 cycles, the series combination made up of C_3 and the parallel branches of (L_1R_1) and C_2 must be in series resonance.

At 10,000 cycles

the reactance X_L of $L_1 = 6.28 \times 10{,}000 \times 0.002$
$= 125.6$ ohms

the reactance X_{C2} of $C_2 = \frac{10^6}{6.28 \times 10{,}000 \times 0.00761}$
$= 2090$ ohms.

The impedance of the parallel branches can be found as follows:

Assume 1000 across xy.

The current I_1 through $L_1R_1 = \frac{1000}{\sqrt{100^2 + 125.6^2}}$
$= 6.23$ amperes.

I_1 lags behind the 1000 volts by an angle θ_1, where

$$\tan \theta_1 = \frac{125.6}{100}$$
$$= 1.256$$
$$\theta_1 = 51.5°$$

i_1 the reactive component of $I_1 = 6.23 \sin 51.5°$
$= 4.88$ amperes.

i_a the active component of $I_1 = 6.23 \cos 51.5°$
$= 3.88$ amperes.

The current through the condenser C_2 is all due to capacitive reactance and equals

$$I_{C2} = \frac{1000}{2090}$$
$$= 0.478 \text{ ampere.}$$

Total reactive current equals $4.88 - 0.48 = 4.40$.

Total active current $= 3.88$ amperes.

Total current $= \sqrt{3.88^2 + 4.40^2}$
$= 5.87$ amperes.

This current lags $\theta_2°$ behind the assumed voltage, where

$$\tan \theta_2 = \frac{4.40}{3.88} = 1.134$$
$$\theta_2 = 48.6°$$

The impedance of the parallel combination $= \frac{1000}{5.87}$
$= 170.4$ ohms.

The resistance of the parallel branches

$$R_p = 170.4 \cos 48.6°$$
$$= 112.7 \text{ ohms.}$$

The reactance of the parallel branches

$$X_p = 170.4 \sin 48.6°$$
$$= 127.8 \text{ ohms.}$$

In order to have series resonance between the condenser C_3 and the parallel combination, we must make the capacitive reactance of C_3 equal to the inductive reactance of the parallel combination.

Therefore

$$X_{C3} = 127.8 \text{ ohms}$$
$$= \frac{10^6}{6.28 \times 10{,}000 \times C_3}$$
$$C_3 = \frac{10^6}{6.28 \times 10{,}000 \times 127.8}$$
$$= 0.125 \ \mu\text{f.}$$

When C_3 is made equal to 0.125 μf. the capacitive reactance and the inductive reactance of the series circuit exactly balance, and the impedance of the circuit at 10,000 cycles is equal to the resistance, 112.7 ohms. This is the lowest impedance it can have at this frequency and therefore it will admit currents of this frequency with the least possible hindrance.

In order to gain an idea of the extent to which the currents of 40,000 cycles are shut out, it is merely necessary to find how much greater is the impedance of the circuit to currents of a frequency of 40,000 cycles as is done in Prob. 96–14.

Prob. 96–14. (*a*) What is the impedance of the complete circuit of the above example to currents of a frequency of 40,000 cycles per second?

(*b*) How many times greater is this than the impedance to waves of 10,000 cycles?

Prob. 97–14. (*a*) Adjust C_2 and C_3 Fig. 368 so that waves of 40,000 cycles per second are transmitted and those of 80,000 cycles per second are suppressed.

(*b*) State the ratio of the impedance at 80,000 cycles to the impedance at 40,000 cycles.

Prob. 98–14. Replace C_3 in Fig. 368 with an adjustable inductance coil L_3 of 80-ohms resistance and sufficient inductance to suppress waves of 10,000 cycles and let through those of 40,000 cycles. When C_2 is properly adjusted, what is the inductance L_3 and the capacitance C_2 under these conditions, L_1 and R_1 remaining the same as in Example 23?

259. Power in Alternating-Current Circuits. The power taken **at any instant** by an alternating-current circuit equals the product of the voltage at that instant times the current at that instant. In the form of an equation it may be written:

$$p = ei;$$

where p = instantaneous power in **watts**;
e = instantaneous voltage in **volts**;
i = instantaneous current in **amperes.**

The power in a single-phase circuit will vary from instant to instant.

The power curve can then be drawn, and each point on it will be the product of the value of the current times the value of the voltage at that instant.

The consideration of average power consumed in an alternating current circuit is divided into two parts:

(1) When the current and voltage are **in phase.**

(2) When the current and voltage are **out of phase.**

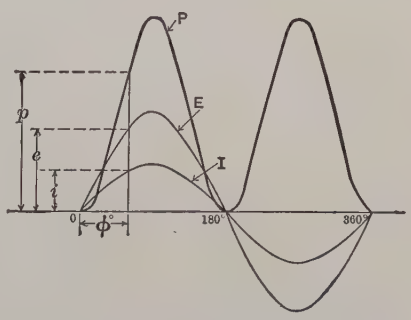


FIG. 370. Each point on the power curve P is the product of the corresponding values on the curves E and I at that instant.

Figure 370 represents a power curve, when the current and voltage are **in phase.**

Any instantaneous value such as p, on the power wave P, is the product of the corresponding value of the voltage e, on the voltage wave, E, times the current i, on the current wave, I.

Note that all the power waves are positive (above the axis), even when the current and voltage are negative. This is because the signs of the current and voltage values change at the same instant, and thus the product is always a product of **two positive**

values or of **two negative** values; either of which products is always positive.

Therefore, when the current and voltage are in phase the power is all positive. That is, no power is flowing backward and being returned to the generator. The average power in such a circuit, is the product of the effective voltage times the effective current. The equation therefore for a non-inductive circuit, or for a circuit containing resistance only, is

$$P = EI.$$

One way of determining this case is to note that since there is no reactance in the circuit all the power must be expended in overcoming what is equivalent to resistance. The power used to overcome resistance at any instant must be i^2R, or;

$$p = i^2R.$$

The average of this power would be av. i^2R or,

$$P = \text{av. } i^2R. \tag{1}$$

But by definition

$$\sqrt{\text{av. } i^2} = I.$$

Therefore,

$$\text{av. } i^2 = I^2;$$

and since

$$P = \text{av. } i^2R;$$

then

$$P = I^2R. \tag{2}$$

But by Ohm's law, the voltage to force a given current through a given resistance is

$$E = IR.$$

Then, substituting this value for IR in equation (2), we have,

$$P = EI.$$

260. Power in Inductive Circuits. Power Factor. Figure 371 represents the power curve, when the voltage and current are $\theta°$ **out of phase.** We have seen that such a case exists, when the circuit contains reactance. The same equation for instantaneous power holds true, — the instantaneous power equals the product of the current value times the voltage value at the same instant, or

$$p = ei.$$

Any point on the power curve, such as p, is the product of the instantaneous values e and i on the voltage and current curves.

But note that the power loops B and D are negative; that is, they are products of negative values of voltage times positive values of current, and vice versa. The power then is not always flowing one way. Part of the time, represented by the small negative loops B and D, it is being returned to the generator. The average power used by such a circuit is then no longer the product of the effective value of voltage times the effective value of the current.

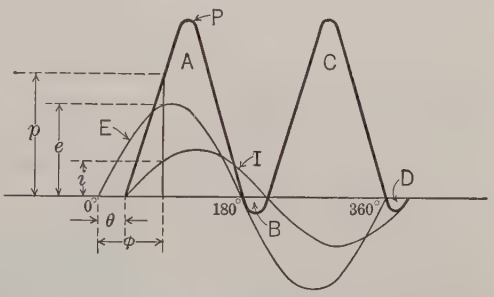


Fig. 371. The power curve P for an inductive circuit. The current I lags the voltage E by $\theta°$.

In Art. 259, Fig. 370, we saw that the power wave for a circuit containing resistance only was a positive wave; there was no backward flow of power. The negative loops in Fig. 371, which indicate that power flows back to the generator part of the time, must therefore be caused by the reactance added to the circuit, since no other change was made.

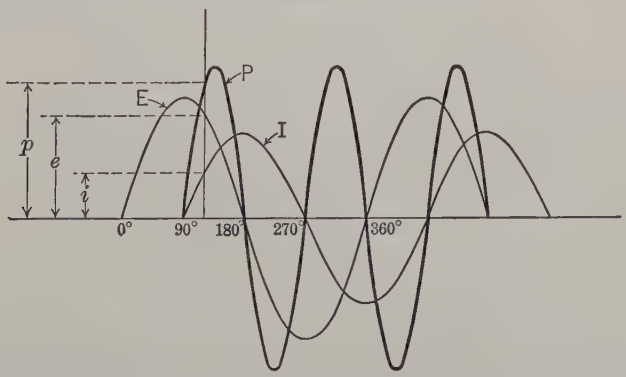


Fig. 372. In a circuit containing inductive reactance only, the positive power loops equal the negative power loops.

We find this is true by referring to Fig. 372, which shows the conditions when there is reactance alone in the circuit. In this case, the current I lags the voltage E by 90° as shown. The power curve for this circuit is also found by multiplying together the instan-

taneous values of e and i, and doing so we find that the power curve P has **equal** positive and negative loops. The reactance is therefore returning to the generator just as much energy as it receives, and **the average power supplied to a pure reactance is zero.**

This makes it much easier to find the power supplied to a circuit containing both resistance and reactance because we now know that all the power is used up in the resistance. By means of a vector diagram, Fig. 373, we can see how to determine this power from the voltage, current, and phase angle.

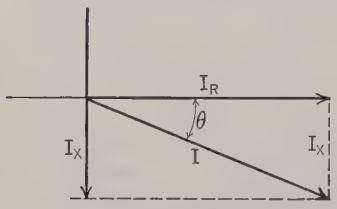


FIG. 373. The component I_R is in phase with the voltage.

Let I represent the effective current lagging $\theta°$ behind the voltage. We may consider this current I as composed of two components, I_R and I_X. The component I_R is in phase with the voltage, and the component I_X is at right angles to the voltage.

Now since I_R is the only component of current, to maintain which power is required, we may say that the average power in the circuit equals the effective voltage times the component of the effective current in phase with voltage. The equation would then be

$$P = EI_R;$$

but

$$I_R = I \cos \theta;$$

therefore

$$P = EI \cos \theta.$$

This is the general equation for the average power consumed in any **alternating-current** circuit.

The term " cos θ " is called the " **power factor** " since it is the factor by which the **apparent power** (amperes times volts) must be multiplied in order to obtain the true power.

When the current is in phase with the voltage, $\theta = 0$ and cos $\theta = 1$. The power factor is then said to be one, or **unity.**

The load which is most likely to produce a lagging current is that which is composed of a number of induction motors.

261. Reactive Component of Current. Since there is no power consumed in forcing the reactive component of current, I_X (Fig. 373), through the circuit, this component is sometimes called the **wattless component** of the current.

The reason why it requires no power to force the reactive or wattless component through the circuit can be understood, if we consider that the power taken by this current is used either to charge a condenser as in the case of a leading current, when of course this charge all flows back into the generator, or to set up a counter electromotive force in a coil, which of course helps send the current back toward the generator, as the e.m.f. reverses.

262. Power Factor Correction by Synchronous Motors. We have seen that for a given amount of power delivered, there will be a minimum line current on a constant potential line when there is parallel resonance, that is, when the leading components of current equal the lagging components. This condition is often approximated by the use of **synchronous motors.**

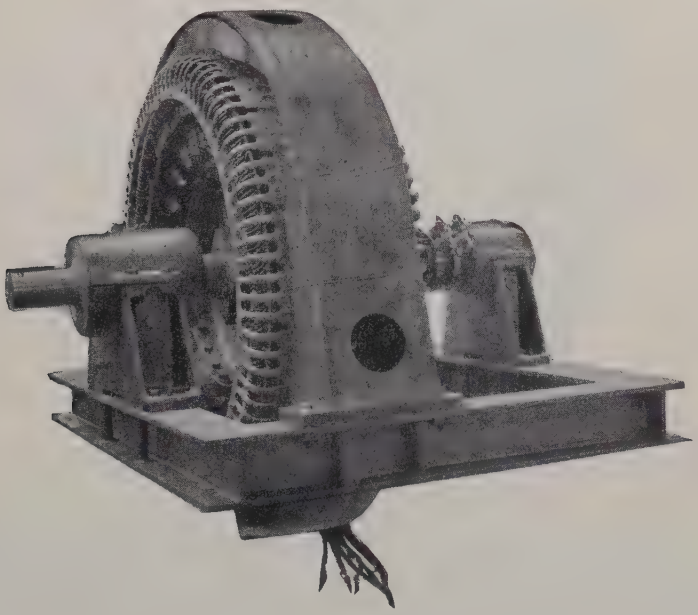


Fig. 374. A synchronous motor. *General Electric Co.*

If an alternating voltage is impressed on the stationary armature winding of an alternating-current generator, and the rotating field is excited by direct current, the generator will operate as a **synchronous motor,** when once started and brought up to synchronous speed. Early models of this type of motor have practically no starting torque as such, and must be started by some auxiliary

device, which may consist of an induction motor or a direct-current motor on the same shaft, or a winding consisting of short-circuited copper bars inserted in the pole faces. In modern designs, the construction of these pole-face, or "damper" windings has been greatly improved, and this improvement, together with the addition of special starting windings on the stationary armature, has resulted in the production of synchronous motors which start without the use of auxiliary motors. Figure 374 shows a modern synchronous motor, and Fig. 375 shows a rotating field with "damper" windings in the pole faces.

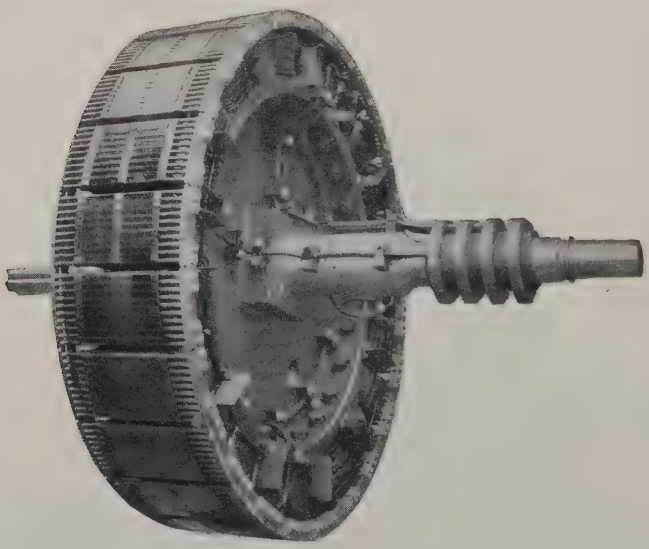


FIG. 375. Rotating field of a synchronous motor showing damper winding in pole faces. *General Electric Co.*

A synchronous motor has the decided advantage of running at absolutely constant speed at all loads which it can carry, providing the frequency of the voltage applied to the armature is constant. If too great a load is applied to it, it will pull out of step with the alternations of the applied current, stop and burn up unless disconnected from the line. Special control apparatus has been developed for use with synchronous motors to protect the windings in case the motor pulls out of step.

Increasing or decreasing the field strength does not change the speed as in the shunt motor, but merely changes the phase angle between the applied voltage and the current. There is one value of

field current at which the current and voltage are exactly in phase and the power factor is unity. This is called the **normal** field current. At all values of field current below this, the power factor is less than unity and **lagging**, and the motor is said to be **under-excited.** At all values of field current greater than normal, the power factor is less than unity but **leading.** The motor is then said to be **over-excited.**

Induction motors, we have seen, generally take a lagging current from the line. By using over-excited synchronous motors in parallel with the induction motors on the same line, we can adjust the field current until the leading component of current taken by the synchronous motor partly compensates for the lagging components taken by the induction motors, and improve the power factor and the regulation of the line. A synchronous motor thus used is often called a **synchronous condenser.**

Of course the motor may be used at the same time to drive machines, provided the current in the armature windings is not allowed to rise above its rated value.

Example 24. On a certain 2200-volt line there are induction motors which take a total current of 175 amperes at 80 per cent power factor. What current will a synchronous motor have to take in order to reduce the line current to a minimum?

Solution.

The induction motors have a lagging power factor of 0.8, so

$$\cos \theta = 0.8$$
$$\theta = 36.9°.$$

This result is used to draw the vector diagram of Fig. 376. The lagging component of induction motor current is a, and

$$a = I_m \sin 36.9°$$
$$= 175 \times 0.6$$
$$= 105.0 \text{ amperes.}$$

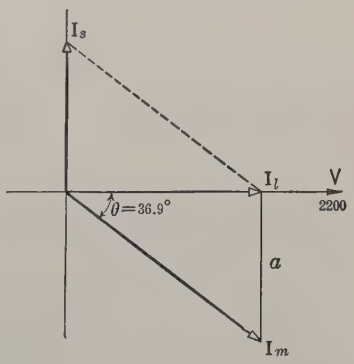


FIG. 376. The vector I_l represents the line current due to a synchronous motor drawing a leading current I_s, and induction motors drawing a lagging current I_m.

The synchronous motor must take a leading current of 105 amperes, represented as I_s.

The resulting line current then equals I_l which has a value of 175 cos 36.9°.

$$0.80 \times 175 = 140.0 \text{ amperes.}$$

Thus by adding an over-excited synchronous motor in parallel with the induction motor we have decreased the line current from 175 amperes to 140 amperes.

Example 25. If it was desired to allow the line current of Example 24 to remain at 175 amperes, but to transmit more power at the same current, how much power could be transmitted to the synchronous motor?

Solution.

The synchronous motor could carry as active current the saving in line current of Example 24 and not increase the line current beyond 175 amperes. The saving equalled 175 − 140 = 35 amperes.

The extra power thus transmitted to the synchronous motor = 2200 × 35 = 77,000 watts, and the line current remains 175 amperes as before. This can be seen from Fig. 377.

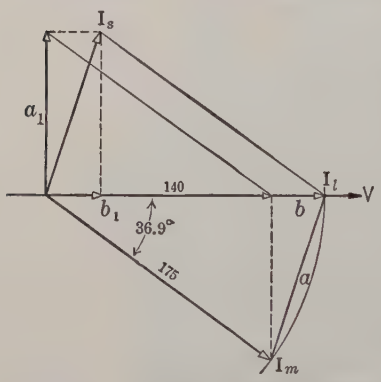


FIG. 377. The line current I_l equals the induction-motor current I_m.

a_1 = 105 amperes leading current taken by the synchronous motor.

b_1 = 35 amperes in-phase current of the synchronous motor.

I_s = the total synchronous motor current.

Note that the resultant current of I_m and I_s is I_l, which is equal to I_m but in phase with the voltage V. Thus no more current is carried by the line, yet 77,000 watts more power is transmitted.

Prob. 99–14. At what power factor will the synchronous motor of Example 25 be operating?

Prob. 100–14. If the synchronous motor of Example 25 is operating at 92 per cent efficiency, what horsepower is it delivering?

Prob. 101–14. A certain plant equipped with induction motors uses 2500 kv-a at 80 per cent power factor. What kv-a synchronous condenser would be required to raise the power factor of the plant to unity? The abbreviation kv-a stands for kilovolt-amperes. Consider losses in condenser to be 100 kw.

Prob. 102–14. (*a*) What was the total kilowatt load in Prob. 101–14 before the condenser was installed?

(*b*) After the condenser was installed?
(*c*) What was the total kv-a load before the condenser was installed?
(*d*) After the condenser was installed?
(*e*) At what power factor was the condenser operating?

Prob. 103–14. If it had been desired to raise the power factor in Prob. 101–14 to 90 per cent only, what kv-a synchronous condenser would have been required? Losses in condenser, 90 kw.

263. Use of Ammeter and Voltmeter to Measure Power. Wattmeter. Since a-c voltmeters and ammeters indicate effective values, when a voltmeter and ammeter are used to measure power, the product of the volts times the amperes, or volt-amperes, does not always give the true power. If the current " leads " or " lags," the angle of lead (or lag) must be known, so that the power factor ($\cos \theta$), may be used to reduce the volt-amperes to watts.

A wattmeter, however, is so constructed that it indicates the product of the effective volts by the effective amperes times the power factor, or the true power in an a-c circuit. Its readings, therefore, do not need to be multiplied by the power factor ($\cos \theta$).

Example 26. There are 5 amperes flowing in an a-c circuit under a pressure of 110 volts. If the current is in phase with the voltage, what power is consumed by the circuit?

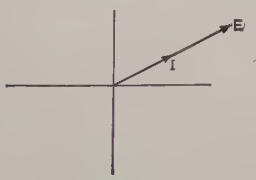


FIG. 378. Vector diagram in which the voltage and current are in phase.

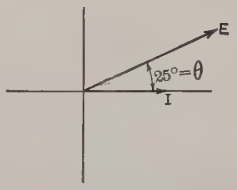


FIG. 379. The voltage leads the current by 25°.

Solution.

Figure 378 shows vector diagram for this case

$$P = EI \cos \theta,$$

and $$\cos \theta = 1.$$

Therefore,
$$P = EI$$
$$= 5 \times 110$$
$$= 550 \text{ watts.}$$

If the voltage leads the current by 25°, what power is being consumed?

Figure 379 shows vector diagram for the case where voltage leads current by 25°.

$$P = EI \cos \theta;$$
$$= 5 \times 110 \times \cos 25°$$
$$= 550 \times 0.906$$
$$= 498 \text{ watts.}$$

Example 27. A 110-volt a-c circuit contains 40 ohms resistance. What is the average power in the circuit?

Solution.

$$I = \frac{E}{R};$$

$$\frac{110}{40} = 2.75 \text{ amperes.}$$

$$P = EI$$
$$= 2.75 \times 110$$
$$= 302 \text{ watts.}$$

Example 28. If the circuit contained 10 ohms resistance and 30 ohms reactance, what would be the average power in the circuit?

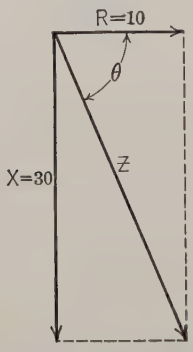


FIG. 380. The resistance and reactance determine the impedance Z and the size of the angle θ.

Solution.

$$Z = \sqrt{R^2 + X^2} = \sqrt{10^2 + 30^2}$$
$$= \sqrt{1000} = 31.6 \text{ ohms.}$$
$$I = \frac{E}{Z} = \frac{110}{31.6} = 3.48 \text{ amperes.}$$

Referring to Fig. 380,

$$\tan \theta = \frac{30}{10} = 3$$
$$\theta = 71.6°$$
$$\cos \theta = 0.316$$
$$P = EI \cos \theta = 110 \times 3.48 \times 0.316$$
$$= 121 \text{ watts.}$$

Prob. 104–14. What is the power factor of a circuit containing 25 ohms resistance and 65 ohms reactance?

Prob. 105–14. What power is taken by the circuit of Prob. 104–14 when connected to a 220-volt a-c line?

Prob. 106–14. A 120-volt 60-cycle circuit contains 40 ohms resistance. (*a*) What current flows? (*b*) What power is consumed?

Prob. 107–14. How much would current in Prob. 106–14 be reduced to, if a coil of negligible resistance and 0.26 henry inductance were placed in series with the 40-ohm resistance?

Prob. 108–14. What power would be consumed by circuit in Prob. 107–14?

Prob. 109–14. A coil of 0.62 henry inductance and 12 ohms resistance is placed on a 25-cycle 110-volt circuit. (*a*) What is power factor? (*b*) What power is consumed by coil?

Prob. 110–14. An ammeter inserted in a 110-volt a-c circuit indicates 15 amperes. Wattmeter reads 1.4 kw. What is power factor and phase difference between current and voltage?

Prob. 111–14. An induction motor supplying a 25-horsepower load is operating at 88% efficiency. It receives power from a 550-volt line at 0.8 power factor.

(*a*) What power is supplied to the motor?
(*b*) What is the line current to the motor?

Prob. 112–14. A synchronous condenser drawing 20 amperes at 0.1 leading power factor is connected in parallel with the induction motor of Prob. 111–14.

(*a*) What line current is taken by the combination?
(*b*) What is the power factor of the combination?

264. Voltage Regulation on Short Alternating-Current Lines.

(1) When power factor of load is unity.
(2) When power factor of load is less than unity.

Case (1) The generator in Fig. 381 is to deliver 20 amperes to the lamp bank L at 110 volts. The problem is to determine the generator voltage necessary, and the regulation of the line.

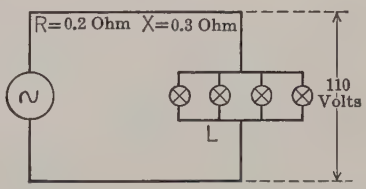


Fig. 381. A generator delivering alternating current to a bank of lamps over a short line.

If the generator were delivering direct current the generator voltage would be computed as follows:

$$\text{Line resistance} = 2 \times 0.20 = 0.40 \text{ ohm}$$

$$\text{Line drop} = 20 \times 0.40 = 8 \text{ volts}$$

$$\text{Generator voltage} = 8 + 110 = 118 \text{ volts}$$

$$\text{Regulation} = \frac{118 - 110}{110} = 7.27 \text{ per cent.}$$

But where a line carries an alternating current, the line wires always possess some inductive reactance as well as resistance. There must therefore be a reactance drop as well as a resistance drop in the line.

$$\text{Line reactance} = 2 \times 0.30 = 0.60 \text{ ohm}$$

$$\text{Reactance drop} = 0.60 \times 20 = 12 \text{ volts.}$$

Construct a vector diagram as in Fig. 382.

V_L = volts at lamps = 110 volts.

V_R = 8-volt resistance drop in line.

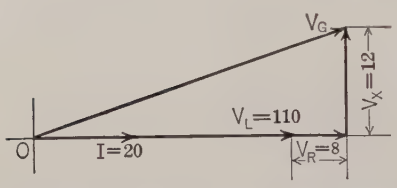


Fig. 382. Vector diagram for finding V_G, the generator voltage. Load at unity power factor.

Note that this is in phase with V_L because the current is in phase with V_L at unity power factor.

V_X = 12-volt reactance drop in line.

This is 90° ahead of V_L and the line current.

V_G = generator voltage.

From the diagram in Fig. 382,

$$V_G = \sqrt{(110 + 8)^2 + 12^2} = 118.6 \text{ volts.}$$

$$\text{Regulation} = \frac{118.6 - 110}{110} = 7.82 \text{ per cent.}$$

Note that for a load at unity power factor, the generator voltage and regulation is little greater than for a direct-current system with the same line current.

Case (2) Power factor of load less than unity.

Replace the lamp bank of Fig. 381 with an induction motor taking the same current (20 amp.) but at an 80 per cent lagging power factor.

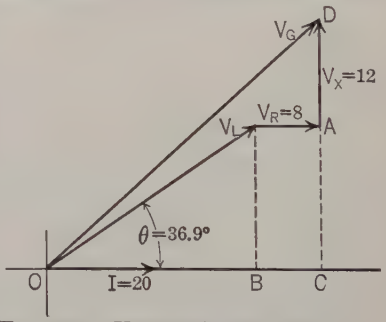


Fig. 383. Vector diagram for load at 0.8 power factor.

A lagging power factor of 80 per cent means the current lags behind the voltage by an angle whose cosine is 0.80, or 36.9°.

Construct the vector diagram as in Fig. 383.

V_L = 110 volts across the motor.

I = current through motor lagging 36.9° behind V_L.

V_R = 8-volt resistance drop in phase with current I and therefore lagging 36.9° behind V_L.

V_X = 12-volt reactance drop, leading the resistance drop 90° as inductive reactance drop always does.

$V_G = V_L \oplus V_R \oplus V_X$ = generator voltage

The value of V_G can be found as follows:

$$V_G = \sqrt{OC^2 + CD^2}.$$

$$OC = OB + BC,$$
$$= 110 \cos 36.9° + 8,$$
$$= 88 + 8 = 96.$$

$$CD = CA + AD,$$
$$= 110 \sin 36.9° + 12,$$
$$= 66 + 12 = 78.$$

$$V_G = \sqrt{96^2 + 78^2},$$
$$= 123.7 \text{ volts.}$$

$$\text{Regulation} = \frac{123.7 - 110}{110} = 12.5 \text{ per cent.}$$

Note that with 80 per cent power factor for the same current, the generator voltage and the regulation are much greater than at unity power factor.

Prob. 113–14. Find the regulation in the above example using a synchronous motor which has the same voltage but takes a leading current of 20 amperes at 0.80 power factor.

Prob. 114–14. What generator voltage would be required to supply the same amount of power at 0.7 power factor (lagging current) to the motor in the above example?

Prob. 115–14. The motor in Fig. 384 takes a current of 15 amperes lagging the voltage across it by 85 per cent power factor. The lamps take 5 amperes each at 115 volts at unity power factor.

(*a*) What is the voltage of the generator?

(*b*) What is the power factor of the load on the generator?

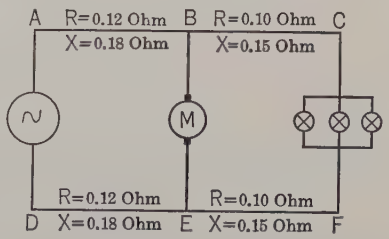


FIG. 384. The a-c generator supplies power to a motor and a lamp bank.

Prob. 116–14. What will be the generator voltage of Prob. 115–14 if the lamps and the motor are interchanged? Assume the motor voltage to be 115 volts and the current through the motor and lamps to be as in Prob. 115–14.

Prob. 117–14. Solve Prob. 115–14, with the motor taking a leading instead of a lagging current.

Prob. 118–14. What will be the 60-cycle reactance per mile of a two-wire transmission line using No. 00 B. & S. copper wire strung 3 feet apart. Use formula on page 332 for inductance.

265. Regulation of Long Transmission Lines. The formulas for the capacitance of both aërial lines and underground cables were given in the last chapter. For short lines the effect of the capacitance is usually negligible and was so considered in the last paragraph, but it must be taken into account in long lines.

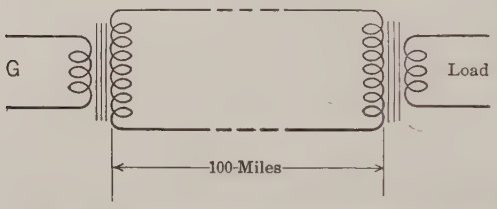


FIG. 385. Conventional representation of a long transmission line with a transformer at each end.

Consider Fig. 385, which represents a 100-mile line, consisting of two No. 00 copper wires spaced 96 inches apart. The capacitance of this line according to the formula on page 353 would be

$$C = \frac{19.4\,l}{10^3 \times \log_{10}\frac{S}{r}}$$

$$= \frac{19.4 \times 100}{10^3 \times \log_{10}\frac{96}{0.182}} = 0.713\ \mu\text{f.}$$

The inductance according to the formula on page 332 will be

$$L = l\left(1.482 \log_{10}\frac{S}{r} + 0.161\right) \text{millihenries.}$$

$$= 100\left(1.482 \log_{10}\frac{96}{0.182} + 0.161\right)$$

$$= 0.420 \text{ henry.}$$

At 60 cycles per second the inductive reactance

$$(2\,\pi f L) = 6.28 \times 60 \times 0.42$$
$$= 158 \text{ ohms.}$$

The resistance of the line is

$$R = 200 \times 0.412$$
$$= 82.4 \text{ ohms.}$$

The capacitance of the two wires is due to the fact that they act like two plates. Only that part of the charge which went out to the far end of the line would flow through the whole wire. That part which charged the near end would not flow an appreciable distance along the wire. The charging current flowing into different parts

of a line would therefore be different for the several parts, being greatest at the generator end, and tapering off to zero at the far end. To compensate for this, it is customary instead of considering the line to be made up of one condenser, to consider it to be made up of one condenser placed at the middle of the line or of two condensers, each having half the capacitance of the line, situated one at each end of the line. Thus the line of Fig. 385 can be treated as equivalent to one condenser of 0.713 μf. capacitance placed at the middle of the line, as in Fig. 386, or to two condensers of $\frac{0.713}{2}$ or 0.356 μf., placed one at each end of the line as in Fig. 387. In the latter case we assume that the charging current for condenser A travels the whole length of the line while that for condenser B travels no appreciable distance along the line. The capacitive reactance is thus the reactance of a condenser of 0.356 μf.

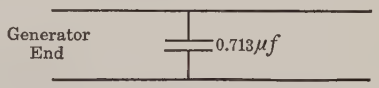


FIG. 386. Capacitance of line in Fig. 385 represented by a condenser at the mid-point.

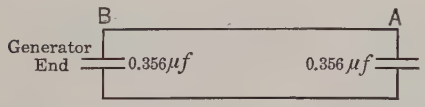


FIG. 387. Capacitance of line in Fig. 385 represented by two condensers placed one at each end of the line.

With this latter assumption let us compute the voltage at the generating end necessary to send a 60-cycle current of 50 amperes to the load end and maintain a voltage of 110,000 volts at the load, unity power factor.

$$X_C = \frac{10^6}{2\pi fC}$$
$$= \frac{10^6}{6.28 \times 60 \times 0.356}$$
$$= 7450 \text{ ohms.}$$

Whether or not any load is put on the load end, if 110,000 volts is maintained there, the line will act as though it had a condenser on the end, and $\frac{110{,}000}{7450}$ or 14.8 amperes will flow. Of course this current will lead the voltage by 90°, since it is due to capacitive reactance only.

When, however, a load current of 50 amperes at unity power factor is demanded by the load, the total line current can be represented by the vector I in Fig. 388. The current I is the vector sum of the load current I_l and the charging current I_C and is equal to

$\sqrt{50^2 + 14.8^2} = 52.1$ amperes leading the voltage by an angle the tangent of which is $\frac{14.8}{50}$ or 16.5°. This corresponds to a power factor of cos 16.5° or 0.959.

The problem thus resolves itself into the finding of the voltage necessary at the generator end to send 52.1 amperes at 0.959 leading

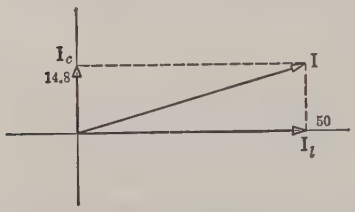


FIG. 388. Vector diagram for finding total line current for line of Fig. 385.

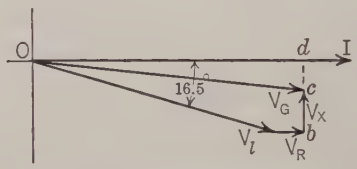


FIG. 389. Vector diagram for finding generator-end voltage of line in Fig. 385.

power factor over a line having 82.4 ohms resistance, and 158 ohms inductive reactance, and have 110,000 volts left at the load end

The line drop due to resistance equals

$$IR = 52.1 \times 82.4$$
$$= 4290 \text{ volts.}$$

This is represented on Fig. 389, by the vector V_R drawn parallel to the current vector I because the IR drop is always in phase with the current which produces it.

The line drop due to reactance is

$$IX = 52.1 \times 158 = 8230 \text{ volts.}$$

This is represented by the vector V_X drawn 90° leading the current vector because the IX_L drop always leads the current by 90°. The voltage V_G at the sending end must be 110,000 $\oplus$ IR $\oplus$ IX, the value of which can be found as follows:

$$V_G = \sqrt{od^2 + dc^2}.$$
$$od = 110{,}000 \cos 16.5° + IR$$
$$= 105{,}500 + 4290$$
$$= 109{,}800.$$
$$dc = 110{,}000 \sin 16.5° - IX$$
$$= 31{,}200 - 8230$$
$$= 22{,}970.$$
$$V_G = \sqrt{109{,}800^2 + 22{,}970^2}$$
$$= 112{,}200 \text{ volts.}$$

Thus it will take 112,200 volts at the generating end to send a load current of 50 amperes at unity power factor over the line and maintain 110,000 at the load end.

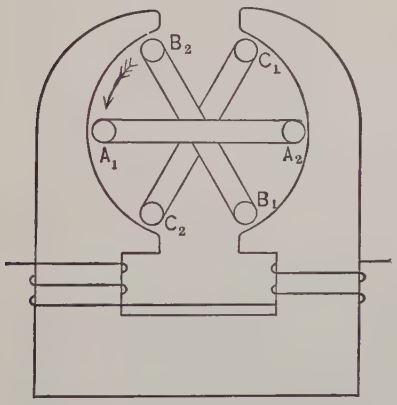


FIG. 390. Conventional representation of a three-phase generator.

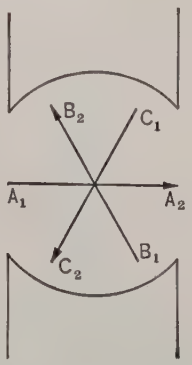


FIG. 391. Vector diagram of voltages in three-phase generator of Fig. 390.

On an unloaded long line there is often the peculiar condition that the voltage at the load end is higher than the voltage at the sending end. How this can be is seen from the vector diagram for Prob. 119–14.

Prob. 119–14. If 110,000 volts are maintained at the load end of the line in the above example, what will be the voltage at the sending end when the line is unloaded?

Prob. 120–14. Compute the voltage needed at the sending end of the line in the above example if the 50-ampere load current has a lagging power factor of 80 per cent.

Prob. 121–14. What would be the voltage at the generator end of the line in Prob. 120–14 if the 50-ampere current had a frequency of 25 cycles per second? Other data as in Prob. 120–14.

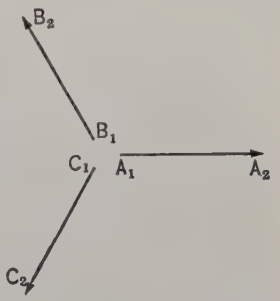


FIG. 392. Vector diagram of voltages produced by a three-phase generator.

266. Three-Phase System. If instead of using a single coil on the armature of an alternator as in Fig. 290, we use three coils equally spaced as in Fig. 390, each of these coils is called a **phase** of the generator. If we consider the voltage induced in coil or phase A from A_1 to A_2 at any instant, we shall see that the voltage induced in coil B from B_1 to B_2 is 120° ahead of the voltage in A, and

that the voltage in coil C from C_1 to C_2 is 240° ahead of the voltage in A. A vector diagram of the voltages would look like Fig. 391 or like Fig. 392. Note that the voltages have a 120° phase angle with one another. The sine waves of these voltages will appear as in Fig. 393. Note that the wave B_1B_2 starts 120° later than the wave C_1C_2, and A_1A_2 starts 120° later than B_1B_2.

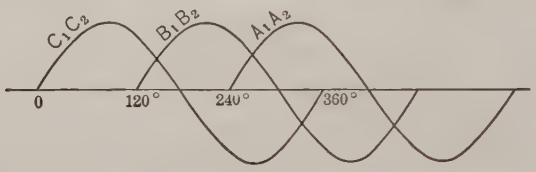


Fig. 393. Sine waves of voltage in the three-phase generator of Fig. 390.

Power might be taken from each of these three armature coils or phases separately by attaching a conductor to each end of the coil. This would mean two line wires for each coil or six wires in all. This is rarely ever done. The three phases are usually joined together

Fig. 394. A modern 220,000-volt, three-phase transmission line.
New England Power Service Co.

and only three line wires connected to them. There are two ways of joining the coils, as explained later, one called the **Delta** or mesh connection, the other the **Wye** or star connection.

The three-phase system is the one in most general use for the transmission of alternating-current power. A typical, high voltage, three-phase transmission line is shown in Fig. 394.

It is not convenient to build generators which deliver the high voltages used in long modern transmission lines, so transformers are used to step up the voltage from the value at the generator to the

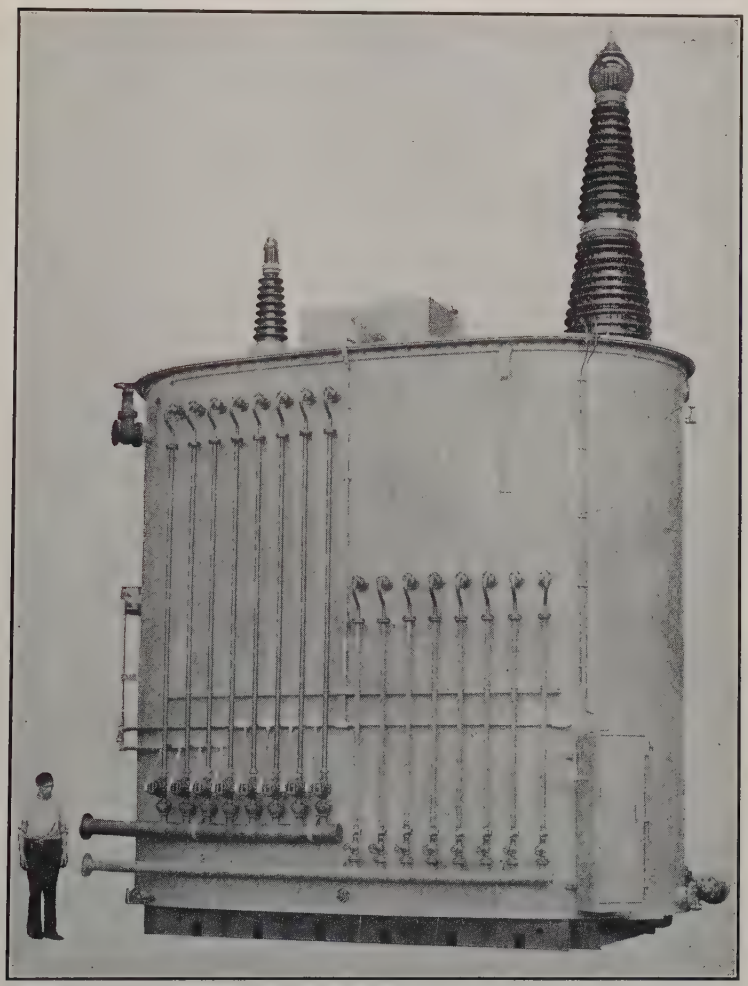


FIG. 395. A modern 55,000-kva. transformer for use with high-voltage transmission lines. *General Electric Co.*

line value. Figure 395 shows a modern 55,000-kv-a transformer used to step a generator voltage of 16,320 volts up to a line voltage of 275,000 volts. At the load end of the line, a similar transformer is used to reduce the voltage so that it can be used more readily.

267. The Delta Connection. Current and Voltage Relations. In the Delta connection the terminal A_2 (see Fig. 390) is joined to B_1; the terminal B_2 is joined to C_1; and the terminal C_2 is joined to A_1, and lead wires are brought out from the three junction points.

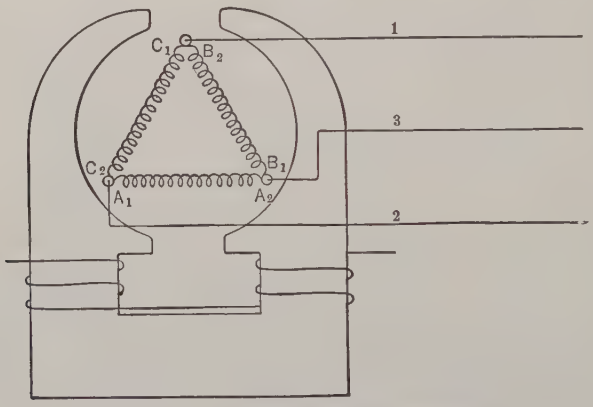


Fig. 396. A delta-connected armature.

This produces a figure like Fig. 396, which gives it the name Delta from the resemblance to the Greek letter Δ corresponding to the English letter *D*. Of course, the coils do not assume this position, but are merely drawn this way as a convenient method of showing the connections.

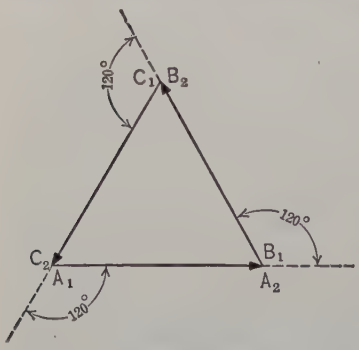


Fig. 397. Voltage diagram of the delta-connected armature of Fig. 396.

The vector diagram for this connection is shown in Fig. 397. It looks as though any two vectors were at an angle of 60° to each other, but on inspection it will be seen that B_1B_2 is 120° ahead of A_1A_2 and that C_1C_2 is 120° ahead of B_1B_2.

The vector B_1B_2 represents the voltage between the line wires 3 and 1, the vector C_1C_2 represents the voltage between the line wires 1 and 2, and the vector A_1A_2 represents the voltage between the line wires 2 and 3.

At first sight it would appear that a current must circulate in the armature when the phases are joined as in Fig. 396 and the voltages are as represented by Fig. 397. The voltages seem to be in such a

relation to one another as to send a current around the delta in a counter-clockwise direction. But note that while there is a voltage A_1A_2 in Fig. 397, inside the coil A_1A_2, tending to send a current from A_1 to A_2, there is a voltage impressed across the coil by the coils B_1B_2 and C_1C_2. Note that this impressed voltage is the sum of the vectors B_1B_2 and C_1C_2 and is exactly equal and opposite to the vector A_1A_2. Thus no voltage exists tending to send a current through A_1A_2. The same is true of coils B_1B_2 and C_1C_2. But notice that when we consider lines 3 and 2, both the coils A_1A_2 and the combination of coils C_2C_1 and B_2B_1 are putting a voltage across the lines. When the phases are balanced, which is the normal condition, the voltages across all phases are equal and at an angle of 120° to one another and the currents delivered to the line wire by the three phases are equal, and each has the same phase relation to its phase voltage as the others have.

Let us consider for simplicity that the current in each coil is in phase with the voltage across the coil, and investigate the current delivered to wire No. 3, under the condition of a balanced load.

The current in line 3 must come from coils A and B because these are the only coils attached to line 3. The current from coil A will flow from A_1 to A_2, which is in the same direction as the voltage through the coil. We can represent this in Fig. 398 by the vector $I_{A_1A_2}$ which is in the same direction as the voltage $V_{A_1A_2}$ in the coil. But for the coil B to deliver current to line 3, the current must flow in the reverse direction to the positive direction of the voltage in B, which is from B_1 to B_2. Therefore the current from coil B is represented by the vector $I_{B_2B_1}$, which is in the opposite direction to the voltage vector of the coil B_1B_2. The vector sum of these current vectors is the current in line 3 and is equal to I_3 in Fig. 398. The value of the line current I_3 equals

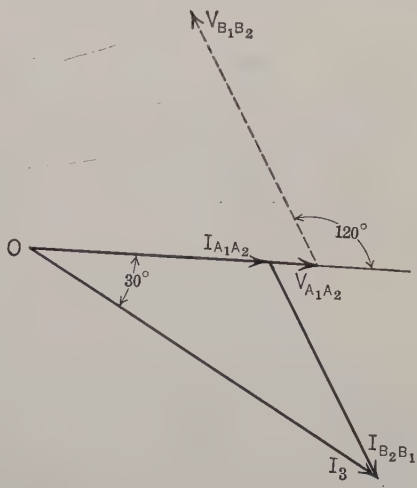


FIG. 398. The vector I_3 represents the current in the line wire 3 in Fig. 396

$$2\, I_{A_1A_2} \cos 30° = \sqrt{3}\, I_{A_1A_2}.$$

This is usually written

$$I_l = \sqrt{3}\, I_\phi \quad \text{or} \quad 1.73\, I_\phi$$

in which I_ϕ is the current in each phase, and I_l the current in each line wire.

Note that although the current in each phase is in phase with the voltage across that phase, still the line current I_3 lags 30° behind the voltage in phase A which is also the voltage between lines 2 and 3.

In a balanced three-phase, delta-connected system, the current in each line wire equals 1.73 times the current in each phase. There is always a 30° phase angle between the line currents and the phase currents supplying it.

Prob. 122–14. Draw the vector diagrams similar to Fig. 398 for the line currents in lines 1 and 2 of Fig. 396.

Prob. 123–14. What current flows in each line wire of a balanced 3-phase system if the generator is Δ-connected and carries 175 amperes per phase?

268. The Star or Wye Connection. Current and Voltage Relations. The star or Wye connection is made by connecting together three like ends (Fig. 390), either A_1, B_1 and C_1, or A_2, B_2, C_2, and attaching a line wire to free ends of each of the three coils. Assume that we join together the ends $A_1B_1C_1$. This will result in a figure like Fig. 399, and a vector diagram like Fig. 400.

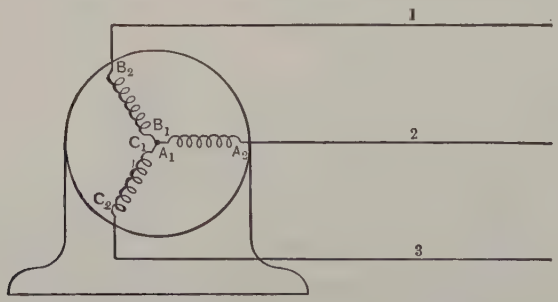


Fig. 399. A star-connected armature.

The current in line 1 would of course be the current in coil B; the currents in lines 2 and 3 would be the currents in coils A and C, respectively. So Fig. 400 can just as well represent the currents in the line wires, if the currents are in phase with the coil voltages. If the currents are out of phase with line voltages, the current vectors

would all shift through the same angle, so their relative positions would remain as in Fig. 400.

The voltage between the lines, as, for instance, between lines 1 and 2, is found as follows.

To go from line 1 to line 2, Fig. 399, we pass through coil B in the direction from B_2 to B_1 which is the reverse of the positive direction. From coil B_1 we pass through coil A in the direction from A_1 to A_2, which is the positive direction. Thus the voltage between lines 1 and 2 is made up of the reverse voltage through coil B and the positive voltage of coil A as shown in Fig. 401.

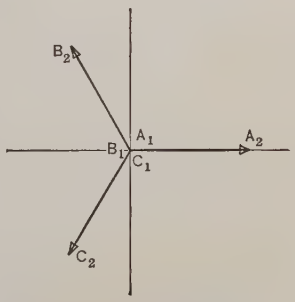


FIG. 400. Vector diagram of phase voltages of armature of Fig. 399.

Vector $E_{B_2B_1}$ drawn in the opposite direction to the vector $E_{B_1B_2}$ represents the voltage from B_2 to B_1 through coil B. The vector $E_{A_1A_2}$ (solid line) is drawn parallel to the original vector $E_{A_1A_2}$ (dotted line) and represents the voltage through coil A from A_1 to A_2. The vector sum of these is the voltage E_{12} from line 1 to line 2.

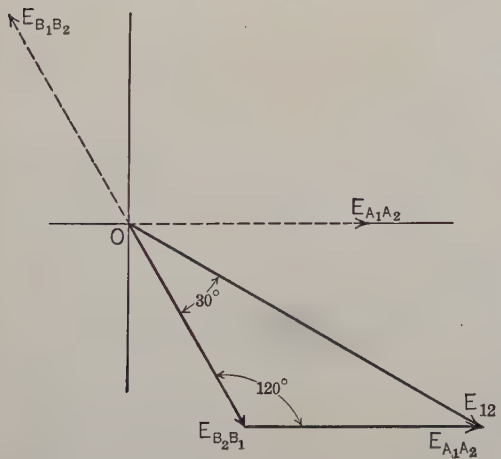


FIG. 401. Vector E_{12} represents the voltage between lines 1 and 2 of Fig. 399.

The value of E_{12} is found as follows:

$$E_{12} = 2\ E_{A_1A_2} \cos 30°$$
$$= \sqrt{3}\ E_{A_1A_2}.$$

Thus the voltage between the lines of a star-connected balanced three-phase system equals $\sqrt{3}$ or 1.73 times the phase voltage.

Prob. 124–14. Draw the vector diagram similar to Fig. 401 of the voltage from lines 2 to 3, and from line 3 to line 1.

Prob. 125–14. What is the line voltage on a balanced 3-phase wye-connected system, if the phase voltage is 1330 volts?

269. Power in a Balanced Three-Phase System. The power delivered **per phase** by a 3-phase generator, whether delta or wye-connected, equals $E_\phi I_\phi \cos\theta$, in which

E_ϕ = voltage across the phase.
I_ϕ = current through the phase.
θ = angle between I_ϕ and E_ϕ.

The total power from a balanced three-phase generator equals

$$3\, E_\phi I_\phi \cos\theta.$$

But in a delta-connected generator I (line current) equals $\sqrt{3}\, I_\phi$ or $I_\phi = \frac{I}{\sqrt{3}}$. Of course E (line voltage) equals E_ϕ.

Thus the total power for a delta-connected three-phase generator equals

$$P = \frac{3\, EI \cos\theta}{\sqrt{3}}$$
$$= \sqrt{3}\, EI \cos\theta$$
$$= 1.73\, EI \cos\theta$$

where

E = voltage between **line** wires.
I = current in **line** wires.
θ = angle between **phase** current and **phase** voltage (**not** between **line** current and **line** voltage).

For a wye-connected generator I (line current) equals I_ϕ, but E (line voltage) equals $\sqrt{3}\, E_\phi$, or $E_\phi = \frac{E}{\sqrt{3}}$. Hence we again have for the total power

$$P = \frac{3\, EI \cos\theta}{\sqrt{3}}$$
$$= \sqrt{3}\, EI \cos\theta = 1.73\, EI \cos\theta.$$

Thus, the power supplied to any balanced three-phase circuit is always given by $P = \sqrt{3}\, EI \cos\theta$, whether the generator is delta or wye-connected. It must be remembered, however, that θ is always the angle between **phase** voltage and **phase** current.

Example 29.

(*a*) Find the current carried by each line wire of the three-phase 110-volt line, Fig. 402, to which are attached in delta 3 groups of lamps each requiring 12 amperes at unity power factor and a three-

phase induction motor taking 20 amperes in each phase at 80 per cent power factor. The motor is connected in delta.

(*b*) Find total power taken by lamps and motor.

Solution.

(*a*) There are several ways of finding the line current. The current taken by the lamps from each line at unity power factor would be

$$1.73 \times 12 = 20.8 \text{ ampere}$$

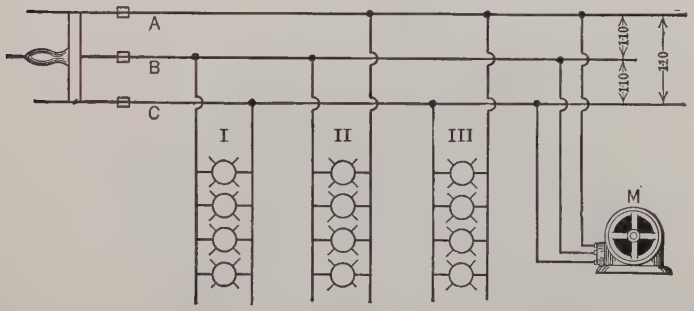


FIG. 402. A three-phase line supplying three groups of lamps and a three-phase motor.

The current taken by the motor from each line wire at 80 per cent power factor would be

$$1.73 \times 20 = 34.6 \text{ amperes.}$$

These line currents would differ in phase by an angle whose cosine is 80 per cent, or 36.9°, as represented in Fig. 403.

The in-phase components of the resultant current would then equal

$$20.8 + 34.6 \cos 36.9° = 48.4 \text{ amperes.}$$

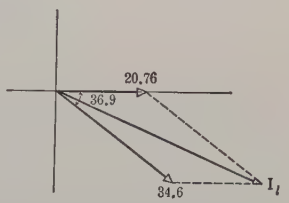


FIG. 403. Vector diagram for finding line current of Fig. 402.

The component at 90° phase would equal

$$34.6 \times \sin 36.9 = 20.8 \text{ amperes.}$$

$$\text{Resultant line current} = \sqrt{48.4^2 + 20.8^2}$$
$$= 52.7 \text{ amperes.}$$

$$\text{Power factor of load} = \frac{48.4}{52.7} = 91.8 \text{ per cent.}$$

$$\text{(b) Total Power} = 1.73 \times 110 \times 52.7 \times 0.918$$
$$= 9.22 \text{ kw.}$$

Check on the total load.

$$\begin{aligned} \text{Power to lamps} &= 3 \times 12 \times 110 \\ &= 3.96 \text{ kw.} \\ \text{Power to motor} &= 3 \times 20 \times 110 \times 0.80 \\ &= 5.28. \\ \text{Total} &= 5.28 + 3.96 \\ &= 9.24 \text{ kw.} \end{aligned}$$

Fig. 404. A three-phase induction motor. *General Electric Co.*

Prob. 126–14. Assuming that the resistance of the lamps in the above example remains constant, what line current flows if the lamp banks are connected in wye across the lines? What total power is supplied?

Prob. 127–14. What power is a generator supplying to a balanced three-phase load, when the line current is 42 amperes and the voltage is 2250 volts between line wires? The power factor of the load is 85 per cent.

Prob. 128–14. A 2300-volt three-wire three-phase transmission line has 30 single-phase distributing transformers supplying 2.5 kw each at 96 per cent power factor, and 24 single-phase transformers each supplying 6 kw at 90 per cent power factor. If the line is balanced, what current must the power station supply to each main?

Prob. 129–14. If the load of Prob. 126–14 was all on a single-phase line at the same voltage between wires, how much current would be supplied to each main?

270. Polyphase Induction Motors. Since single-phase induction motors require special starting devices, most induction motors are three-phase. Figure 404 shows a three-phase induction motor. The power is brought to the three terminals usually through a starting compensator shown in Fig. 405 which allows about half voltage to be applied until the motor reaches sufficient speed to make it safe to throw the full voltage across it. In order to reverse the direction of rotation it is necessary merely to interchange the lead wires coming to any two connections.

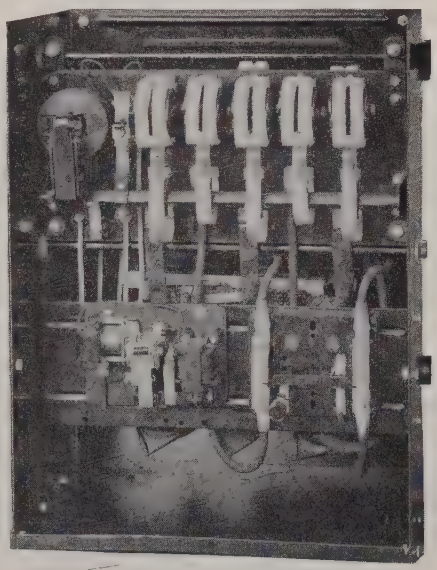


FIG. 405. An automatic starting compensator for an induction motor. *General Electric Co.*

The rotor of the motor as explained in Art. 248 is of the squirrel-cage type shown in Fig. 338. The current set up in the bars of the rotor by the revolving field flux of the stator reacts on the revolving flux and produces a torque.

The following figures indicate how this revolving flux is produced. Figure 406 shows the connections from the star-connected armature of the generator to the star-connected stator of the motor. The rotor of the motor is left out for the sake of clearness. In the practical machine there are no salient poles in an induction motor, the coils being set in slots in the frame, and only local areas of the frame are made successively North and South poles. For the sake of clearness these areas are shown in the accompanying figures as salient poles.

Figure 407 shows that when the armature currents in the three phases of the generator are as shown in Fig. 406, the flux in the motor has the position indicated. Note that there is no current in phase C of the generator at this instant, and, therefore, the field coils C and C_1 produce no flux. The current in phase B is such as to make B a North pole and B_1 a South pole. The current in phase

A is in the reverse direction to that in phase B and this makes A_1 a North pole and A a South pole. This produces a motor field as in Fig. 407 (b).

After the generator armature has moved 60° and has reached the position shown in Fig. 408, there is no current in phase B, so the

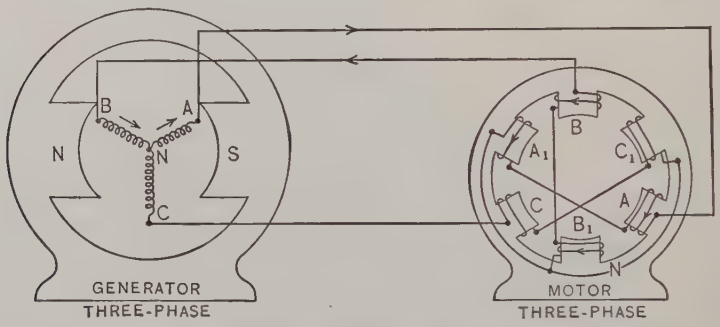


FIG. 406. A star-connected generator supplying a star-connected induction motor.

poles B and B_1 produce no flux. The current in phase A is still in the same direction as before, so A_1 is still a North pole and A a South pole. In phase C the current is now flowing in such a direction as to make pole C a North pole and C_1 a South pole. This

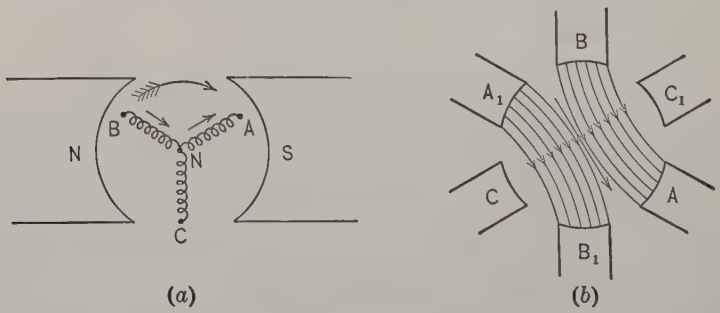


FIG. 407. The direction of the motor field at the instantaneous position shown in Fig. 406.

produces a field in the direction shown in Fig. 408 (b). Note that the flux has moved 60° in a counter-clockwise direction. In the same way in Fig. 409 it will be seen that when the currents in the armature coils have passed through another 60° of their cycles the direction of the flux in the motor has also shifted another 60° in a

counter-clockwise direction. The motor field is thus rotating in synchronism with the alternations of the currents in the armature of the generator and in the field coils of the motor. Although the motor seems to have six poles, it really has but two pole-areas at any

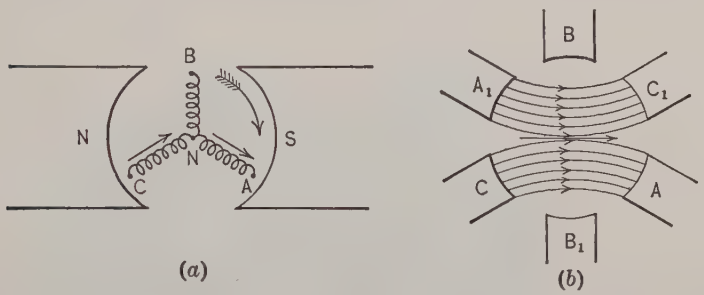


FIG. 408. The direction of the motor field after the generator armature has turned 60°.

one time and is therefore a two-pole motor. It happens in the figures that the motor field rotates in the reverse direction to the generator armature, but this is due to the way the connections happened to be made as will be seen from the following problems.

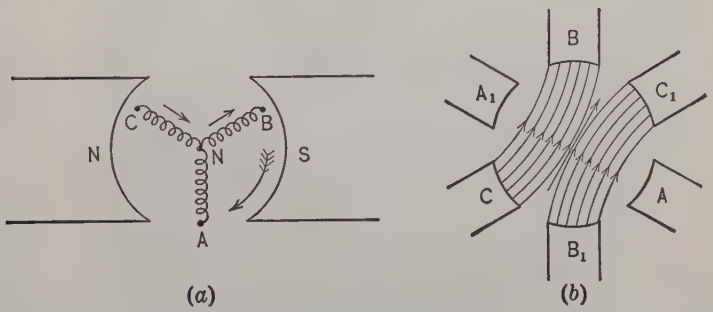


FIG. 409. The direction of the motor field after the generator armature has turned 120°.

Prob. 130–14. Draw diagrams similar to Fig. 409 (*a–b*) for three later instants of the cycle, at 60° intervals.

Prob. 131–14. Interchange the leads *A* and *B* in the motor of Fig. 406, connecting lead *A* at Pole *B* and lead *B* at Pole *A*. Construct three diagrams similar to Figs. 407, 408 and 409. In what direction now is the field rotating?

Prob. 132–14. Interchange the leads *A* and *C* as placed in Prob. 131–14 and repeat the problem, noting direction of rotation.

271. Synchronous Converters. When only small amounts of alternating-current power are to be converted into direct-current power, we may use a device such as the copper-oxide rectifier shown in Fig. 454. For converting larger amounts of power, a machine of the type shown in Fig. 410 is commonly used; it is called a synchronous converter. Within the past decade, however, the mercury arc rectifier (see Fig. 457) has been developed to the point where it is replacing the synchronous converter in many applications.

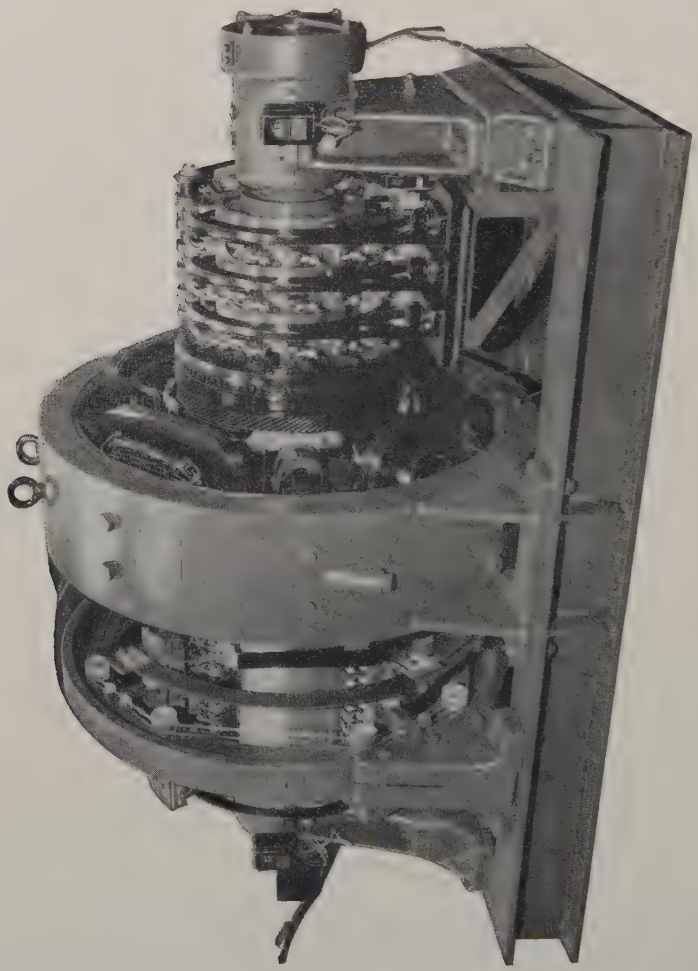


Fig. 410. A synchronous converter for changing a-c power to d-c power. *General Electric Co.*

Synchronous converters are merely double-current generators as described in Chapter VI. We have seen that in every direct-

current armature there is induced an alternating e.m.f. which must be converted into direct e.m.f. by a commutator if we wish to use the machine as a direct-current generator. If we bring taps from the winding out to collector rings we will have alternating voltage across the rings.

In a synchronous converter we put both a commutator and a set of collecting rings on a regular direct-current armature. We supply alternating voltage to the collecting rings, which causes the machine to run as a synchronous motor. All we have to do is to put brushes on the commutator and take off direct-current. The direct voltage is always a given number of times the alternating voltage, being 2.82 times for a converter with six rings, which is the usual number. The voltage cannot be directly controlled by changing the field. The strength of the field determines the power factor at which the converter will operate. In this respect the synchronous converter acts like a synchronous motor.

In Fig. 410, the d-c commutator is on the left-hand end of the machine; a-c power is supplied through the six slip rings at the right. The stationary field is self-excited from the d-c end.

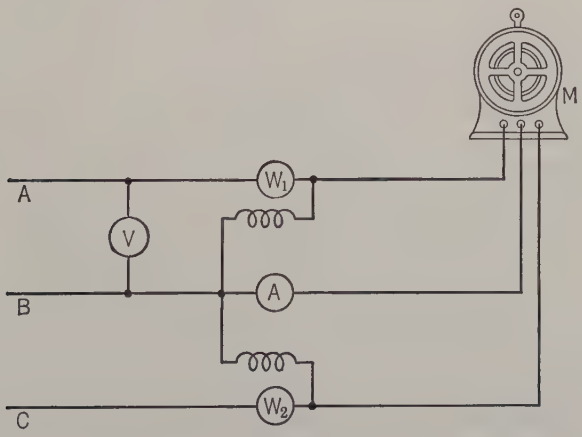


FIG. 411. Connections for the measurement of three-phase power by means of two wattmetters.

272. Measurement of Power and Power Factor in a Three-Phase System. The power in a three-wire, three-phase system can be measured by two wattmeters. In order to obtain the power factor it is necessary also to have an ammeter and a voltmeter. Figure 411 shows how the two wattmeters, the ammeter and the voltmeter must be attached in order to measure the power taken by

the motor M. Note that the wattmeter W_1, has its current coil connected in one lead (A) and the voltmeter coil between lead A and lead B.

The current coil of wattmeter W_2 must be placed in line C, so that its voltmeter terminals may be attached to line C and line B. The algebraic sum of these two wattmeter readings is the effective three-phase power taken by the motor.*

The apparent power is found by connecting the voltmeter V and the ammeter A as in Fig. 411 and multiplying the product of their indications by 1.73. If the system is only slightly out of balance the average readings of an ammeter placed in each lead successively may be taken as the current, and the average voltage between leads as the pressure. The power factor may then be found from the equation

$$\text{Power factor} = \frac{\text{Effective power}}{\text{Apparent power}}$$

$$= \frac{\text{Sum of wattmeter readings}}{1.73 \times \text{ave. line volts} \times \text{ave. line current}}.$$

Prob. 133–14. Two wattmeters connected like W_1 and W_2 in Fig. 411 indicate 1860 and 1320 watts respectively. A voltmeter reads 230 volts between lines and an ammeter in one line wire reads 10 amperes. Assume that the line currents are balanced.

(*a*) What power does the motor take?
(*b*) At what power factor is the motor running?

SUMMARY OF CHAPTER XIV

An ALTERNATING CURRENT flows back and forth through a circuit.

A CYCLE consists of one complete flow back and forth. A cycle is divided into 360 degrees.

The FREQUENCY is the number of cycles completed in one second; symbol, f.

* The two wattmeters should both be connected in the same manner to each line; that is, if the + or right-hand side of the ammeter and the voltmeter terminals of W_1 are connected to the motor side of the line, then both the ammeter and voltmeter + or right-hand terminals of W_2 should be connected to the motor side of the line. If both instruments now indicate up-scale, add the two indications. If the pointer of one wattmeter tends to indicate negative values, reverse either the current or the voltmeter connections and subtract the reading of one wattmeter from the reading of the other.

An ALTERNATING E.M.F. or an alternating current has four characteristic values:

(1) The instantaneous value, symbols e and i.
(2) The maximum value, symbols E_m and I_m.
(3) The average value, symbols E_{av} and I_{av}.
(4) The effective value, symbols E and I.

There are three ways of representing an alternating current:

(1) By curve; generally the sine curve;
(2) Vector diagram, if of sine-wave form;
(3) Trigonometric equation.

All alternating currents considered in this chapter are assumed to follow the sine curve.

Any INSTANTANEOUS VALUE depends upon the phase and is found by the equations:

$$i = I_m \sin \phi,$$
$$e = E_m \sin \phi,$$

where ϕ = angle of phase.

The AVERAGE VALUE of alternating currents and voltages equals 0.637 times the maximum value.

The EFFECTIVE VALUE, which is the value always meant when the terms voltage and current are used alone, is that value in amperes of an alternating current which will produce the same heating effect as a direct current of the same number of amperes. The effective value equals the square root of the average of the squares of all the instantaneous values, and is equal to 0.707 times the maximum value.

The current may either be IN PHASE with the voltage, LAG BEHIND the voltage, or LEAD the voltage. The angle of lead or lag is represented by the letter θ.

AN INDUCTION-MOTOR load usually causes the current to lag. This lag may be decreased by the use of over-excited synchronous motors in parallel with the induction motors.

POWER. When the current is IN PHASE with the voltage,

$$i = I_m \sin \phi,$$
$$P = EI$$

When the current LAGS BEHIND the voltage,

$$i = I_m \sin (\phi - \theta),$$
$$P = EI \cos \theta,$$

($\cos \theta$ is called the POWER FACTOR).

When the CURRENT leads the voltage,

$$i = I_m \sin(\phi + \theta),$$

$$P = EI \cos \theta.$$

The cause of a lag in an alternating current is the inductive reactance of the circuit.

INDUCTIVE REACTANCE is the opposition offered by self induction to the flow of an alternating current, and is measured in ohms; symbol, X_L. $X_L = 2\pi fL$.

The cause of any lead in an alternating current is the capacitive reactance of the circuit.

CAPACITIVE REACTANCE is the opposition offered by capacitance to the flow of an alternating current, and is measured in ohms; symbol, X_C.

$$X_C = \frac{1}{2\pi fC}.$$

REACTANCE is any combination of inductive and capacitive reactance; symbol, X.

$$X = X_L - X_C.$$

With inductance alone in the circuit the current lags 90 degrees; with capacitance alone, the current leads 90 degrees.

$$\text{Current} = \frac{\text{Voltage}}{\text{Reactance}},$$

but the current is always 90 degrees out of phase with the voltage.

IMPEDANCE, symbol Z, is the combination of reactance and resistance, and is found from the following equation:

$$(\text{Impedance})^2 = (\text{Resistance})^2 + (\text{Reactance})^2,$$

or

$$Z = \sqrt{R^2 + X^2}.$$

Voltage to force a current of I amperes through an impedance of Z ohms equals the product of current times impedance, i.e.,

$$E = IZ,$$

or

$$I = \frac{E}{Z}.$$

RESONANCE occurs in SERIES circuits when the capacitive reactance equals the inductive reactance. In this case the circuit offers minimum impedance to the current.

RESONANCE occurs in PARALLEL circuits when the capacitive reactive components of the currents in the parallel branches equal the inductive reactive components. If one branch contains negligible resistance a parallel resonant circuit offers maximum impedance to the current.

FILTER circuits are composed of an arrangement of series resonant and parallel resonant parts.

The angle of lead or lag, in the case of a circuit containing impedance, can be found from the equation,

$$\tan \theta = \frac{X}{R}.$$

Voltage and current values in parallel and series circuits are found by adding vector quantities. See summary on page 418.

Alternating current is at present distributed either by **SINGLE-PHASE** or **THREE-PHASE SYSTEMS.**

In a **THREE-PHASE SYSTEM** the voltage across each phase is at an angle of 120 degrees to the voltage across the other phases.

When **WYE-CONNECTED, AND BALANCED,** the voltage across any phase equals $\sqrt{3}$ times the voltage between one line wire and the neutral.

The current in each path in the armature is the same as the current in one phase.

When **Δ-CONNECTED AND BALANCED,** the voltage across any phase equals the voltage across any one path in the armature.

The current in each line-wire equals $\sqrt{3}$ times current in each armature path.

POWER in a balanced three-phase circuit is found by the equation

$$P_3 = 1.73\ EI \cos \theta.$$

SYNCHRONOUS CONVERTERS are synchronous motors having a direct-current armature and a commutator. Alternating voltage is applied to the collecting rings and direct voltage taken from the commutator.

PROBLEMS ON CHAPTER XIV

Prob. 134–14. A coil of 60 ohms impedance and 70 per cent power factor is connected across a 230-volt, 60-cycle line. What power is consumed in the coil?

Prob. 135–14. What is the inductance of the coil of Prob. 134–14? What is its resistance?

Prob. 136–14. At what voltage would the coil of Prob. 134–14 take the same power if the frequency is 25 cycles?

Prob. 137–14. If a 40-ohm resistance is connected in series with the coil of Prob. 134–14 and the combination is supplied from a 220-volt, 60-cycle line, what power is taken from the line?

Prob. 138–14. What is the line current in Prob. 137–14 and what is the phase angle between this current and the line voltage?

Prob. 139–14. What is the power factor of the series circuit in Prob. 137–14?

Prob. 140–14. An induction coil of 45 ohms impedance and 60 per cent lagging power factor is placed in parallel with a choke coil of 32 ohms impedance and 7 per cent lagging power factor. What are the impedance and the power factor of the combination?

Prob. 141–14. A telephone receiver of 80 ohms resistance and 0.008 henry inductance is connected in parallel with another receiver of 30 ohms resistance and 0.012 henry inductance. What is the impedance of the parallel combination at 1000 cycles?

Prob. 142–14. What is the power factor of the combination in Prob. 141–14?

Prob. 143–14. If the power consumed in the first receiver of Prob. 141–14 is 0.01 watt, what is the total power consumed in the combination? Frequency = 1000 cycles.

Prob. 144–14. At what frequency will the impedances of the receivers of Prob. 141–14 be the same? What is the impedance of the parallel combination at this frequency?

Prob. 145–14. Two induction motors each take 12 kv-a at 230 volts from a generator. Each motor has a power factor of 78 per cent, lagging current. What is the total kv-a output of the generator? What power does the generator deliver?

Prob. 146–14. What current is supplied from the generator in Prob. 145–14, if the system is single phase? If the system is balanced three-phase, what is the line current?

Prob. 147–14. In order to improve the power factor of the system of Prob. 145–14, one of the induction motors is exchanged for a synchronous motor which takes a leading current. If the power factor of the synchronous motor is kept at 90 per cent, what apparent load in kv-a must it take in order to make the power factor of the line unity? What power will the synchronous motor be taking?

Prob. 148–14. Each main of a three-wire three-phase 115-volt distributing system can carry 30 amperes. How many 60-watt 115-volt lamps can be attached to this system if properly balanced?

Prob. 149–14. How many 60-watt 115-volt lamps could be attached to a single-phase 115-volt system, using the same size wires as in Prob. 148–14?

Prob. 150–14. How many 60-watt lamps could be used on the distributing system of Prob. 148–14, if it were used as a three-wire single-phase line and the load were properly balanced?

Prob. 151–14. A balanced load of incandescent lights and three-phase induction motors is carried at the end of a three-wire feeder which takes 250 kw from the station switchboard with 75 amperes per wire and 2300 volts between wires. What is the power factor of this feeder?

Prob. 152–14. The distributing transformers on the feeder of Prob. 151–14 and the feeder itself require for excitation, altogether,

20 kv-a at 65 per cent power factor. How much power is consumed by the lamps and motors together, and at what power factor?

Prob. 153–14. A three-phase induction motor taking a lagging line current of 52 amperes with a power factor of 79 per cent is connected in parallel with a three-phase synchronous motor taking a leading line current of 68 amperes with a power factor of 64 per cent. What current flows in each wire of the three-phase line which supplies the two motors?

Prob. 154–14. The total power supplied to the two motors of Prob. 153–14 is 80 kw. What is the voltage between lines of the three-phase system?

Prob. 155–14. A filter is to be constructed by using a parallel combination of a coil having 0.01 henry inductance and 20 ohms resistance, and an adjustable condenser C_1. A second condenser C_2 is placed in series with this parallel combination.

(*a*) What values must C_1 and C_2 be made in order to let through currents of 40,000 cycles per second and suppress those of 90,000 cycles per second?

(*b*) What is the impedance of the combination to currents of each of these frequencies?

Prob. 156–14. (*a*) Replace the condenser C_2 in Prob. 155–14 with an adjustable induction coil of 30 ohms resistance. Adjust the condenser C_1 to suppress currents of 40,000 cycles per second and adjust the inductance of the coil to let through currents of 90,000 cycles per second.

(*b*) What is the impedance of this combination to currents of each of these frequencies?

CHAPTER XV

VACUUM TUBES AND GASEOUS CONDUCTION

The last decade has been one of phenomenal development in the construction and application of vacuum tubes and gaseous conduction devices. They now occupy such an important position in a wide variety of industries that it is as necessary for the electrical engineer to know the laws governing their operation as it is to know Ohm's Law and its various applications.

273. Metallic Conduction and Ohm's Law. Before taking up the study of vacuum tubes and gaseous conduction devices, it will be well to review our theory of the conduction of electricity in metals. We have seen that a conductor is a material containing a large number of free electrons, which are small negative particles of electricity. These free electrons move rapidly in all directions within a conductor, colliding with one another and with the molecules of the metal, but there is no general drift of electrons in any one direction.

The molecules of the metal, although relatively fixed in position, are in a state of rapid vibration, the amount of vibration of the molecules being a measure of the temperature.

When an electromotive force is applied to a conductor, the random motion of the electrons continues, but there is in addition a tendency for the electrons to move toward the positive potential. This results in a drift of electrons along the conductor and it is this drift that we know as an **electric current.** The molecules of the metal exert such an attraction on the electrons that they keep them in the metal and do not allow them to fly off into the surrounding medium. Thus the electric current is confined to the conductor.

As the electrons move along the conductor under the influence of the applied electromotive force, their paths are obstructed by the molecules of the metal, and many collisions occur. Since the electrons lose velocity by striking molecules, the average motion along the conductor is retarded and we say that the conductor has **resistance.** Furthermore, the normal vibration of the molecules in the metal is increased by the impacts of electrons moving along the wire, and we observe this by the fact that the temperature of the

metal increases. This accounts for the fact that an electric current always heats the conductor through which it is passing.

If the applied electromotive force is removed, there is no force acting on the electrons to keep them moving along the wire, and any velocity of drift they might have acquired is rapidly lost by colliding with molecules of the metal. This accounts for the fact that an electromotive force must be continually applied in order to keep an electric current flowing.

We found that the electromotive force required to keep the current flowing is equal to the product of the current in amperes times the resistance in ohms. This we called Ohm's Law; it, together with Kirchhoff's Laws, can be used to solve all problems concerned with the distribution of direct current and voltage throughout any system of metallic conductors.

274. Non-Metallic Conduction. We have already studied one form of non-metallic conduction, designated as electrolytic conduction. In an electrolyte there are **ions,** that is, atoms or molecules which have gained or lost one or more electrons. A positive ion is an atom or molecule which has lost one or more of its normal number of electrons; the negative ion contains one or more extra electrons.

In electrolytic conduction, these ions move through the liquid toward the electrodes; at the electrodes the ions become neutral atoms or molecules by receiving or delivering electrons, thus changing the potential of the electrodes. This type of conduction is very different from metallic conduction in which the molecules do not move along, but in which the electrons pass through the fixed framework of molecules which forms the conductor.

We shall see later that both ions and electrons are concerned in the conduction of electricity through gases. The laws of gaseous conduction are important not only because of the insight they provide into the nature of gases and vaporized solids, but also because of their many practical applications.

275. Thermionic Conduction. Edison Effect. In the course of his experiments with electric lamps, Edison discovered that if a cold metal plate was placed in an electric lamp bulb, a current could be passed through the evacuated space between the plate and the heated filament if a voltage was applied between them. Furthermore, it was found that, while the current could be made to flow in the direction from plate to filament, no current would flow in the opposite direction, even with very high voltages applied. Hence, if an alternating voltage were applied between the plate and filament of this

device, current would flow only during the time the plate was positive, and the direction of current would always be from plate to filament. This process of producing a direct current from an alternating voltage is known as **rectification,** and the device used is a **rectifier.**

The type of conduction we are here concerned with is called **thermionic conduction.** This name comes from the fact that the action depends upon the ability of a **heated** filament to emit electrons. The electrons act as negative ions transferring electric charge between the plate and the filament just as the ions of an electrolyte do in the case of electrolytic conduction.

Thermionic conduction was first put to practical use by Fleming who found that a tube containing a heated filament and a plate would rectify even the high-frequency currents used in radio telegraphy. He was therefore able to use the device as a radio detector (see Art. 282) in the form known as the " Fleming valve." It was for many years the most satisfactory detector available.

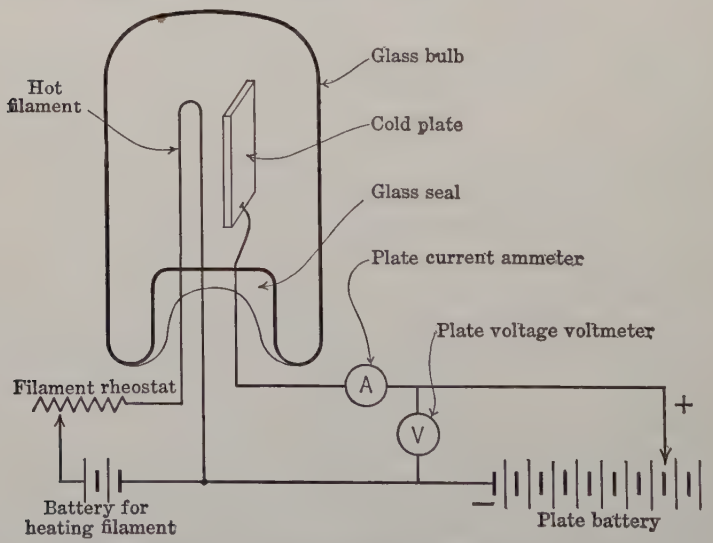


FIG. 412. A two-element thermionic tube. Electrons evaporated from the hot filament are forced over to the cold plate.

Our understanding of the unique laws governing thermionic conduction is largely derived from experiments with two-element tubes similar to the Fleming valve. Figure 412 shows a tube of this type connected in a circuit arranged so as to permit us to measure

the current flowing from plate to filament as the voltage between the two elements is varied. A rheostat is connected in series with the filament so that the filament current, and hence the temperature of the filament, can be varied. One side of the filament is connected to the negative pole of the plate battery; the positive pole of the plate battery is connected through an ammeter to the plate. The plate battery is arranged with taps brought out at each cell so that the voltage between plate and filament can be varied, and a voltmeter is connected to read this voltage. Figure 413 shows the characteristic curve when the measured plate current is plotted against the corresponding plate voltage.

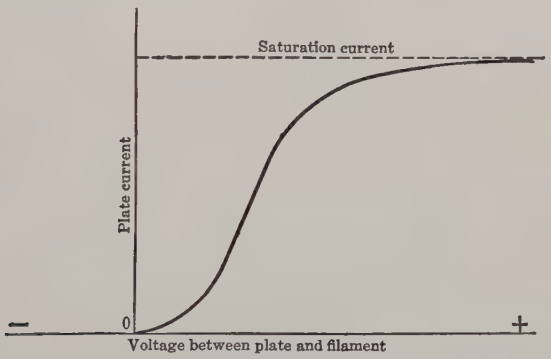


FIG. 413. The relation of the plate current to the voltage between the filament and the plate at a given filament temperature.

The device evidently does not obey Ohm's Law. If it did, the curve between voltage and current would be a straight line, and doubling the voltage between the plate and the filament would double the current from the plate to the filament. It does not have a constant resistance, but its resistance varies according to the current passing through the tube.

Further study of this curve shows other things which require explanation. In the first place, there is a current which flows when the plate is positive, but no current when it is negative with respect to the filament. This is of course the property which makes the tube useful as a rectifier. Secondly, the current rises slowly at first and then more rapidly as the voltage is increased, and finally reaches a maximum or "saturation" value which is not increased by further increase of voltage.

The curve shown in Fig. 413 is for a fixed value of filament temperature. If the filament current is adjusted to several successive

values, a series of curves can be obtained as shown in Fig. 414. Each of these curves is for a different value of temperature, designated as T_1, T_2, and T_3, where T_1 is the lowest temperature and T_3 the highest. It will be noted that the curves for different values of filament temperature coincide for low values of plate voltage, but that the saturation value of current reached in each case is different, and increases with increase of filament temperature.

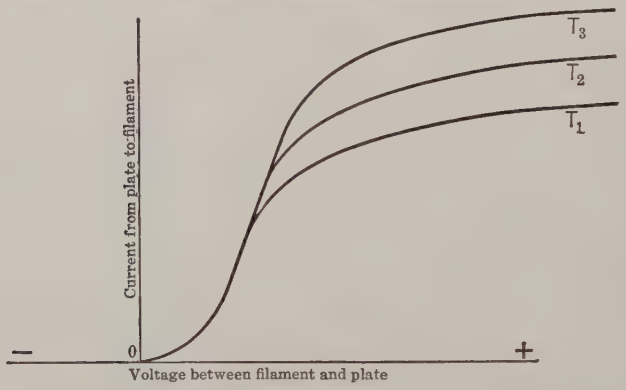


FIG. 414. The effect on the saturation current of changing the filament temperature.

276. Richardson's Law of Thermionic Emission. For an explanation of the above action in a two-element tube, we go back to the electron theory of metals. We found that the metal of the filament contained many free electrons which moved rapidly about in all directions. When we heat the filament, we supply energy to it. Part of this energy is used to make the molecules of the metal vibrate faster, while the remainder causes the free electrons to move faster. In fact, if the temperature is increased sufficiently, some of the electrons will move so rapidly that they will escape from the body of the metal and fly out into space.

This effect is exactly analogous to the evaporation of a liquid. When a liquid is heated, some of the molecules by reason of their heat motion, acquire enough speed to be able to escape from the surface. In exactly the same manner, we may evaporate electrons from an incandescent filament.

If the filament is made negative with respect to the cold plate, these electrons are repelled by the filament and attracted by the plate. Some of them accordingly move across to the plate. This constitutes an electric current which is measured by the ammeter.

The direction of this current as measured by the ammeter is in the opposite direction to that in which the electrons actually move. This, as we have seen, is due to the fact that the ammeters show the direction in which a **positive** charge of electricity flows, and the electrons flowing are, as we know, negative charges of electricity. This scheme was adopted before the electron theory was known, and before the charge of an electron was found to be negative. Of course it makes no difference in which direction we draw the arrow indicating the direction of current provided we are consistent. That is we always draw the arrow in the direction in which a positive charge would flow. The electrons always flow in the opposite direction to the arrows which represent the flow of positive charges. The motion of the electron, however, is always from the hot filament to the cold plate, and never in the other direction; for the cold plate does not give out any electrons by which a current could be set up in the opposite direction.

If the plate voltage is small, only a few of the electrons evaporated from the filament are forced over to the plate. The remainder either return to the filament, or become part of a cloud of electrons surrounding the incandescent filament. This cloud of electrons is known as the **space charge,** and it is responsible for the relatively slow increase in current found in the first part of Fig. 413, as compared with the more rapid increase in current over the middle portion of the curve. The space charge is made up of a large number of negatively charged particles. Since like electrical charges repel one another, the electrons in the cloud outside the filament repel the electrons which are trying to get out of the filament, and hence oppose the force set up by the voltage between plate and filament. As the plate voltage is increased, however, many of the electrons in the space charge as well as new ones from the filament are drawn over to the plate. Thus the space charge effect is decreased, and, at higher voltages, a given change in plate voltage will produce a larger change in current.

If the potential between filament and plate is raised to a value sufficiently high, **all** the electrons will pass over to the plate as fast as they are evaporated from the filament. Then no matter how much the voltage increases beyond this, the current cannot increase, for there are no more free electrons to be moved over to the plate. This accounts for the saturation value of current which was found in the curve of Fig. 413, that is, the maximum current which can flow in the tube at the given value of filament temperature. If the

filament temperature is increased, a larger number of electrons per second is evaporated. Then the maximum current which can be passed through the tube is increased, or the saturation current becomes greater. This gives rise to the series of curves shown in Fig. 414.

The evaporation of electrons described above is known as **thermionic emission,** and a device which utilizes such an effect is a **thermionic tube.** It is **not gaseous** conduction, for there is no gas which plays a part in the device. In fact, the more completely the bulb is exhausted of air, the better the device works. Thermionic tubes are accordingly freed as far as possible from gas; that is, they operate in a **hard** vacuum. The conduction is hence across an evacuated space by reason of electrons which are evaporated from the filament and which move from the filament to the plate through empty space.

The law which governs the evaporation of electrons from a filament is of the same form as the law which governs evaporation from a liquid. The rate of evaporation can be measured by determining the saturation current of the tube. Richardson was the first to state the law governing evaporation from a heated filament by giving an expression for the saturation current. His equation has since been modified slightly and is given by Dushman* in the form

$$i_s = \frac{AT^2}{2.718^{\frac{K}{T}}} \text{ amperes per square centimeter,}$$

where

i_s is the **saturation** current in amperes per square centimeter,
A is a constant equal to 60.2,
T is the temperature of the filament in centigrade degrees absolute, = centigrade reading + 273 degrees,
K is a constant depending upon the material of the filament.

For pure tungsten wire, the value of K is about 52,600. In using the formula, we must be careful to note that it applies only to the portion of the filament which is actually at the temperature T, and that the ends which are cooled by the supports will not emit to the same extent. Also, the result given by the above formula must be multiplied by the surface area of the filament in order to find the total current; only the area which is at the temperature T is considered as active emitting area.

* "Electron Emission from Metals as a Function of Temperature," S. Dushman, Physical Review. Vol. 21, June, 1923.

Example 1. A tungsten filament 2 inches long and 0.002 inch diameter is at a temperature of 2200 degrees absolute. If this filament is part of a two-element tube, what is the maximum current, that is, the saturation current, which can be passed through the tube? Assume that the entire filament is at the given temperature.

Solution.

$$i_s = \frac{AT^2}{2.718^{\frac{K}{T}}}$$

$$A = 60.2$$

$$K = 52{,}600$$

$$i_s = \frac{60.2 \times 2200^2}{2.718^{\frac{52{,}600}{2200}}} = \frac{291.3 \times 10^6}{2.718^{23.9}}$$

$$= \frac{291.3 \times 10^6}{2.398 \times 10^{10}} = 0.0113 \text{ amp. per sq. cm.}$$

$$= 11.3 \text{ milliamperes per sq. cm.}$$

$$\text{Area} = 2 \times 3.14 \times 0.002 \times 2.54^2 = 0.081 \text{ sq. cm.}$$

$$\text{Current through tube} = 0.081 \times 11.3$$

$$= 0.915 \text{ milliampere}$$

Prob. 1–15. A thermionic tube is constructed with a tungsten filament 4.2 inches long and 0.006 inch in diameter. The filament is at a temperature of 1950° C. except for a length of $\frac{1}{8}$ inch at each end which is cooled by the support and may be considered not to emit. A voltage of 800 volts causes saturation current to flow. Compute the value of this saturation current.

Prob. 2–15. A tungsten-filament vacuum tube has a saturation current of 18 milliamperes. The filament is made of wire 0.007 inch in diameter and operates at a temperature of 2400° absolute. What is the active length of the filament?

277. Thermionic Tubes as Amplifiers or Repeaters. Tube Characteristics. With the exception of Fleming's application of the two-element tube as a detector, there was no practical application of thermionic conduction devices to any extent until after the introduction of a third element by De Forest. This element consisted of a wire mesh or grating placed between the filament and the plate. The appearance of this element suggested the name " grid " which is now in general use. In a three-element tube constructed as in Fig. 415, the plate completely surrounds the filament and the grid is built in the form of a spiral supported in the space between the plate and the filament, but insulated from both of them.

If we apply a voltage between the plate and the filament (plate positive) of the tube of Fig. 415, it will behave simply as a two-

element tube. Of course, the electrons evaporated from the filament will have to pass through the grid in order to reach the plate, but the grid is designed to permit passage of electrons through it. If we now apply another voltage between the grid and the filament, we find that the plate current changes. The plate current increases if the grid is made positive with respect to the filament, and decreases if the grid is made negative. When the grid is positive, it helps the plate to pull electrons away from the filament. Some of the electrons will strike the grid wires and there will hence be a small current flowing in the grid circuit, but because of the open construction of the grid, most of the electrons pass through and go on to the plate, and the plate current is thus increased. If the grid is negative, however, it opposes the action of the plate and tends to prevent electrons from leaving the filament and thus decreases the number of electrons which can reach the plate. In other words, the plate current decreases when the grid is made negative with respect to the filament.

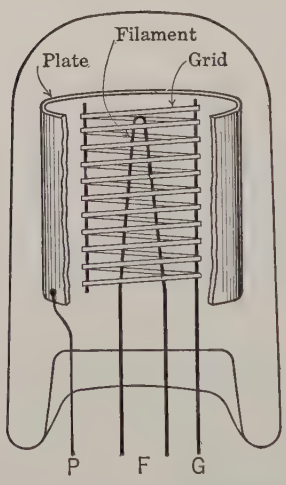


FIG. 415. A three-element thermionic tube.

The condition of operation in which the plate is positive, and the grid is negative, with respect to the filament, is of great importance in the operation of three-element tubes. Since the grid is negative, any electrons approaching the grid wires are repelled and must either go on to the plate or return to the filament. Thus no electrons can reach the grid and **no current flows in the grid circuit.*** This is very important, for it means that as long as the grid is kept negative, no power is supplied to it. The average negative potential of the grid is usually referred to as the **bias.**

We have stated above that if the grid is made positive, the plate current increases; if the grid is made negative, the plate current decreases. With a negative bias on the grid, the plate current will

* Actually some electrons have sufficient velocity to reach the grid in spite of the repulsion, and positive charges which are present in the space due to small traces of gas are attracted to the grid. Due to these effects and the leakage of current over insulating supports, there is always a small current in the grid circuit. It is so small, however, that for all practical purposes it is usually neglected.

be less than would flow if the grid had no potential applied. If this negative potential is decreased slightly, we find that the plate current increases because the opposition of the grid to the flow of electrons is decreased. Thus we find that we can change the plate current by merely changing the grid potential. We could have caused this change of plate current by changing the plate voltage, leaving the grid voltage constant, but due to the fact that the grid is located much nearer the filament than the plate, it is able to influence the flow of electrons from the filament with a much smaller change of voltage than would be required at the plate. Thus a small change in grid voltage produces the same change in the current through the tube as a large change in plate voltage. Moreover, as long as the grid voltage is kept negative, no power is required in the grid circuit in order to control a large change of power in the plate circuit. Any device which requires a small amount of power to control a large amount of power is called an **amplifier.** In telephone service, it is customary to call this device a **repeater.**

The filament and plate of a three-element tube are frequently called the **cathode** and **anode,** respectively. The plate is always positive with respect to the filament, and the name " anode " refers to the positive pole in any device, while " cathode " refers to the negative (see Art. 169). A tube containing filament, grid, and plate is called a **triode** (three electrodes); if the tube contains only a filament and plate it is called a **diode** (two electrodes). Figure 416 shows two forms of the conventional symbol used to represent a triode in drawing circuit diagrams. Figure 416 (*a*) is used to represent a tube of the type shown in Fig. 415, in which the electrons are emitted directly from a heated filament. In modern practice, however, the heating current required to produce emission is usually an alternating current. If alternating current is used in an ordinary filament, the filament temperature will tend to follow the variations in heating current, and the emission will hence vary and thus produce a " ripple " in the plate current. To avoid this effect, the emitting surface is made in the form of a hollow cylinder, and a separate " heater " is placed inside the cylinder but insulated electrically

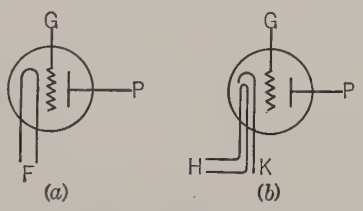


FIG. 416. Conventional symbols used to represent triodes in circuit drawings. (a) Triode with directly heated cathode; (b) Triode with separate heater for cathode.

from it. The cathode assembly thus made has a large heat capacity so that its temperature cannot change rapidly, and variation of plate current is avoided. Figure 416 (*b*) shows the conventional symbol for the heater type of tube. The plate is represented by part P and the grid is represented by the wavy line marked G. The emitting cylinder is used as the cathode and is marked K; the heating current is supplied at the terminals marked H. Figure 417 shows a triode of the heater type.

FIG. 417. Type 56 triode. Standard type of triode with a separate heater for cathode. *R.C.A. Radiotron Co., Inc.*

278. Characteristics of Three-Element Tubes. Amplification Factor. The operation of a triode may best be understood from a study of its characteristic curves. These are curves which show how the plate current of the tube varies as the plate voltage and grid voltage are varied. Figure 418 shows the circuit connections which may be used to obtain the characteristics. The filament heating current is supplied from a battery and is controlled by a rheostat R, using the ammeter A_2 to measure the heating current. This current must of course be kept constant throughout the test in order to keep the temperature, and hence the emission, constant. Another battery equipped with

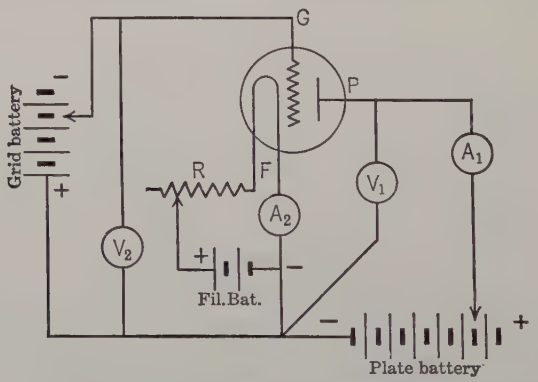


FIG. 418. Circuit used in measuring characteristics of a triode.

taps is used to supply a plate voltage which can be varied; this voltage is measured by the voltmeter V_1 and the current in the plate circuit is indicated by the ammeter A_1. A third

battery, also tapped, supplies grid potential which is measured by the voltmeter V_2, but no ammeter is required in the grid circuit because the grid is kept negative so that it cannot draw current. Note that both the grid and plate circuits are connected back to the filament on the negative side of the filament battery. This is done to insure that the grid is kept negative with respect to the entire filament. If the grid is connected to the positive side of the filament, and the negative grid voltage is smaller than the voltage

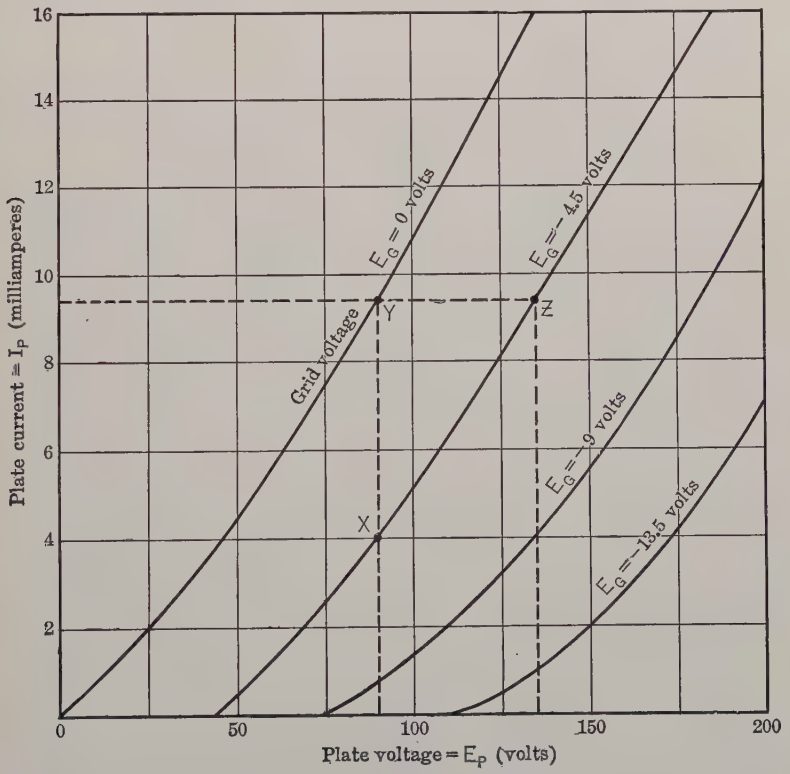


FIG. 419. Plate characteristics of a triode. Each curve is for a different value of grid bias.

across the filament, part of the filament will be more negative than the grid. In other words, the grid will be at a positive potential with respect to part of the filament and hence will draw current from that part.

With the tube connected as in Fig. 418, we can measure the variation of plate current as the grid voltage and the plate voltage are

varied. Since there are three quantities which vary, it is impossible to show the results in a single curve. We keep one quantity constant, as for example the grid voltage, while the others vary over their entire range, and repeat the procedure for a number of different values of the fixed quantity. We thus obtain a " family " of curves as shown in Fig. 419. Note that each curve represents the variation of plate current against plate voltage for a fixed value of grid voltage, but that the grid voltage varies from curve to curve. These curves are usually referred to as the **plate characteristics** of the tube.

The left-hand curve in Fig. 419 is taken with no voltage on the grid and therefore is equivalent to operating the tube as a diode. It is merely the lower part of the curve of Fig. 413. Now suppose we have 90 volts impressed on the plate of the tube and zero voltage on the grid. Referring to Fig. 419, we see that this will produce a current of 9.4 milliamperes in the plate circuit, operating at the point Y. If we now impress a voltage on the grid of -4.5 volts, keeping the plate voltage constant, we must go to the curve corresponding to -4.5 volts on the grid. The operating point therefore moves from Y to X, and the plate current decreases to 4 milliamperes. Moving along the line YX does not change the plate voltage, so the entire change of plate current from 9.4 milliamperes to 4 milliamperes is due to the change in grid voltage.

Let us now hold the grid voltage constant at -4.5 volts and increase the plate voltage until the plate current is again 9.4 milliamperes. Since the grid voltage is constant, we must move along the curve corresponding to -4.5 volts on the grid, and we find that at the point Z on this curve the plate current is again equal to 9.4 milliamperes. The point Z, however, corresponds to a plate voltage of 135 volts. With no voltage on the grid, it required only 90 volts to force 9.4 milliamperes through the tube, but with -4.5 volts on the grid, it required 135 volts to pass the same current through the tube. In other words, a **change** of 4.5 volts in the grid circuit is equivalent to a **change** of $135 - 90$, or 45 volts in the plate circuit. Since the plate voltage must change ten times as much as the grid voltage, it is evident that a change in grid voltage is ten times as effective as a change in plate voltage. This ratio of the **change** in plate voltage required to counteract a **change** in grid voltage is called the **amplification factor** of the tube. The amplification factor is usually represented by the Greek letter μ (Mu), and in modern tubes its value runs from about 3 to several hundred.

279. Performance of Triodes in Amplifiers. The values of plate current, plate voltage, and grid voltage plotted in Fig. 419 can also be plotted as in Fig. 420. In the latter case, curves of plate current against grid voltage are plotted for various constant values of plate voltage. From these curves, we can read directly the change in plate current which occurs when the grid voltage changes. A

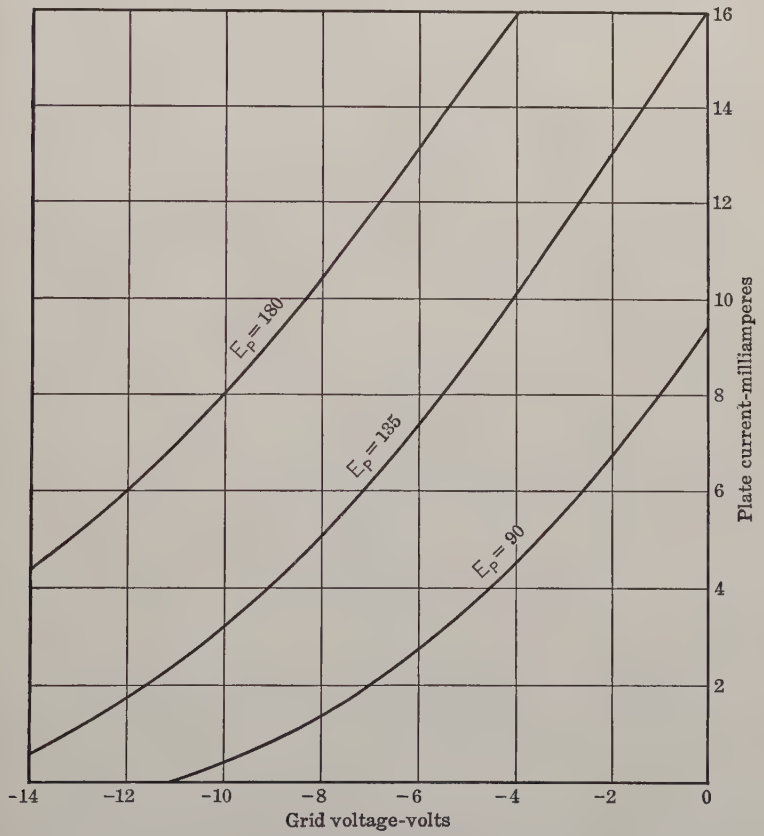


FIG. 420. Transfer characteristics of a triode. The data are the same as in Fig. 419, but each curve is now for a different value of plate voltage.

family of curves of this type is sometimes called the transfer characteristic of the tube, because it shows how the effect of varying grid voltage is transferred to the plate circuit in the form of a varying plate current.

In Fig. 421, a heater-type triode is supplied by a plate battery of constant voltage equal to E_p. A negative bias equal to E_g is im-

pressed on the grid, in series with a small alternating voltage e introduced through a transformer. The curve of plate current against grid voltage for this tube is plotted in Fig. 422. If no alternating

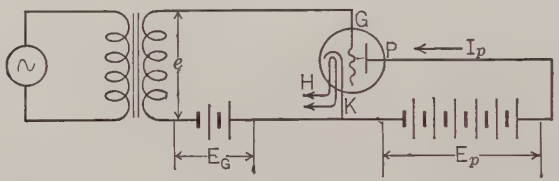


FIG. 421. A heater-type triode with an alternating grid voltage applied in series with a negative bias.

voltage is impressed, the tube operates at point X on the curve, corresponding to a grid voltage E_g and a plate current I_p (plate voltage constant). Suppose now that we apply the alternating

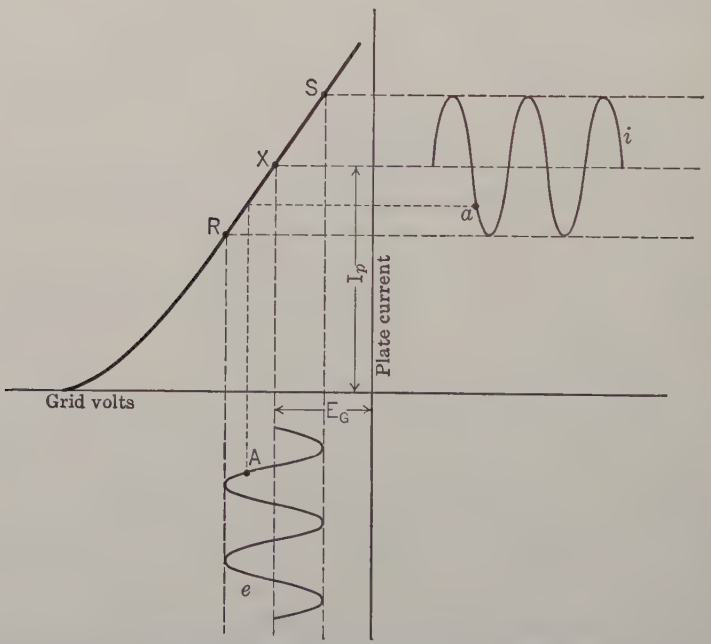


FIG. 422. Transfer characteristic showing how an oscillating grid voltage produces an oscillation of plate current.

voltage e in series with the grid-battery voltage. These two voltages merely add at each instant with the result that while the grid voltage always remains negative, it oscillates about the bias voltage with

an amplitude of oscillation equal to the amplitude of the alternating voltage. The plate current in turn always remains positive but oscillates about the value I_p in synchronism with the alternating voltage on the grid; the amplitude of the plate-current oscillation is determined from the curve as shown in Fig. 422. The point A on the grid-voltage wave corresponds to the point a on the plate-current wave. Similarly, every point on the grid-voltage wave determines a point on the plate-current wave.

It is important that the portion of the curve included between the maximum and minimum values of grid voltage be a straight line. In Fig. 422, this refers to the section between R and S. If this section is straight, the plate-current variation will exactly follow the grid-voltage variation. If it is curved, the shape of the plate-current wave will be different from that of the grid-voltage wave and is said to be distorted. It is very important in radio work that distortion be avoided if high fidelity of reproduction of speech and music is to be obtained.

280. Plate Resistance. Determination of Amplifier Gain. The amount of amplification which a tube will produce can be determined from the plate characteristics. In Art. 278, we saw that we could find the amplification factor of a tube from the plate characteristics. In order to determine the voltage gain of an amplifier tube, we must also know the internal resistance, or plate resistance, of the tube. If the tube conduction obeyed Ohm's Law, the tube resistance would simply be the plate voltage divided by the plate current. Of course, for the case where the plate current and plate voltage are constant, the resistance given by Ohm's Law will produce correct results, but in actual operation as an amplifier, the plate current is varying. We therefore use a value of resistance which gives correctly the relation between the **change** of plate voltage and the **change** of plate current. The **plate resistance** of a tube is defined as the **change in plate voltage** which produces a given change in plate current, divided by that **change in plate current.**

Thus, in Fig. 423, points X and Y represent slightly different values of plate voltage and plate current on a curve of constant grid voltage. From the definition given above, we see that the plate resistance,

$$R_p = \frac{E_{p1} - E_{p2}}{I_{p1} - I_{p2}}.$$

If the plate characteristics of a vacuum tube are known, the plate resistance of the tube can be computed by the above method. Care

must be taken to select points very close together and located on a portion of the curve which is straight. Usually, however, the plate resistance is known from measurements made by the manufacturer.* These measurements are made by introducing a small alternating

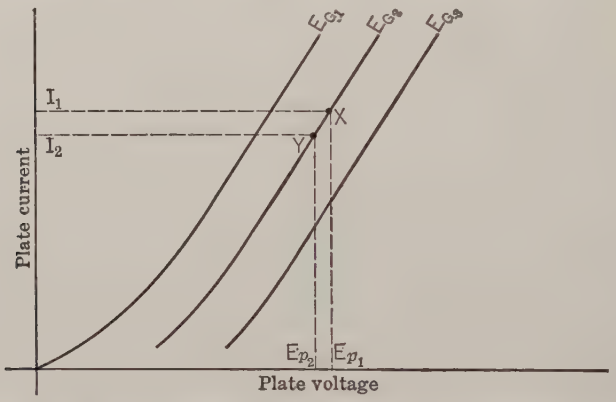


FIG. 423. The plate resistance of a triode can be determined from two points on the plate characteristic.

voltage in the plate circuit and taking the ratio of the alternating voltage to the resulting alternating current.

Suppose we have a tube of amplification factor μ, and plate resistance R_p, connected in the circuit of Fig. 424. In addition to the internal resistance of the tube, there is a plate load resistance R in

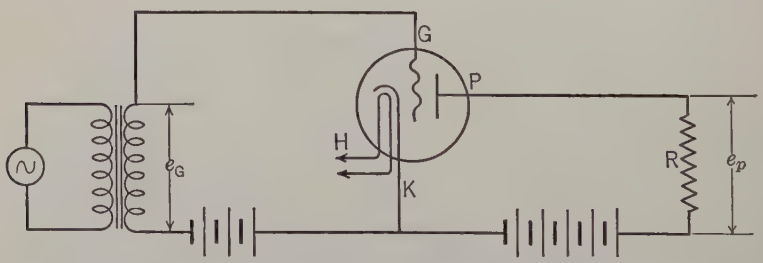


FIG. 424. An alternating grid voltage applied to the triode produces an alternating current in the plate load resistance R.

the circuit. The useful output of the tube is the voltage drop across R due to the **alternating current** in the plate circuit. If we call the useful alternating voltage output of the tube e_p, and the alternating

* Complete data on all standard receiving tubes are available in the " Radio Tube Manual," published by R.C.A. Radiotron Co., Inc.

grid voltage e_g, the gain of the tube is the ratio $\frac{e_p}{e_g}$. This ratio is determined as follows.

The amplification factor μ tells us the relative effectiveness of a change in grid voltage, as compared with a change in plate voltage, in producing a change in plate current. Thus, if the grid voltage change is 3 volts, the effect is the same as though the plate voltage changed 3μ volts. Then as far as the alternating current flow in the tube is concerned, we can replace the circuit of Fig. 424 by the equivalent circuit of Fig. 425. The total resistance of the plate circuit consists of the plate resistance r_p in series with the plate load-resistance R. The alternating voltage acting in this circuit is simply μ times the alternating grid voltage e_g.

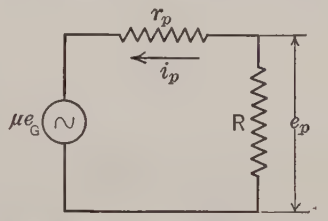


FIG. 425. Equivalent circuit for the plate loop of a triode carrying an alternating current output.

Using the equivalent circuit, the gain of the tube can be computed by applying Ohm's Law. The alternating plate current in Fig. 425 is given by

$$i_p = \frac{\mu e_g}{r_p + R}.$$

Hence the drop across the load resistance R is

$$e_p = i_p R = \frac{\mu e_g}{r_p + R} R.$$

The gain of the tube is therefore,

$$\frac{e_p}{e_g} = \mu \frac{R}{r_p + R}.$$

In Fig. 426, the gain as given by the above equation is plotted against the plate load resistance R. When R is small, a considerable increase in gain is secured by an increase in R, but as R gets bigger, there is less advantage to be obtained by further increase. We see also from this curve that the gain approaches the amplification factor μ as a limiting value, but never quite reaches it. A gain equal to the amplification factor can never be obtained because of the internal resistance of the tube. No matter how large we make R, some of the alternating voltage in the plate circuit will be lost in the internal drop.

Prob. 3–15. A triode having the plate characteristics shown in Fig. 419 is operated with a grid bias of -4.5 volts, and with 150 volts on the plate. What is the plate resistance under these operating conditions?

Prob. 4–15. If the tube of Prob. 3–15 is supplying a plate load resistance of 8500 ohms, what is the voltage gain in the tube? Amplification factor = 10.

Prob. 5–15. A certain triode has a plate resistance of 11,000 ohms and is supplying a plate load resistance of 14,000 ohms. If an alternating grid voltage of 3 volts produces an alternating voltage drop of 28 volts across the 14,000-ohm resistance, what is the amplification factor of the tube?

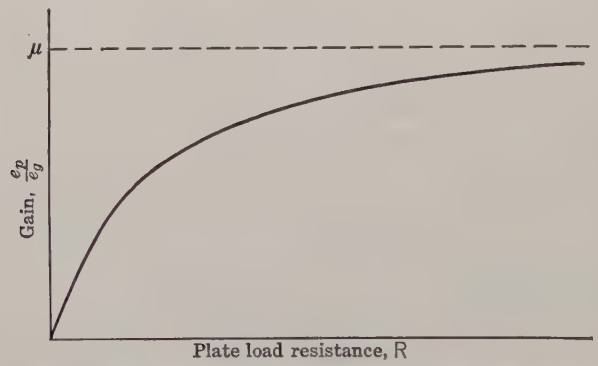


FIG. 426. Variation of amplifier gain with plate load resistance.

281. Typical Amplifier Circuit Using a Triode. Figure 427 shows a typical circuit using a triode for amplification. The voltage which is to be amplified, e_1, is applied to the grid circuit through the step-up transformer T_1. The secondary winding of this transformer has one end connected to the grid G and the other end to the common point O. The negative voltage required for biasing the grid is obtained from the drop across the resistance r which is connected between the cathode K and the common point O. Due to the direct-current component of plate current flowing through r, the cathode is at a positive potential with respect to the point O. This is the same as making the grid negative with respect to the cathode because the grid is connected to O through the transformer winding. Since no current flows in the grid circuit, there is no d-c voltage drop in the transformer winding and the grid is hence at the same d-c potential as the point O, and is hence negative with respect to the cathode. A large condenser C_1 is connected across r in order to prevent the alternating current in the plate circuit from passing

through the bias resistor r. If the reactance of the condenser is small compared to the resistance r, most of the alternating current in the plate circuit will flow in the condenser rather than in the bias resistor. This is desirable, because if the a-c component of plate current flows through the bias resistance, it produces an a-c voltage drop which opposes the a-c voltage induced in the transformer secondary and hence reduces the signal voltage applied to the grid. Under these conditions, the amplifier is said to be degenerative.

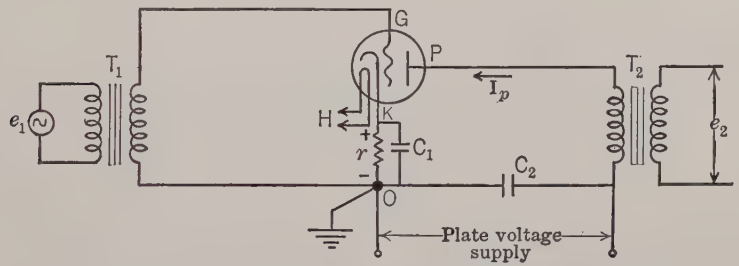


FIG. 427. Typical triode amplifier circuit showing the use of a grid-bias resistor.

The total plate current of the tube flows through the primary winding of the transformer T_2. Since the primary current must change its value in order to induce a voltage in the secondary of the plate transformer, it is evident that only the a-c component of plate current is effective in producing the output voltage e_2 of the amplifier. The d-c component produces no effect in the output. It is for this reason that in calculating the amplifier gain in the preceding section, we considered only the a-c component of plate current.

The plate voltage may be supplied from a battery or from a rectifier with a smoothing filter in its output (see Art. 290). It is not necessary for the a-c component of plate current to flow through the plate power supply, so a condenser C_2 of low reactance (high capacitance) is connected across the power supply to conduct the a-c component of current from the cathode to the plate transformer. This provision is quite important where a number of tubes are supplied from the same plate power supply, for it serves to prevent voltage variation in the plate circuit of one tube due to the variation of plate current taken by another tube.

The voltage gain of the entire amplifier circuit in Fig. 427 is found by multiplying the gain in the tube, as calculated in Art. 280, by the step-up ratios of the transformers. Thus if each transformer

increases the voltage by a factor of 3, for example, and the tube gain is 25, the overall gain of the amplifier is $3 \times 3 \times 25 = 150$.

Prob. 6–15. A triode requires a grid bias of -6 volts at a plate current of 3 milliamperes. How many ohms is required in a grid-bias resistance used as in Fig. 427?

Prob. 7–15. If the bias resistance of Prob. 6–15 is in parallel with a condenser of 20 μf. capacitance, and the alternating current in the plate circuit is at a frequency of 1000 cycles, what percentage of the a-c component of plate current passes through the bias resistance?

282. The Thermionic Tube as a Rectifier. Detection. Since the plate of a thermionic tube is cold, it does not emit electrons, and the tube hence cannot conduct when the plate is negative with respect to the filament. The tube can conduct in only one direction, from plate to filament, and can therefore be used as a rectifier.

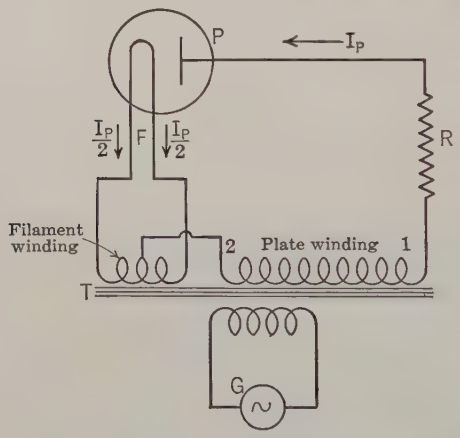


Fig. 428. A half-wave diode rectifier circuit.

In Fig. 428, an ordinary diode is connected so as to supply a rectified current to the load resistance R. A transformer T receives power from the alternator G. One secondary winding is used to heat the filament of the diode. The output circuit starts at the plate P of the diode, goes through the load resistance R, through another secondary winding on the transformer, and returns to the filament at a tap on the mid-point of the filament-heating winding. When point 1 of the plate winding of the transformer is positive current flows through R, then through the tube from plate to filament. Half of this current passes through one filament lead and half through the other, as it returns to the mid-point of the filament winding where the connection is made back to point 2 of the plate winding.

When the voltage in the plate winding reverses in polarity during the next half cycle, point 2 becomes positive, and the filament is hence at a positive potential with respect to the plate. The tube does not conduct during this half cycle, but again carries current in the next half cycle when point 1 becomes positive. The result is

that the current in the load R flows only during the half cycles when the point 1 of the plate winding is positive, and this tube is called a **half-wave rectifier.**

In Fig. 429, the top wave (a) shows the time variation of voltage in the plate winding. Wave (b) shows the resulting half-wave rectified current which flows. Since the current flows only half of the total time, its average value instead of being 0.637 times the maximum value is only half of this, or 0.318 I_m. This is the value that a d-c ammeter placed in series with the load would read.

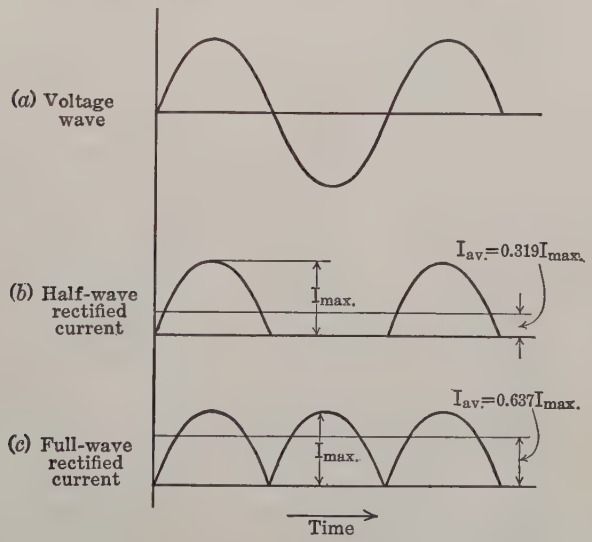


FIG. 429. Wave forms of output currents in half-wave and full-wave rectifier circuits.

It is often desirable to obtain a **full-wave** rectified current as shown in part (c) of Fig. 429. This can be accomplished by the arrangement shown in Fig. 430. The tube used is a double diode having a single filament and two plates. The two plates are connected to opposite ends of the plate winding of the transformer. Thus, when plate 1 is positive, plate 2 is negative, and in the next half cycle both plates reverse in polarity. Consequently, each plate carries a half-wave current, but the current in plate 1 flows when the current in plate 2 is zero, and vice versa. The load resistance R is connected between the filament and the mid-point 0 of the plate winding. During one half-cycle, current flows from 0 to plate 1, and in the next half-cycle from 0 to plate 2. Note that in each

case, current flows **away** from point 0, so that the current I in the load R is always in the same direction, as shown. The average value of a full-wave current is 0.637 times the maximum value and is therefore twice that of a half-wave current.

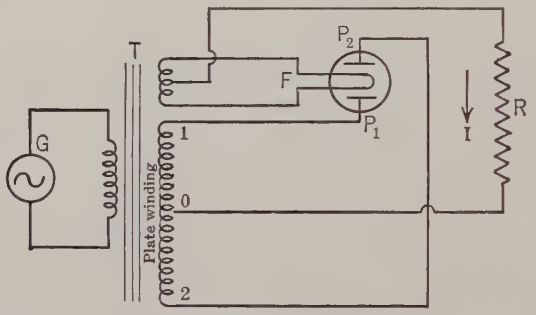


Fig. 430. A double diode used in a full-wave rectifier circuit.

In radio receivers, the " detector " is primarily a rectifier acting on the signal picked up by the antenna. This signal is an alternating voltage of very high frequency, the amplitude of which varies with time in accordance with the wave form of the lower frequency sound wave being transmitted. The high frequency is called the **carrier** frequency, and the process of varying its amplitude at some other frequency is called **modulation.** Carrier frequencies used in radio transmission go from 50,000 cycles to about 30 megacycles (1 megacycle = 1,000,000 cycles) with experimental work being done on higher carrier frequencies.

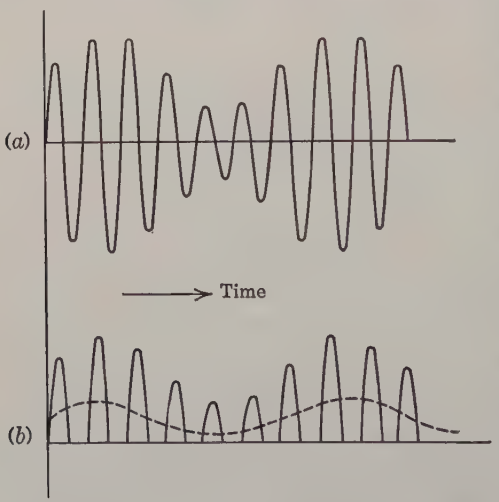


Fig. 431. (a) A carrier frequency modulated by an audio frequency. (b) A modulated carrier after rectification. The dotted line shows the variation of average value.

Figure 431 (*a*) shows the form of a high frequency modulated by a low frequency sine wave. The voltage is alternating at the

high rate, but its amplitude varies at the lower rate. If a voltage of this form were applied to a telephone receiver or a loud-speaker, no sound would be heard because the frequency is too high. The highest frequency that the average person can hear is about 18,000 cycles per second, and this is much below usual carrier frequencies. If we could hear a sound wave produced from a voltage of the form shown in Fig. 431 (*a*), it would be merely a tone of almost constant frequency which simply varied in loudness at the frequency of the modulating wave.

In order to hear the sound wave, it is necessary to rectify the signal as shown in Fig. 431 (*b*). When the latter wave is applied to a telephone receiver, the diaphragm responds to the variation of the **average** value of the rectified signal. This average value is shown by the dotted line in Fig. 431 (*b*), and it will be noted that the variation of the average is the same as that of the original modulating frequency which was applied to the carrier. The average value of the carrier wave before rectification was zero, so that it had no effect on the telephone receiver.

Various types of detector circuits are used in radio receivers. In many modern receivers, an ordinary diode is used. This diode is usually built into the same bulb with an amplifier tube, so that the same cathode can be used for both. Many receivers use triodes with negative grid bias high enough to prevent current flow during the negative half cycles of signal voltage. Thus, in Fig. 420, if a tube having the characteristics shown is operated with 90 volts on the plate and a d-c grid bias of -11 volts, it will produce a half-wave plate current variation when an alternating voltage is applied to the grid. If the signal voltage is positive, it opposes the negative grid bias so that the resulting grid voltage is less than -11 volts and the tube will conduct. When the signal voltage is zero, the grid bias reduces the plate current to zero, and we say that the tube "cuts off." When the signal voltage becomes negative, it adds to the grid bias, making the total grid voltage more negative, and no current can flow. In the biased detector, therefore, plate current can flow only while the signal voltage opposes the grid bias, and the plate current is a half-wave rectified current.

Figure 432 illustrates the method of detecting by the use of a grid leak and condenser, a method which was widely used in many of the older radio receivers. In both the grid-leak detector and the biased detector described above, there is the advantage over the diode detector that the same tube acts both as an amplifier and a

detector. The diode simply rectifies, and its output must be amplified by another tube. The grid-leak detector is more sensitive than the biased detector, and is particularly efficient on weak signals.

In Fig. 432, the signal is picked up by the antenna and is carried into the grid circuit by means of the air-core transformer T_1. A variable condenser C_2 is connected across the secondary winding of the transformer and is adjusted until the circuit is resonant to the frequency of the incoming carrier. This process of "tuning"

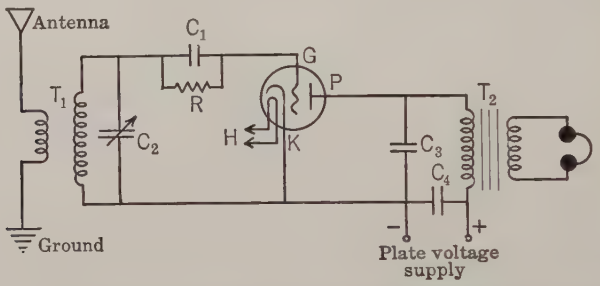


Fig. 432. Typical circuit showing a triode used as a grid-leak detector in a radio receiver.

brings the voltage of the desired frequency up to a maximum value and suppresses voltages at other frequencies. The desired signal is applied across the grid and cathode of the tube through the parallel combination of the condenser C_1 and the resistor R. The resistance R is called the grid leak and usually has a value of from 1 to 5 megohms; the condenser C_1 is usually of about 0.00025 microfarad capacitance.

At radio frequencies, the reactance of the condenser C_1 is very low (about 600 ohms at 1,000,000 cycles) so the alternating grid potential is practically that produced across the transformer secondary. During the half cycles when the grid is negative, the plate current merely follows the variation of grid voltage as in an amplifier. The action is quite different, however, when the grid becomes positive, which it can do because there is no separate bias applied to the tube. During each positive half cycle, the grid attracts electrons and a current hence flows in the grid circuit. From this, we see that the radio-frequency voltage actually causes a rectified current to flow in the grid circuit, with the grid and filament acting as the two elements of a diode. We found from a study of Fig. 431, however, that the average value of a modulated wave which

has been rectified varies at the modulating frequency and not at the carrier frequency. In the detector of Fig. 432, the condenser C_1 has been chosen to conduct the carrier frequency easily, but at the modulating frequency its reactance is much higher. Hence the rectified current in the grid circuit tends to flow through the grid leak R, and in doing so it causes a voltage drop at the desired modulating frequency. Although the grid current is small, the resistance R is large, so the voltage drop is appreciable.

We have thus produced in the grid circuit by rectification, a voltage which is varying at the audio-frequency of the desired sound wave. This voltage now becomes the grid voltage of the triode, and is amplified and causes an audio-frequency variation of plate current.* Of course, the original radio-frequency voltage is also amplified and causes a radio-frequency variation of the plate current, but this component of current is filtered out of the output as shown below.

The plate current of the tube passes through the primary of the iron-core transformer T_2; the resulting output is received by phones connected across the secondary of the transformer. If a loudspeaker is to be operated, it is usually necessary to amplify the signal further by additional amplifier stages, similar to that shown in Fig. 427, before sufficient power can be obtained.

In passing the plate current through the primary of a transformer, we can neglect the steady component of current because it has no effect on the secondary voltage. We do not wish to have the radio-frequency component of current appear in the output, however, so the condenser C_3 in Fig. 432, is connected between plate and cathode to by-pass it. The inductive reactance of the transformer is high at the radio-frequency, and the capacitive reactance of C_3 is low. The high frequency therefore tends to go through the condenser rather than the output transformer. At the same time, the condenser C_3 is made small enough that its reactance to the audio-frequency is higher than that of the transformer, so that most of this current goes through the output transformer. The condenser C_4 in Fig. 432, across the plate power supply is similar to the one shown in Fig. 427. Its purpose is to keep the audio-frequency out of the power supply.

* By properly adjusting the values of C_1 and R, the above action takes place, and in addition a small charge is maintained on the condenser plates which serves to give the grid a negative bias. This improves the action of the tube as an amplifier and still permits the grid circuit to act as a diode rectifier.

283. Vacuum Tube Oscillators. An ordinary triode can be made to act as a generator of continuously alternating current by a connection of the form shown in Fig. 433.* In this connection, it will be noted that the plate current flows through the part of coil L_1 which is included between terminals 1 and 2. Any change in plate current will induce a voltage in the remainder of the coil between terminals 2 and 3. This induced voltage is applied to the grid of the tube and is amplified. If the phase relations are such that this amplified voltage causes plate current to flow and thus induces a grid voltage aiding the grid voltage already existing, the device will oscillate. When the connections are correct, the frequency of oscillation adjusts itself so that the phase relations are correct to maintain the oscillation. We call this a feed-back circuit because the plate output is coupled so that it produces a voltage in the grid circuit. The action really consists of a continuous series of surges of power between the grid and plate circuits at the frequency of oscillation.

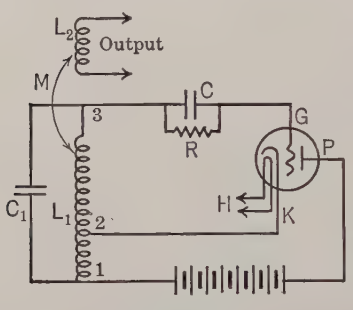


FIG. 433. Circuit connections of an oscillator using a triode as a source of high frequency oscillations.

In order to obtain a sine-wave output, the parallel circuit consisting of C_1 and L_1 is adjusted for resonance at the oscillation frequency. If a circuit of this type is "excited," as for example, by charging the condenser and letting it discharge through the inductance, the current in the circuit will oscillate at the resonant frequency, but the amplitude of the oscillations will decrease with time. By connecting this oscillating circuit with a tube as shown in Fig. 433, the oscillating circuit receives an impulse at each cycle, and the oscillations continue at a constant amplitude instead of decreasing. We can then couple an output coil L_2 so that the oscillating current in L_1 induces an output voltage by mutual induction. By amplifying this output voltage with an ordinary amplifier, considerable power can be developed at high frequencies.

The action of the grid leak and condenser in the grid circuit is very similar to that found in the grid-leak detector. Due to the flow of grid current during each positive half cycle, a series of

* Many types of circuits are used in building oscillators. For a more complete discussion, see "Radio Engineering," by F. E. Terman.

surges is set up in the plate circuit, and these are fed back to the grid circuit to maintain the oscillations. The average voltage across the grid leak R also supplies a negative grid bias which makes the tube act more efficiently as an amplifier.

284. Multi-Grid Vacuum Tubes. Many types of modern vacuum tubes are built with more than one grid. The first additional grid introduced is placed in the space between the plate and the first grid, usually called the control grid. A grid thus located is known as a **screen grid**; it is introduced in order to overcome a disadvantage present in the ordinary triode.

A triode consists of three metallic elements located near one another in space. There is, therefore, capacitance between each pair of elements. These capacitances are very small, as is shown by the following values for the type 27 tube.

Capacitance from grid to plate = 3.3 micromicrofarads.
Capacitance from grid to cathode = 3.5 micromicrofarads.
Capacitance from plate to cathode = 3.0 micromicrofarads.
1 micromicrofarad = 10^{-12} farad.

The trouble in using a triode arises in the small capacitance between the grid and the plate. If the plate voltage is varying at a high frequency as in a radio receiver, a varying voltage is set up on the grid due to the varying plate voltage. If the phase relations are correct, the tube will oscillate and produce distortion of the signals being amplified. In order to prevent this, older types of receivers were equipped with various complicated neutralizing circuits. These circuits introduced voltages into the grid circuits which were equal in magnitude and opposite in phase to the voltages introduced through the grid-plate capacitances, and thus prevented oscillation.

It was found possible to eliminate the oscillation and thus simplify the circuits by the introduction of a screen grid. Effectively, the screen grid reduces the grid-plate capacitance to a value low enough to prevent oscillation. In the type 24 tube, for example, this capacitance is only 0.007 micromicrofarad, which is very low compared to the 3.3 micromicrofarads in the type 27 tube.

The screen grid is operated at a positive potential slightly less than that of the plate, so it draws some current, but because of its open construction, most of the electrons pass through it and reach the plate. The potential of the screen grid is practically constant even though its current varies with the control grid voltage. This

is due to the fact that there is no load impedance between the screen grid and the source of voltage and hence no variable impedance drop to cause a variation of screen grid potential. Since the screen grid is much nearer the control grid than the plate, it has a much greater effect on the control grid potential than the plate. By maintaining a constant potential on the screen grid, therefore, the control grid is "shielded" or "screened" from the effect of the varying plate potential. This type of tube is commonly called a **tetrode,** since it contains four elements.

We have previously stated that since the plate of a vacuum tube is cold, no electrons are emitted from it. This is true as far as thermionic emission is concerned. In the operation of a vacuum tube, however, the electrons attracted from the filament to the plate are accelerated to high velocities. If these electrons strike the plate with sufficient speed, they may knock free electrons of the plate material out into space. The action is very similar to the splash which occurs when water is struck by a stone.

The electrons thus knocked out of the plate are said to be emitted by secondary emission, because this effect is secondary to the emission from the heated cathode. In ordinary diodes and triodes, no harm is caused by this emission because the secondary electrons are immediately drawn back to the plate by its positive potential. In the tetrode, however, the screen grid is at a positive potential very nearly equal to the plate potential. The screen grid therefore attracts these electrons, especially if the plate potential during its variation becomes less than the screen grid potential. If any appreciable number of the electrons produced by secondary emission are drawn over to the screen grid, the plate current is decreased and distortion occurs. In order to prevent distortion in a tetrode from this cause, it is necessary to limit the plate current variation so that the plate is always more positive than the screen grid and thus recovers most of its emitted electrons.

This limitation can be removed by the introduction of another grid, called the **suppressor grid,** between the screen grid and the plate. The suppressor is ordinarily connected to the cathode and it thus introduces a strong negative potential in the region near the plate. This potential drives the secondary electrons back to the plate and thus prevents them from reaching the screen grid. The suppressor is also built with open construction so that it does not interfere with the normal passage of electrons from filament to plate.

A tube of this type has five elements, and is called a **pentode.** The five elements are: cathode, control grid, screen grid, suppressor grid, and plate. Figure 434 shows a typical amplifier stage using a pentode; the negative grid bias required for the control grid is supplied by means of a bias resistor used as in Fig. 427.

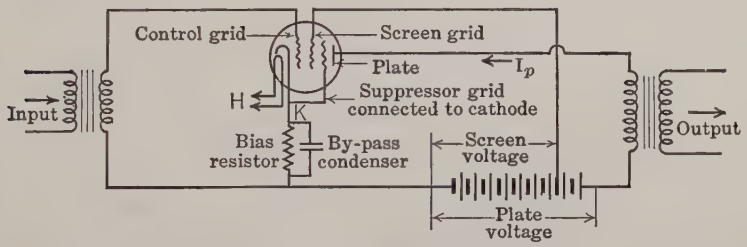


FIG. 434. A pentode amplifier circuit.

285. The X-Ray Tube. The modern X-ray tube consists of only two elements, a plate and a heated filament. In spite of the formidable appearance of the tube itself and its associated apparatus, the tube is merely a diode of special design, and its operation in a circuit is the same as any other diode.

Figure 435 shows the construction of the Coolidge X-ray tube. A heavy metallic rod supports the filament and acts as one filament lead; the other filament lead passes through the center of the support. The filament itself is in the form of a spiral mounted inside an open cylinder on the end of the supporting rod. This cylinder is called the focusing shield; it is at the same potential as the filament and therefore repels the electrons as they leave the filament. It is shaped, however, so as to prevent the emitted electrons from spreading out in space, and it thereby causes the electron stream to hit the plate in one spot.

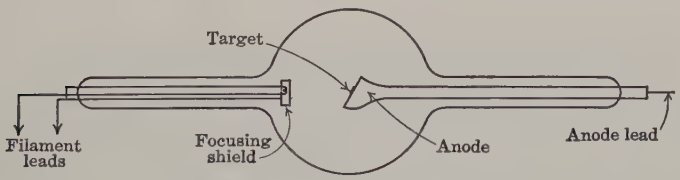


FIG. 435. A Coolidge X-Ray tube.

The plate or anode of the tube is in the form of a heavy rod of copper, with the end toward the filament flared out in a bulb of

enlarged diameter. This bulb is then cut off at an angle so that a flat surface is seen when viewed from the filament. In the middle of this flat surface, directly opposite the filament, there is imbedded a piece of tungsten, or an alloy of tungsten and other metals. This is the " target " upon which the electrons coming from the filament are aimed or focused. Practically all the electrons reaching the plate strike this target, and we shall see later that the target is the source of the X-rays.

An electron passing from filament to plate under the influence of the applied voltage is accelerated throughout the entire distance, and even with a moderate voltage applied, its speed is enormous by the time it reaches the plate. The final speed increases as the voltage is increased, and for voltages up to about 10,000 volts, the speed can be computed from the equation

$$v = 0.595 \times 10^8 \sqrt{V}$$

in which

v = velocity in **centimeters per second**
V = voltage between plate and filament in **volts.**

Using this equation, we find that in a tube operating at 10,000 volts, the electrons reach the plate with a speed of about six billion centimeters per second. Expressed in more familiar units, this speed is about 37,000 miles per second, which is sufficient to carry the electron one and one-half times around the earth in one second.

Most X-ray tubes are operated at voltages between 20,000 and 100,000 volts, with a few operating at as high as 800,000 volts. At these voltages, the electron speeds are even greater, and at the higher voltages, the speed approaches the speed of light (186,000 miles per second).

We would expect that an electron traveling at such an enormous speed would cause an unusual disturbance upon hitting the target. The action is quite complicated, but the general effect is that the atoms of the metal which forms the target are thrown into a state of violent oscillation. These oscillations, which are at very high frequencies, give rise to the radiation known as X-rays.

The penetrating power, or " hardness," of the X-ray depends upon the frequency of the oscillations set up in the target. The frequency increases as the voltage is increased. An ordinary vacuum tube used in a radio receiver produces some X-rays, but these rays are very " soft " because the tube voltage is relatively low. Never-

theless, in some special vacuum-tube applications, the X-rays produced are a disturbing factor and must be eliminated by operating the tube at a lower voltage.

Although an X-ray tube is primarily a diode, its operating conditions are quite different from those of most diodes. In order to obtain high speed electrons, it is necessary to maintain a high voltage across the tube. This can only be done by operating the tube at its saturation current, so that all the electrons from the filament are drawn over to the plate. For a given filament temperature, therefore, the current through the tube is at the saturation value, and cannot be increased by increasing the voltage. The current through the tube is actually controlled by adjusting the filament current. This controls the filament temperature and thus controls the emission rate. It is necessary to control the current through the tube because although the hardness of the X-rays depends on the voltage, the amount of radiation depends on the current.

It is customary in modern practice to supply the plate voltage of an X-ray tube from a separate diode rectifier tube, or from two diodes connected to supply a full-wave rectified current. Since the X-ray tube is also a diode, it will also rectify and thus supply its own d-c plate voltage. This method is sometimes used for the sake of compactness and lower cost.

A high-speed electron has considerable kinetic energy just before it hits the plate of a diode. When the electron is stopped by the plate, its kinetic energy reappears, partly in the form of X-rays, but mostly in the form of heat generated in the plate. The rate of supplying heat to the plate, in watts, is given by the product of the plate voltage times the plate current. In X-ray tubes and large radio tubes the power which goes into heat in the plate may be very large and some provision must be made to keep the plate cool. One purpose of the heavy, copper anode support in the X-ray tube of Fig. 435 is to absorb and carry away the heat generated in the target. If this is not done, the target temperature will increase to the point where it also emits electrons, and the tube will conduct in both directions. In some designs of X-ray tubes the anode support is made of a continuous tube through which water is circulated to carry away heat.

Prob. 8–15. An X-ray tube draws a saturation current of 6 milliamperes when a steady voltage of 8500 volts is impressed between the filament and the anode. What is the velocity of the electrons reaching the target?

Prob. 9–15. What power is used in heating the anode of the tube in Prob. 8–15?

Prob. 10–15. If the voltage across the tube in Prob. 8–15 is raised to 15,000 volts, what power is used in heating the anode?

Prob. 11–15. A type 27 triode draws a plate current of 5 milliamperes when operated with a plate voltage of 180 volts, and a grid bias of −13.5 volts. What power is lost in heating the plate of this tube?

Prob. 12–15. What is the speed of the electrons striking the plate of the type 27 tube of Prob. 11–15?

286. Photoelectric Effect. Photoelectric Cells. In a thermionic tube, the electrons which are the means of conducting current between the filament and the plate are produced by emission from the heated filament. The free electrons of the metal were given heat energy, which increased their velocities and thus allowed some of them to escape into the space around the filament.

Ordinary light waves are also a form of energy. If light waves are allowed to fall on the specially prepared cathode of a tube known as a **photoelectric cell,** some of the free electrons of the cathode will receive sufficient energy from the light waves to permit them to escape from the cathode. These electrons can then be drawn over to an anode carrying a positive potential just as the emitted electrons in a thermionic tube are carried over to the plate. Of course, the number of electrons obtained by photoelectric emission is much smaller than the number emitted from a heated filament, and the current is therefore very small, but by passing this current through a high resistance and amplifying the voltage drop, an output of magnitude sufficient for many practical purposes is obtained.

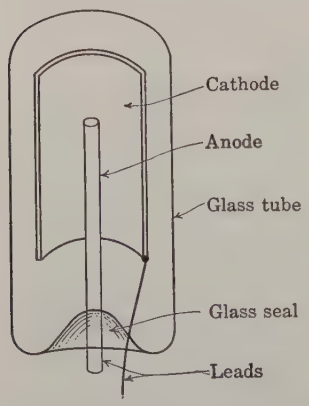


Fig. 436. A photoelectric cell. Electrons are emitted by the action of light on the cathode.

A common type of photoelectric cell is illustrated in Fig. 436. The cathode is a piece of silver bent into a section of a cylinder. The active coating is a very thin layer of a metal such as caesium, potassium, lithium, etc. These metals are all known as alkali metals and have very pronounced photoelectric properties. The anode is simply a piece of wire

placed near the cathode surface but located so that the light is not obstructed.

When a voltage is impressed across the cell, with the anode positive, a current flows if light reaches the surface of the cathode. For a **constant voltage applied,** the **current is proportional to the intensity of the light** on the cathode. If the light intensity is kept constant and the voltage is increased, the current increases until all the electrons emitted from the cathode are drawn over to the plate, after which no further increase in current can be obtained unless the light intensity is increased. Thus a photoelectric cell has a saturation current similar to the saturation current of a thermionic tube. The value of the saturation current can be changed by changing the light intensity. Figure 437 shows the current-voltage curves of a

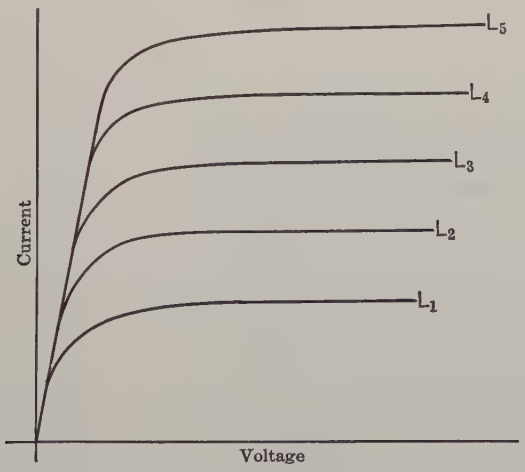


FIG. 437. The relation of photoelectric-cell current to voltage at different values of light intensity.

photoelectric cell for a number of values of light intensity; L_1 represents the lowest value of light intensity and L_5 the highest. Note the similarity between these curves and those for the thermionic tube in Fig. 414.

Since the current through a photoelectric cell is proportional to the intensity of the light falling on its cathode, the cell is an important device for the measurement of light intensity. Within recent years, however, its use has been extended to many fields outside of laboratories. Practically the entire motion picture industry now depends upon photoelectric cells for the reproduction of the speech

and music which is recorded on the "sound track" of the film. The two types of sound tracks now in common use are shown in Figs. 438 and 439. Figure 438 shows the variable-density sound track in which the instantaneous amplitude of the sound wave is represented by the density of the track at each point. The type of sound record shown in Fig. 439 actually shows the amplitude of the sound wave at each instant by the width of the black part of the track; this is known as a variable-area sound track.

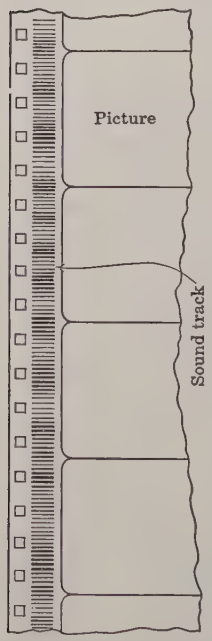


FIG. 438. A variable-density sound track on motion picture film.

In either case, the sound is reproduced by passing a narrow beam of light through the sound track of the moving film, and allowing the light that finally gets through to strike the cathode of a photoelectric cell. The variations of the photoelectric cell current correspond to the variations in the light which gets through the track. These current variations are made to control the grid voltage of the first tube of an amplifier, which supplies the loud-speakers located behind the screen.

FIG. 439. A variable-area sound track on motion picture film.

Figure 440 shows a type of circuit which may be used to couple the photoelectric cell to the input of an amplifier. The plate battery supplies current to both the thermionic tube and the photoelectric cell. Starting from the positive terminal of the plate battery, the photoelectric-cell current, I_a, goes through the cell from anode to cathode and returns through the resistance R to the negative side of the battery. Since the photoelectric-cell current is flowing down through the resistance R, the end of R nearer the grid of the amplifier tube is positive. A grid battery is therefore introduced as shown to

oppose the average voltage drop in R and thus maintain a negative grid bias. If the light reaching the photoelectric cell varies, the

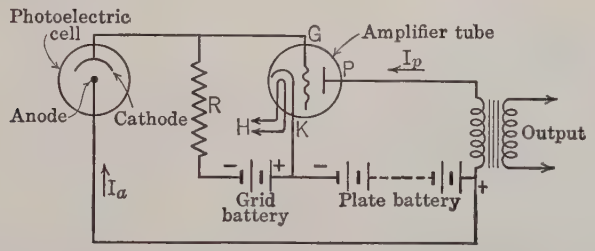


FIG. 440. Circuit for controlling a triode amplifier by means of a photoelectric cell.

current through the cell will vary, and the resulting variation of the voltage drop across R will be amplified by the triode.

Photoelectric cells also find application in many types of control devices. By sending a beam of light across the path of a series of moving objects and into a photoelectric cell, we can count the number of pieces that go by a point, detect incorrect location or dimensions, sound an alarm if a vital part is missing, etc. We use them to turn on street lamps at sundown and turn them off at sunrise. A person approaches a door and interrupts a light beam placed across his path; by the time he reaches the door a mechanism operated by a photoelectric cell has opened the door for him. The list of useful applications of this device is already long and is growing every day. Figure 441 shows a commercial type of photoelectric control unit. The tube in the

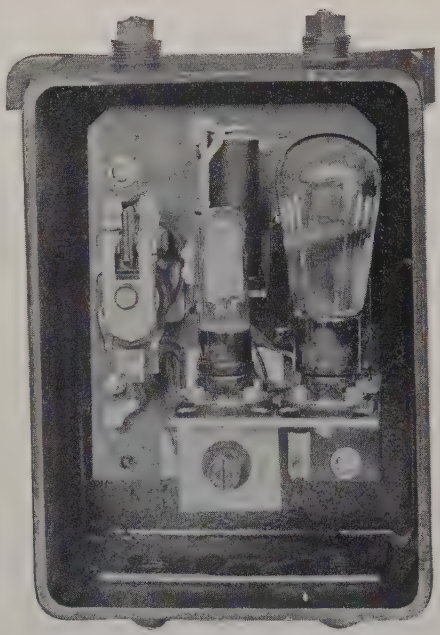


FIG. 441. Photoelectric control unit. Relays are operated by means of the photoelectric cell and amplifier tube. *General Electric Co.*

center is the photoelectric cell; its thermionic amplifier tube is at the right. The relays shown are operated by the interruption of a light beam and these relays are heavy enough to control large amounts of power.

Fig. 442. Weston Photronic Cell. *Weston Electrical Instrument Corporation.*

Figure 442 shows another device which operates from light energy. The form shown is called a Photronic Cell.* This cell has the remarkable property that no external source of power is required; with a microammeter connected across its terminals, current readings are obtained merely by allowing light to strike the cell. Essentially, the cell consists of a thin metal disc on which there is deposited a film of light-sensitive material. Contact is made with the film and with the metal disc, and these two leads go directly to the meter. The entire assembly is at atmospheric pressure instead of being in an evacuated bulb. Although no satisfactory explanation of its action is available, it is being widely used in illumination meters and photographic exposure meters.

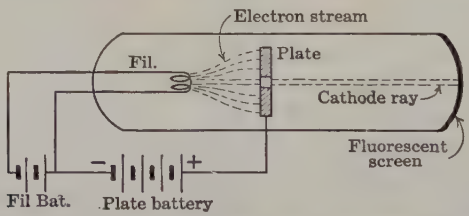


Fig. 443. Electrons hitting the hole in the plate pass through and form the cathode ray.

287. The Cathode-Ray Tube. The simplest form of cathode-ray tube is that shown in Fig. 443. Note that it is a diode, but that the plate has a hole cut through it directly opposite the filament. When a positive potential is applied to the plate, the electrons emitted by the filament are drawn over to the plate at high speed. Most of these electrons will hit the plate

* "Photronic" is a copyrighted name used to designate the photoelectric cells and devices manufactured exclusively by the Weston Electrical Instrument Corp.

just as in an ordinary diode, but a certain number of them will be aimed at the hole in the plate and will travel through the hole. On the other side of the plate, there is no force acting on the electrons and they continue to travel at high speed until stopped by the walls of the tube. There is therefore a beam or ray of electrons coming through the hole in the plate; this ray is called a **cathode ray.**

Cathode-ray tubes are made in the form shown in Fig. 444. The diode arrangement which produces the beam of high-speed electrons (called an "electron gun") is located in the narrow end of the

Fig. 444. A cathode-ray tube. *R.C.A.-Victor Co., Inc.*

tube; in most modern tubes, additional electrodes are provided which permit adjustment of the intensity of the beam and also permit it to be focused so that all the electrons strike a small spot on the wall at the enlarged end of the tube. We observe the point at which the cathode ray strikes the wall by coating the inside of the glass with a fluorescent material. This material will glow at the spot where the cathode ray strikes it, and if we deflect the beam by some means, the spot of light will travel over the end of the tube and thus show the position of the beam at each instant.

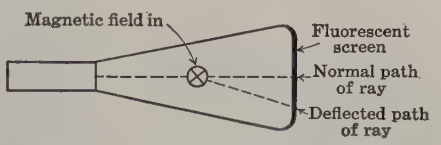


Fig. 445. The cathode ray is deflected by a magnetic field.

The beam of electrons can be deflected in two ways. If we set up a magnetic field perpendicular to the normal path of the ray, it will be deflected as shown in Fig. 445. The explanation of this is that the moving stream of electrons is actually a small electric current and the force acting on the beam of electrons is the same as that acting on a wire carrying current and located in a magnetic field. The direction of the force can be found by applying the left-hand rule (see Art. 133, Chapter IX), but we must remember that the direction of current flow is opposite to that of the electron motion. Thus, in Fig. 445, the electrons move from left to right, and this is equivalent to a flow of current from right to left.

The cathode ray can also be deflected as shown in Fig. 446. A

pair of metal plates V is placed inside the tube and located so that the beam passes between them. If a voltage is maintained across these plates, the negative electrons of the beam are attracted toward the positive plate and the path is thus changed. Another pair of deflecting plates H can be located at right angles to the first pair, and by applying voltage to these, the ray is deflected horizontally as well as vertically.

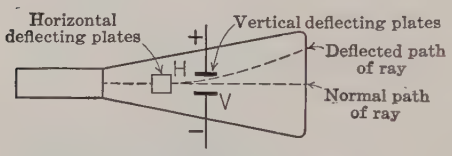


FIG. 446. Deflection of the cathode ray by voltage applied to parallel plates.

By applying variable voltages to both pairs of deflecting plates, the spot of light caused by the beam hitting the fluorescent screen can be made to follow a path which is really a curve of one of the voltages plotted against the other. If we apply a sine wave of voltage on the vertical deflecting plates, and a voltage which varies uniformly with time on the other pair, a sine curve will be traced on the fluorescent screen. The cathode-ray tube is used in this manner in the **cathode-ray oscillograph.** Figure 447 shows a commercial type of cathode-ray oscillograph used for studying the wave shapes of variable voltages and currents.

FIG. 447. A cathode-ray oscillograph. *R.C.A.-Victor Co., Inc.*

The action of a cathode-ray oscillograph in tracing waves can be made clear by a simple analogy. Suppose we represent the cathode-ray beam by a stream of water coming from the nozzle of a hose, and represent the fluorescent screen by the wall of a barn. If we direct the stream of water so that it hits the wall at one spot, we have the equivalent of the undisturbed cathode-ray beam. Now, suppose we move the nozzle rapidly up

and down so that the water "draws" a vertical line on the wall, and then, while we are still moving the stream up and down, move it at a uniform speed sideways. Instead of simply getting a vertical line, we will draw a curve which shows how the nozzle was moved at each instant of time.

In order to get the uniform horizontal motion of the cathode-ray beam, we apply to the horizontal deflecting plates a voltage of the form shown in Fig. 448. With this voltage applied, the beam moves horizontally at a uniform rate until the maximum voltage is reached. It then jumps back to the original position and repeats. If the frequency of this "saw-tooth" wave is made the same as that of an alternating voltage applied to the vertical deflecting plates, a stationary sine wave will appear on the fluorescent screen. Since the motion of the beam across the screen is the same for each cycle, it simply draws the same curve over and over again, and the resulting wave apparently stands still.

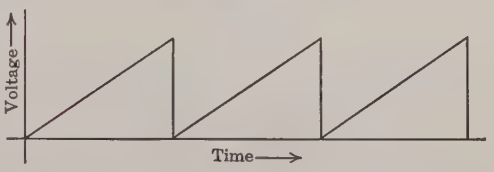


FIG. 448. Form of voltage wave applied to horizontal deflecting plates to secure uniform time axis on cathode-ray oscillograph.

288. Television. One of the most promising of the recently proposed systems of television is that described by V. K. Zworykin. Figure 449 shows the elements of the system in simplified form. The operation depends upon the use of a tube called the Iconoscope (meaning "image-observer"), which is essentially a cathode-ray tube containing a special photosensitive surface instead of a fluorescent screen. This surface is a mosaic of millions of droplets of a caesium-oxide-silver mixture deposited on a thin sheet of mica. The back of the mica sheet has a continuous coating of metal on it, called the signal plate.

A picture to be transmitted is focused on the mosaic plate by means of a lens. Since each droplet on the plate is composed of photoelectric-cell cathode material, electrons will be emitted from the plate in proportion to the intensity of light at each point. These electrons are drawn off to a positively charged metallic coating on the inside of the glass tube. Each droplet on the mosaic plate is made of conducting material, so there is a small capacitance between each droplet and the signal plate. When the emitted electrons are drawn away, each of these condensers is left positively charged with

respect to the signal plate, and the amount of charge on a droplet is proportional to the light intensity falling on it. The voltage between the signal plate and any point on the mosaic surface is therefore proportional to the intensity of light at the point, and the image is represented by a variable voltage distribution on the surface of the plate.

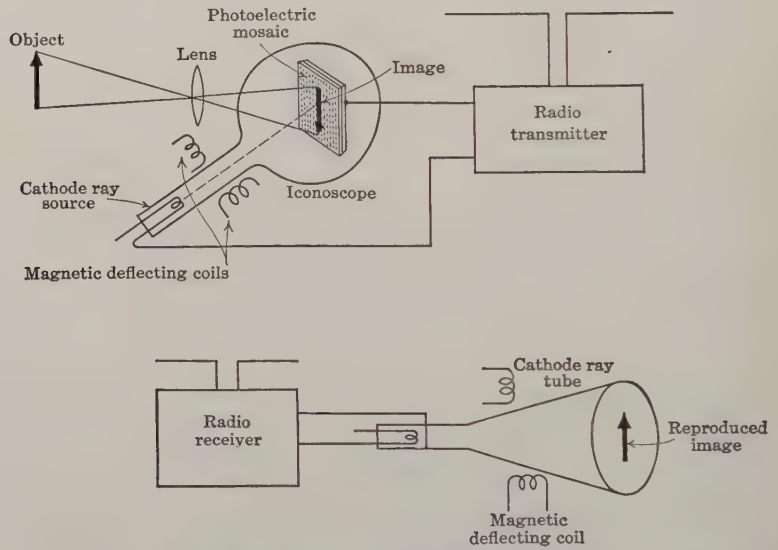


FIG. 449. Simplified diagram of a television transmitter and receiver.

If the cathode-ray beam is now swept horizontally and vertically over the surface of the mosaic, a process called "scanning," the electron current in the beam will restore the electrons lost by photoelectric emission, and the amount of current which flows in the beam will be proportional to the voltage at each point on the plate. Thus the variation in light intensity on the plate is now represented by the variation of current in the scanning beam. This variation can be amplified and transmitted to the receiver.

At the receiving end, a radio receiver picks up the transmitted voltage variations and these are amplified and applied to electrodes in a cathode-ray tube so as to control the current in the cathode-ray beam. The beams in both the iconoscope and the receiving tube are swept by pairs of magnetic deflection coils. Special synchronizing signals are transmitted along with the picture signals so that the beams at the transmitting and receiving ends move together at all times. Since the intensity of the receiving beam is changing exactly

as that of the transmitting beam, and the two are always in the same position, the variation of light intensity on the fluorescent screen will reproduce the variation of light intensity on the mosaic plate of the iconoscope, and the image is thus transmitted. Since the beams are swept very rapidly in both directions so as to scan the entire picture area in about $\frac{1}{24}$ second, a continuous series of images can be transmitted and the motion of the object is also reproduced.

289. Gaseous Conduction. Ionization. We have thus far considered only those tubes which depend for their operation on the conduction of electricity between anode and cathode by means of a stream of electrons evaporated from a heated cathode surface. These tubes are thoroughly evacuated, and in many of them chemical " getters " are flashed inside the tube after pumping is finished, in order to rid the space of all traces of gas.

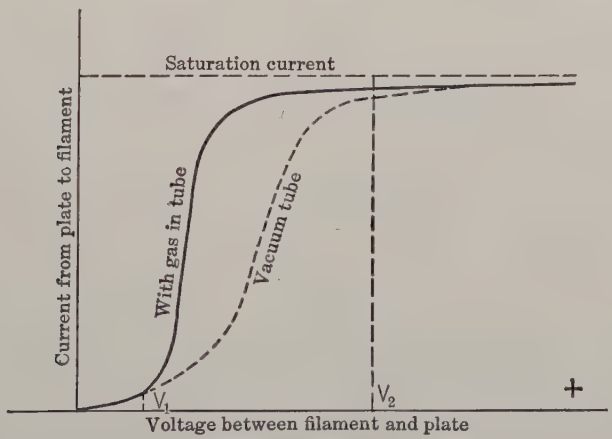


FIG. 450. The voltage-current curve of a diode is changed by the addition of gas to the tube.

It is now necessary to consider the effects of introducing small quantities of gas into these tubes. The gases commonly used are the inert gases argon, neon, and helium, and mercury vapor.

If a small amount of one of these gases is put in an ordinary diode, one of the first things we notice is that the characteristic voltage-current curve of the tube is altered. Figure 450 shows the curve before and after a small amount of gas is introduced. Note that until the voltage V_1 is reached, the two curves are identical. The curves then diverge until after the voltage V_2 is reached when they

again become identical, and the saturation current of the tube is the same in both cases.

We can explain these effects as follows. The gas in the tube is composed of molecules which contain positive nuclei and negative electrons. If a gas molecule is struck by one of the rapidly moving electrons on its way to the plate, the result may be that one of the electrons of the molecule is detached. The electron thus produced is exactly like the one which came from the filament, and becomes part of the stream of electrons traveling toward the plate. It may reach the plate directly, or it may in turn strike another molecule on the way and produce another free electron. This action is going on all through the space between the filament and the plate.

The molecule of gas which has lost an electron is now left with an excess positive charge, and we therefore call this molecule a **positive ion.** The process of producing ions in a gas is called **ionization.** Since the ions produced are positive in sign, the action of the applied voltage on them is opposite to the action on negative electrons, and the ions therefore travel toward the filament instead of the plate. The current through the tube is therefore composed of two components: first, the stream of negative electrons moving toward the plate; second, the stream of positive ions moving toward the filament. These are both equivalent to a positive current from plate to filament, so the two components must be added to find the total current.

It was noted in Fig. 450 that the behavior of the gas tube was the same as the vacuum tube until the voltage V_1 was reached, and after that the current in the gas tube increased more rapidly with further increase of voltage. This is due to the fact that ionization cannot take place until the free electrons have enough kinetic energy to dislodge one of the electrons in the molecule. Since the speed of the electron, and hence its energy, depends upon the voltage across the tube, we see that there must be a certain minimum voltage applied before ionization can occur. This value of voltage is called the **ionization potential,** and its value depends upon the gas used. The ionization potential for argon is 15.7 volts, and for mercury vapor it is 10.4 volts.

At first sight, it might seem that the increased current in a gas tube for voltages above the ionization potential was due to the additional number of electrons and positive ions produced by collision. There is, of course, an increase of current from this cause, but the increase is so small that it is inappreciable. This is proven

by the fact that the saturation current of the gas tube is the same as that of the vacuum tube. The saturation current is determined by the total number of electrons emitted from the filament and this number depends only on the material, size, and temperature of the filament. If, in addition, a large number of electrons is produced by ionization of the gas, the current at saturation must be higher; the fact that we cannot measure the difference in saturation currents in the two cases indicates that the number of electrons produced by ionization is small compared to the number of electrons emitted.

At values of voltage between the ionization potential and the voltage which produces saturation, the current in the gas tube is higher than that in the vacuum tube because of the neutralization of space charge by the positive ions. It will be recalled that a cloud of electrons, called the space charge, surrounds the hot filament, and this negative charge tends to prevent electrons from leaving the filament. The positive ions on the way to the filament mix with the cloud of electrons and thus neutralize the effect of the space charge. This allows a given plate voltage to draw more electrons away from the filament, so the current increases as shown in Fig. 450.

If additional gas is introduced beyond the amount required to neutralize space charge, the characteristics obtained are very much different from those described above; this type of gaseous conduction will be described in Art. 293.

290. Gaseous Rectifiers. The hard-vacuum diode is used to a considerable extent as a rectifier of high-voltage waves. It is very useful in these applications, particularly because of its ability to stand the high reverse voltage which is across the tube during the half cycle when current does not flow. A diode with gas introduced will not stand as high a reverse voltage because the gas tends to ionize with voltage applied in the reverse direction, and the reverse voltage must be kept at a lower value in order to prevent appreciable conduction in the reverse direction. However, for moderate voltages, the gas rectifier is more efficient than the hard-vacuum tube because its internal voltage drop is smaller. For the same reason, the voltage regulation at the output is also better when using a gas rectifier.

These rectifiers are used extensively to convert alternating-current power to direct-current power for use in the vacuum-tube circuits of radio receivers, amplifiers, etc. Figure 451 shows a conventional type of circuit, using a full-wave rectifier, and passing its output

through a " smoothing " filter in order to remove the fluctuations and secure a steady, d-c output voltage. In the smoothing filter, the inductances pass the d-c component of the rectified current, but their reactance to the fluctuating component is high. The condensers will not pass the d-c component of current, but their reactance to the fluctuating component is low. In operation, therefore, this circuit allows the d-c component of current to flow into the load,

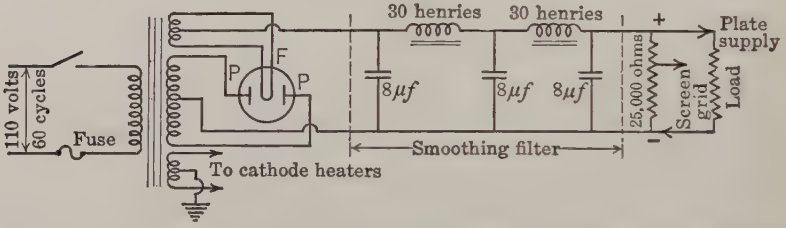


Fig. 451. Full-wave rectifier and smoothing filter for converting alternating current to direct current.

but blocks the fluctuating part of the current and sends it back to the tube through the condensers. Across the output, a tapped resistor is connected which permits us to obtain several values of voltage for use on plates and screen grids; this resistance is called a voltage-divider.

Sometimes the output of a rectifier can be used directly without smoothing, as in the operation of motors, relays, and similar equip-

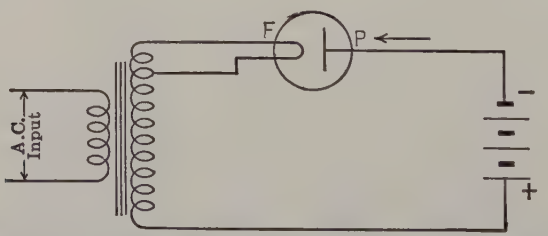


Fig. 452. Half-wave rectifier circuit for battery charging.

ment from rectified alternating current. A more common application is in the charging of storage batteries as shown in Fig. 452. The plate of the rectifier is connected to the negative pole of the battery and the open end of the transformer secondary to the positive pole. A few turns of the transformer secondary are used to supply filament voltage. Note that the current must flow from plate to filament in the rectifier and it hence flows from the positive

pole to the negative pole inside the battery. This is the direction of current required to charge the battery. One of the tubes used for this purpose is a gas diode called a "Tungar." The name comes from the use of **tungsten** in the filament and **argon** as the gas used to neutralize space charge. Figure 453 shows a rectifier of this type provided with two tubes so that two sets of batteries can be charged from the same unit. Taps are brought out from the transformer to permit variation of the charging rate and also to provide for charging from one to six batteries in series in each output circuit.

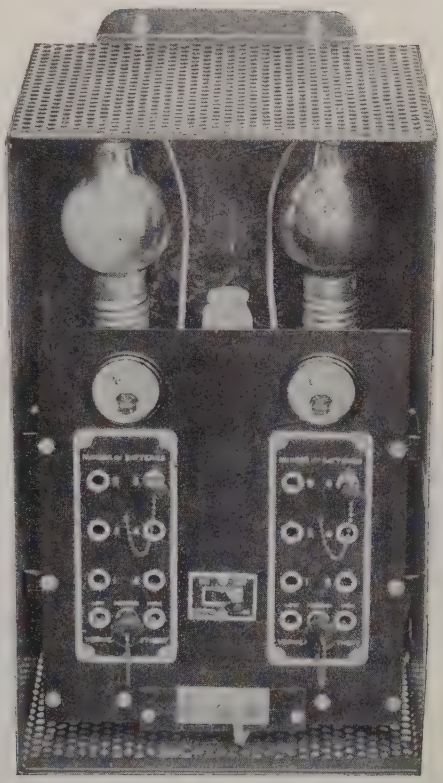


FIG. 453. A "Tungar" battery charger. *General Electric Co.*

A more recent type of rectifier for battery-charging service is the copper-oxide rectifier illustrated in Fig. 454. In this device, the rectifying element is a copper disc on which a film of copper oxide has been formed by heating. If metal plates (usually lead) are pressed against the two opposite faces of this disc, it is found that current flows more readily in the direction from the copper oxide to the copper than in the opposite direction, and the device will therefore rectify. In practice, a number of these discs are arranged in series-parallel circuits so as to secure sufficient voltage and current capacity. They are also provided with metal fins between each pair of discs. These fins, shown in Fig. 454, provide a large radiating surface for the dissipation of heat.

291. Mercury Arc Rectifier. Mercury vapor is frequently used for the neutralization of space charge in diodes as described above. This is accomplished by merely placing a few drops of mercury in

the evacuated space. Under these conditions, the mercury diffuses through the space in the form of a vapor and acts just like any other gas.

In the mercury arc rectifier, the mercury is the source of emitted electrons as well as positive ions. The heated filament is replaced by a pool of mercury as shown in Fig. 455. When this tube is conducting, most of the current through the tube is concentrated on a small spot on the surface of the mercury pool. The heat gen-

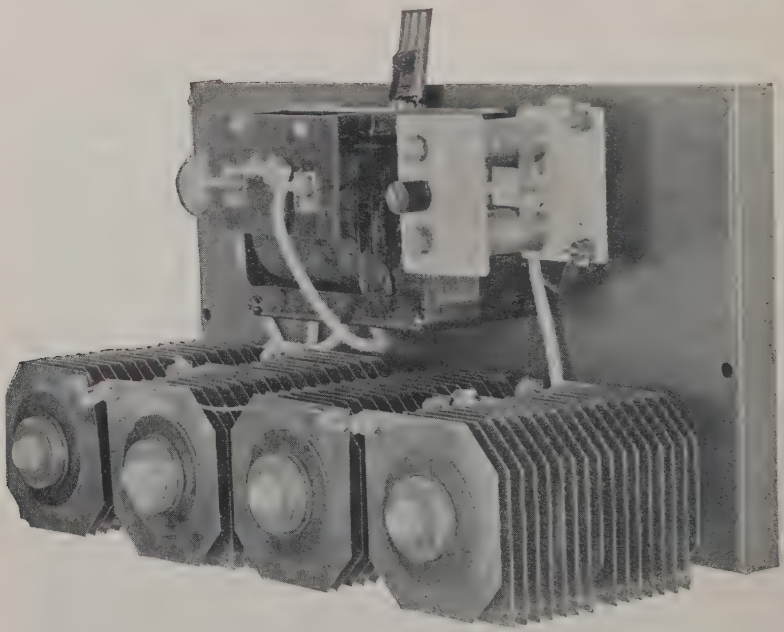


Fig. 454. A copper-oxide rectifier. *General Electric Co.*

erated at this spot cause profuse emission of electrons from the mercury so that this tube is capable of conducting a very large current. Since the electrons necessary for conduction are produced only at the cathode hot spot, this tube is a rectifier. It must be used as a full-wave rectifier, however, for if the discharge stops due to reversal of voltage, the emission of electrons stops. Even with the full-wave connection shown in Fig. 455, it is necessary to use reactors in the load circuit in order to keep the current flowing through the tube for a short time after the alternating voltage on one side of the tube reaches zero. This maintains the discharge until the voltage on the other side of the tube reaches a value high

enough to continue the flow of current. The reactor in each branch is merely an inductance coil wound on an iron core. Any tendency for the current in this reactor to decrease sets up an induced voltage in a direction to continue the flow of current.

Since the emission of electrons depends upon the flow of current through the tube, special means must be provided for starting the flow. This is done by the use of a starting anode, also shown in Fig. 455. The switch S is closed, and the tube is rocked until the mercury in the cathode arm of the bulb flows over and touches the mercury in the starting anode. The current which then flows is limited in value by the resistance R. When the tube is restored to its upright position, the mercury contact breaks, causing hot spots at the two mercury surfaces and thus liberating electrons. These electrons are drawn over to whichever running anode (A_1 or A_2) happens to be positive, and the main discharge then continues between the running anodes and the cathode. After starting, the switch S is opened.

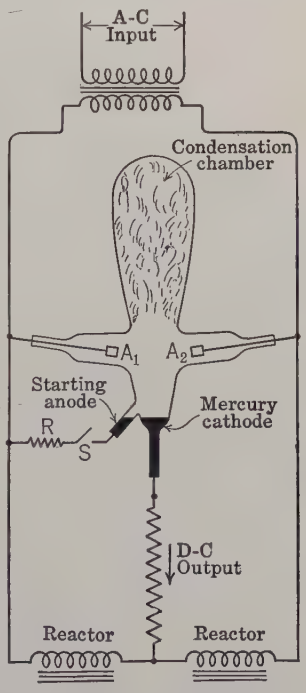


FIG. 455. Full-wave rectifier circuit using a mercury-arc tube.

During operation, the inside of the tube is saturated with hot mercury vapor caused by evaporation from the cathode pool. This vapor rises to the top of the tube where a large condensing chamber is provided. Here the vapor strikes the walls of the tube, which are relatively cool, and is condensed back to metallic mercury; the condensate then runs down the walls of the tube to the cathode pool.

The operation of any gaseous conduction device is usually accompanied by the production of a characteristic light. A mercury rectifier produces a characteristic greenish-blue light which is useful for illumination in certain kinds of work. Mercury tubes are also used as lamps, therefore, and although the same principle of operation applies, the design is entirely different from that of a rectifier because of the difference in purpose.

Due to the fact that a large current can be rectified by means of

the mercury arc, this type of rectifier is being used more and more to replace synchronous converters and motor-generator sets as a means of converting from alternating current to direct current. In the electrification of railways, it is desirable to transmit power over long distances at high voltage, so alternating current is used for transmission. We can then change to direct current for operating the traction motors by using a converter, motor-generator set, or a mercury arc rectifier.

The rectifiers used in this service operate from three-phase alternating current. It is customary to convert this to six-phase power before rectifying. The output then consists of six overlapped sine waves, as shown in Fig. 456, with a resulting output voltage as shown by the heavy line. Note by comparison with Fig. 429 how the amount of ripple in the voltage wave is reduced by this method. Figure 457 shows a modern mercury arc rectifier rated to deliver 3000 kilowatts at 625 volts. Metallic construction is used throughout, and the unit is built with an auxiliary vacuum pump, and a pump for circulating cooling water.

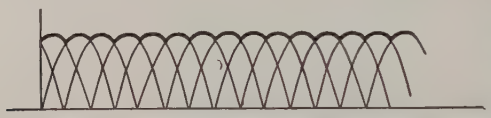


FIG. 456. Output-voltage wave form of a six-phase rectifier.

292. The Grid-Controlled Gas-Filled Thermionic Rectifier. The utility of a gas diode can be greatly increased for some applications by the introduction of a grid between filament and plate similar to the grid found in a vacuum triode. The effect of this grid, however, is quite different from that in the ordinary triode.

If the grid voltage is made sufficiently negative, the tube will not conduct current from plate to filament because the negative grid is more powerful in driving electrons back to the filament than the positive plate is in attracting them. Now, if we gradually reduce the value of grid bias, we will reach a point where there is sufficient attraction for electrons to be drawn to the plate, and a thermionic current flows. If the grid voltage is still further reduced, the electrons move toward the plate at higher speeds. When the average speed of the electrons reaches that corresponding to the ionization potential of the gas in the tube, ionization takes place and the current suddenly jumps to a higher value due to the neutralization of space charge. At the same time, a current will flow in the grid because positive ions are attracted toward the grid as well as the filament. Care must be taken to place resistance in series with the grid and

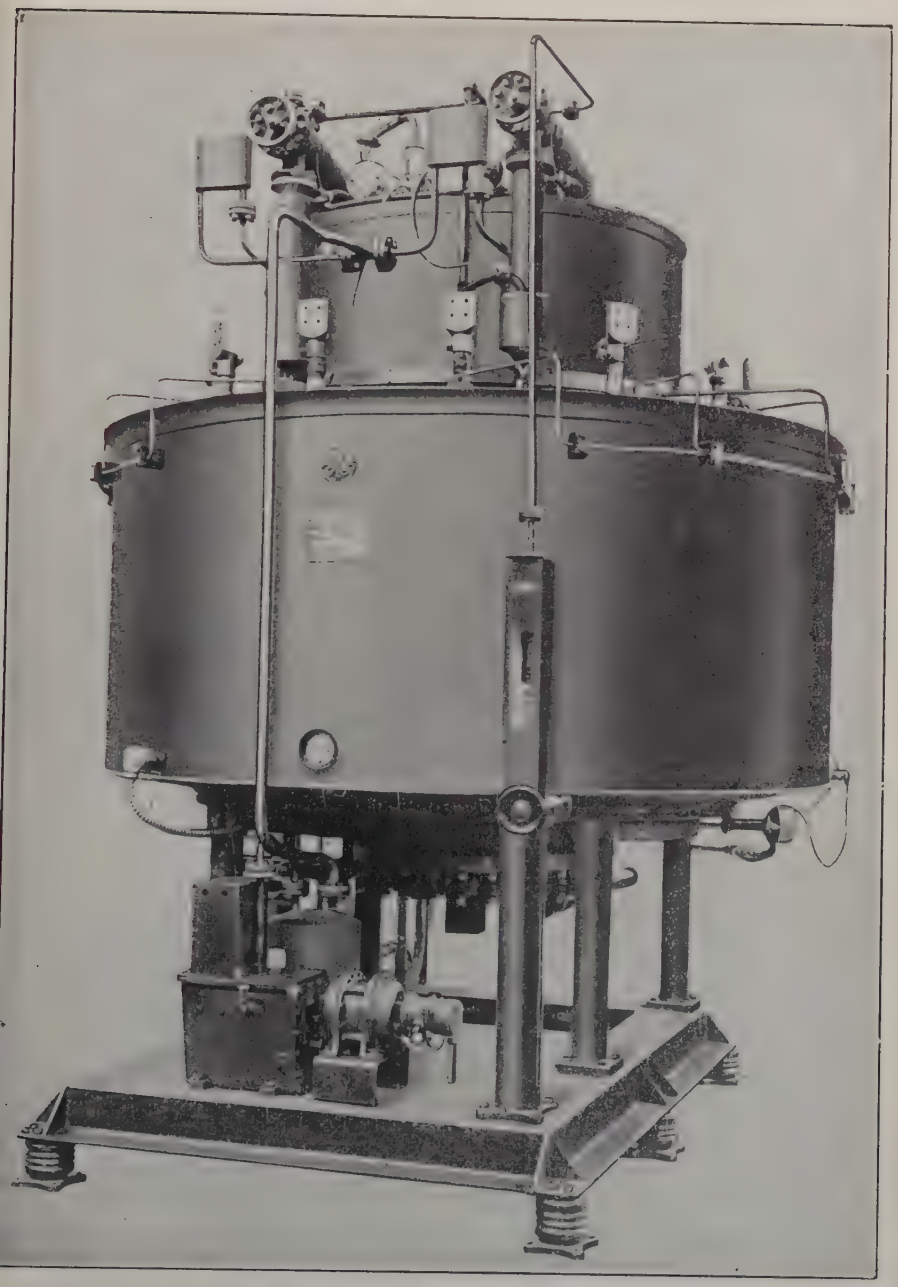


FIG. 457. A 3000-kw, 625-volt, mercury-arc rectifier. *General Electric Co.*

plate of this type of tube to prevent excessive current from flowing and overheating the conductors which pass through the glass seal.

If, after starting the discharge from plate to filament, we now increase the negative voltage on the grid, we find that it has no effect on the plate current. In other words, we can stop the tube from conducting in the first place by maintaining a sufficient negative bias on the grid, and we can start it conducting by reducing the value of this negative bias, but once we allow conduction to start in the plate circuit, the grid is no longer able to exert any control, even though it is made more negative than it was originally. The only way the negative grid can regain control is to reduce the plate voltage to zero, either by opening a switch in the plate circuit, or by using a plate voltage which is alternating and hence goes through zero twice in each cycle.

This peculiar effect is caused by the positive ions which are present due to ionization of the gas. These ions migrate toward the grid wires under the action of the grid's negative potential, and cluster around the grid in a cloud just like the negative electrons of the space charge cluster around the filament in a thermionic tube. The grid wires are therefore surrounded by a sheath of positive charges which neutralize the negative potential of the grid and render it powerless to control the flow of electrons to the plate.

Tubes of this type are available under the name " Thyratron " (General Electric Co.), and " Grid-Glow Tube " (Westinghouse Elec. and Mfg. Co.). They are useful in a wide variety of applications because of the relatively large amounts of power which can be controlled by means of the grid. This is not the direct type of control found in thermionic tubes, but usually depends upon the use of an alternating plate voltage which has a zero value twice in each cycle. If we wish to stop the flow of current, we make the grid negative and as soon as the alternating plate voltage reaches zero the grid regains control.

Figure 458 shows a control panel using Thyratrons for the control of electric welding. Instead of depending upon the judgment of an operator to determine how long a welding current should flow, the use of these tubes permits us to predetermine the number of half cycles during which current will flow. At the beginning of the weld, the grids " trip " their tubes and allow current to flow. An auxiliary circuit times the flow of welding current and at the end of the predetermined time, this circuit puts a negative bias on the

grids of the main control tubes. The welding current stops at the next zero point of voltage.

FIG. 458. Panel for welding control using Thyratrons. *General Electric Co.*

293. Gaseous Discharge Tubes. The large number of gaseous discharge tubes which are now used for illumination, particularly in advertising displays, depend for their operation upon an entirely different way of using ions and electrons to carry current through a

gas. These tubes have no heated cathodes, but are simply pieces of gas-filled glass tubing, bent into the shapes desired, and having electrodes at each end. The characteristic glow is obtained when a high alternating voltage is impressed across the electrodes.

In any gas, there is always a certain number of free electrons. If a voltage is applied across a region containing gas, the free electrons move toward the positive electrode. At low voltages, the only current which flows is that due to the movement of free electrons, and the value of this current depends upon the rate of production of free electrons in the space. These are produced very slowly and the current is hence very small.

As the voltage across the discharge tube is raised, the speed of the electrons becomes higher. Some of these electrons will reach a speed great enough to ionize a gas molecule, and thus additional free electrons can be produced. The pressure of the gas is an important factor in this process, however, because it determines the mean free path of an electron. At any given pressure, the mean free path is the average distance that the electrons travel between collisions with gas molecules. As the pressure is decreased, the molecules of gas become spaced farther apart, and the mean free path hence increases. If this distance is great enough to permit an electron to reach the ionizing velocity, most of the collisions will result in ionization. Furthermore, the electrons produced by ionization will also travel toward the positive electrode and produce further ionization. At the same time, the positive ions move toward the negative electrode, with the result that the total current through the tube is increased due to the increased electron flow and also due to the flow of positive ions.

While this process of producing free electrons is going on, there is also a loss in the number of electrons and of positive ions in the gas. If a positive ion and an electron come close together in the space, they will unite to form a neutral molecule. The molecule thus formed is in violent oscillation and emits light waves of a frequency depending upon the gas used. This results in a glow, the color of which is determined by the particular gas in the tube. On the other hand, a positive ion may dislodge an electron from the cathode and thus secure an electron to form a neutral molecule. This process is called recombination. In a tube carrying a steady current, the ionizing rate must therefore be sufficient to supply the current taken by the positive electrode and also to supply the electrons which later disappear by recombination.

If the gas pressure is too high, the distance between collisions is so small that the electrons cannot reach the speed required for ionization. If the pressure is too low, so that the mean free path is large, an electron may be able to travel the entire distance to the positive electrode without hitting a molecule. In either of these cases, very little ionization takes place and the current is small.

For intermediate values of pressure, however, the mean free path is large enough to permit the electron to reach ionizing speed, and is small enough to permit one electron to ionize several molecules before it reaches the positive electrode. These are the conditions required to obtain the cumulative ionization which is essential for the maintenance of a gaseous discharge. The applied voltage must, of course, be large enough to accelerate the electrons to the speed required for ionization.

In order to produce cumulative ionization we must have, first, sufficient voltage to bring electrons up to ionizing speed, and second, ample opportunity for these high-speed electrons to collide with neutral molecules. When the discharge is said to be cumulative, we mean that any given amount of ionization tends to produce more ionization in the body of the gas. If this process is allowed to continue unchecked, the discharge tube will be damaged by the large current which will flow. It is therefore customary to operate such tubes with a fixed resistance in series to limit the current to a safe value. When an alternating voltage is applied as in operating the gaseous discharge tubes used in signs, the reactance of the high-voltage winding of the supply transformer is usually sufficient to limit the current.

294. The Spark. As we saw in the preceding section, if the gas pressure in a discharge tube is increased beyond the highest value for which cumulative ionization can take place, the mean free path of an electron becomes too small for it to reach the ionizing speed, and the discharge cannot be maintained.

Another type of discharge can be produced, however, by increasing the voltage further. A point will be reached at which sufficient speed is given an electron to allow it to ionize a molecule after traveling only a short distance. Under these conditions, ionization will also be very rapidly cumulative, but the discharge cannot be maintained because the voltage drops rapidly as the current increases and ionization then ceases. This is a violent, disruptive type of discharge which we call a **spark.** If the voltage across the gas

recovers rapidly, the process repeats, and it is thus possible to obtain a continuous series of sparks.

The sparks with which we are most concerned are those which occur in air at atmospheric pressure. On the basis of experimental evidence available at present, it seems probable that the mechanism by which a spark occurs in air is somewhat different from that described above. In order to produce a spark we must stress the air at some point to 30,000 volts per centimeter. This value increases with the barometric pressure but under standard conditions it is 30,000 volts per centimeter.

It has been found by experiment, however, that there is always a time lag in this process. That is, the spark does not occur as soon as the stress in the air reaches 30,000 volts per centimeter; a short time elapses before the action takes place. Furthermore, the amount of time required is found to be larger than that required for ionization to take place. Measurements made on the velocity of ions in air point to a mechanism which accounts for this behavior.

The effect is apparently caused by a peculiar affinity which the oxygen molecules of the air have for any free electrons which happen to be present. As a result, air under normal conditions contains practically no free electrons. Due to various causes, on the average about 10 free electrons per second are produced in each cubic centimeter of normal air, but these immediately attach themselves to oxygen molecules, forming negative ions, or ionic molecules. In every cubic centimeter of air there are about 1000 of these ionic molecules of oxygen under normal conditions.

If this air is now stressed at 30,000 volts per centimeter, the first thing that happens is that these ionic molecules are removed from the space by the action of the applied voltage. This does not produce ionization, however, because the oxygen molecule is too heavy to reach ionizing speed in the time it takes to travel over its mean free path. As a result, the air is quickly cleared of all charged particles so that for a time, there is no way of producing ionization. Any free electrons which are now produced, however, will immediately be accelerated by the voltage before they have had opportunity to get attached to oxygen molecules, and it is these electrons which start the ionization which cumulates in the form of a spark. Thus a time delay must occur before enough free electrons are produced to start the ionization.

If the voltage is applied between parallel plates, and the edges of these plates are protected in some manner, the critical voltage

will be reached when the voltage applied is 30,000 times the separation in centimeters. When this voltage is reached, the air will break down and a spark will pass between the parallel faces of the plates. With electrodes of other shapes, however, this simple rule will not apply, for the stress will not be uniform. It is always necessary to bring the stress at some point up to the critical value of 30,000 volts per centimeter, and usually this will produce breakdown. At the point where the critical stress is reached, the air ionizes and becomes conducting. This point is usually in the layer of air next the electrode. This layer then becomes conducting so that the full voltage is applied across the remainder of the space. The next layer of air is now ionized and becomes conducting, thus increasing the stress on another layer. Usually this process continues until the air breaks down completely and a spark passes.

A calibrated spark gap may be used for measuring very high voltages by noting the separation when the gap breaks down. For a given separation, the easiest gap to break down is the needle gap, for the stress is highly concentrated at the points of the needles. A gap between small balls will break down at a lower potential than the same gap between large balls. The Standardization Rules of the American Institute of Electrical Engineers give the following spark-gap settings, for example, for 160,000 volts breakdown at 25° C. and 29.9 inches barometer:

for 125-mm spheres, spacing = 110 mm;
" 250-mm " " = 90 mm;
" 500-mm " " = 83 mm.

The rate of application of voltage is an important factor in determining the breakdown of air across a gap. When the voltage is applied gradually, the necessary time is available for the ionization to take place by layers, and hence for the discharge to spread across the gap. On the other hand, for very suddenly applied potentials, of short duration, the effect of using sharp-pointed electrodes is lost. That is, breakdown does not occur merely because the stress is concentrated near the electrodes, because there is no time available in which ionization by layers can take place. Consequently, for these sudden surges, all gaps of the same separation will break down at approximately the same potential. This principle is used to advantage in the design of lightning-arresters.

It should be noted that the distribution of electric stress in a gap can be simply mapped only when no current is passing. For

instance, the distribution of voltage drop per centimeter between filament and plate of a thermionic tube cannot be simply drawn. The presence of the electrons in the space, that is, the space-charge effect, causes the stress to be modified and increases the stress near the positive electrode.

Prob. 13–15. A sphere 120 centimeters in diameter is situated 8 centimeters above a flat metal plate. What voltage will be necessary between sphere and plate to cause a spark to pass? Assume that the curvature of the sphere increases the stress 30 per cent in the layer of air next to the sphere.

295. Corona. It does not always follow that breakdown of air at one layer between electrodes will result in a complete breakdown. When the layer of air adjacent to the electrode is ionized, the full stress is brought to bear on the remaining space. Whether the stress on the next layer of this remaining space is thus brought to the critical value depends upon the shape of the electrodes and their separation. With small electrodes far apart, it will often happen that in the new distribution of stress when the surface layer of air is ionized, there will be no new point further out from the wire where the stress is sufficiently raised to continue the spread of the discharge.

In such cases we have a brush discharge or corona. Such a case occurs particularly on the wires of a high-tension transmission line. For the voltage at which many of these are used, the stress at the surface of the wire is often above the critical stress, and yet the configuration is such that breakdown from wire to wire does not occur.

For instance, let the distance between the two conductors of a high-tension line be from 150 to 200 inches. Assume that the wire is one-half inch in diameter. If corona occurs, the stress must be at least 30,000 volts per centimeter at the surface of the wire. Let us assume a stress of 32,000 volts per centimeter at the surface of the wire.

At a distance of one-half inch from the wire the electric stress would be about 10,000 volts per centimeter.

This would not be enough to break down the air and ionize it. Thus the air would be ionized to a distance of less than one-half inch out from each wire, and the remaining air between each conductor would remain as an insulator. The length of non-ionized air would therefore be decreased by less than one inch out of the 150 to 200 inches which the conductors are spaced. This would scarcely affect the stress or its distribution around the wire.

In actual use each wire is surrounded by a luminous envelope of ionized air. The production of ions and their discharging on the wire constitute a leakage current from the wire. This occasions a loss in the transmission line. Due, however, to the great advantages in the use of high voltages, a small amount of corona loss can often be tolerated.

Prob. 14–15. The leakage current from each wire of a transmission line due to corona is one ampere, the line being 100 miles long. Assuming two wires only with 110,000 volts applied, how much loss does this involve? The line current at full load is 100 amperes. The line wires have a resistance of 0.4 ohm per mile. Dropping the voltage to 88,000 volts would stop corona entirely. From the standpoint of efficiency of transmission, would it be advisable to make this change? The same amount of power is to be transmitted in both cases.

296. Arcs. It requires an enormous electric stress to break down air at atmospheric pressure, but once the air is disrupted and a spark is passed, even a fairly small voltage can maintain a current across the gap.

The sudden explosive flow of current when air is broken down by a high stress is called a spark. The steady current which follows at low voltage drop is called an **arc.** Figure 459 shows a three-phase arc at 1,000,000 volts, with the conductors spaced at the points of a triangle 9 feet long on each side.

The arc has certain distinctive properties which cause it to differ from a discharge through gas at low pressure. It is confined to a relatively narrow path between the electrodes instead of being uniformly spread out. It therefore produces an intense heat at a small point on the electrode. If the arc is long continued, this point is brought to incandescence and melted or even vaporized. When this occurs, the molecules of vapor are charged when they leave the electrode and assist in carrying the current. The positive ions when they bombard the incandescent metal cause it to emit electrons copiously. There is also a thermionic emission by which electrons are evaporated from the surface of the negative electrode. In the air space through which the arc passes, there is intense heat which renders the air molecules easily ionized. For all these reasons, an arc which has continued long enough to heat the electrodes is easily maintained and large amounts of current can pass at low voltage. Fifty volts will maintain an intense arc between carbon electrodes in air.

An arc has a negative characteristic. The higher the current passed, the lower is the drop between electrodes. The reason for this is easily seen. The greater the current carried, the larger the cross-section of the arc becomes. The resistance is therefore much less for a heavy current. In fact, the conductivity of the arc path increases at a greater rate than simple proportionality to the current.

FIG. 459. A three-phase, 1,000,000-volt arc. *General Electric Co.*

There are many kinds of arcs. Arcs between carbon electrodes are much used for lighting. In such a case, most of the light comes from the incandescent electrodes themselves. The arc stream is relatively much less luminous. In certain arc lights, however, known as flaming arcs, the light from the arc stream itself is increased by impregnating the carbon electrodes with certain salts.

An arc light must be operated on a constant-current circuit or with a resistance in series for stabilizing. Due to its negative

characteristic, an arc connected to a constant-potential circuit will be very unstable. The more current it takes, the less is its drop, and it hence acts like a complete short circuit. A resistance in

FIG. 460. Arc welding. *General Electric Co.*

series will give a rising characteristic for lamp and resistance and hence stabilize the arc. For a street-lighting circuit, arcs are connected in series and supplied by a transformer which automatically maintains a constant current, thus avoiding possible instability.

Arcs are also much used for welding metals. In the process which is most commonly used for welding iron and steel as shown in Fig. 460, one electrode is the material to be welded and the other is an iron rod held in the hand. The rod is touched to the work to start the arc, and then withdrawn slightly to form an arc about one-fourth of an inch long. Only 30 to 50 volts are required to maintain the arc. A current of 50 to 200 amperes is used, depending upon the nature of the work. The arc raises the temperature of the material to be welded to fusion temperature. Small globules of the " pencil " or electrode held in the hand are dropped in a molten condition into the weld. Considerable skill is required in order to produce uniform work.

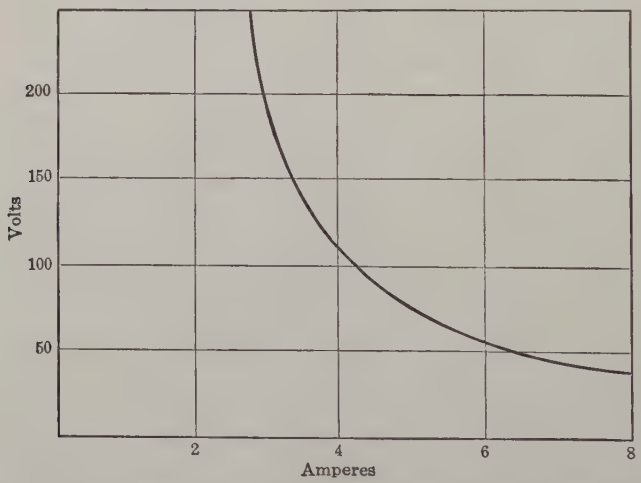


FIG. 461. The relation between the current and voltage of an arc in air; metal electrodes.

Besides the useful applications of arcs, there are many places where they must be dealt with by the electrical engineer in connection with switching and line protection. Their nature should therefore be thoroughly understood.

It has been mentioned that an arc has a negative characteristic. In Fig. 461 is shown the characteristic for an arc in air between metal electrodes. It will be noted that as the current is decreased, a point is reached where the voltage rises very rapidly indeed. It requires a certain current to maintain an arc. If the current falls below this value, the arc will suddenly go out. The current necessary to maintain the arc depends upon its length, the electrode material, the electrode temperature and several other factors.

When a switch is opened in a power circuit, an arc follows the separation of the blades. If the voltage of the source is high or if there is much inductance in the circuit, the arc may be drawn out very long, in fact, to several feet on high-voltage power circuits. If the source is capable of maintaining the arc at its maximum length, it will continue until interrupted in some manner, or until it has fused and destroyed the switch. If the arc is drawn out, however, until the current passing becomes too small to maintain it, the arc will suddenly go out. It is this sudden extinguishing of the arc which produces a rapid change of current and a high voltage when an inductive circuit is opened. Thus when the field circuit of a large machine is suddenly opened by a switch, it is not the instant when the contacts separate that is dangerous, for a hundred volts or so will start and maintain the arc, but it is the instant when the arc breaks.

An arc stream which is situated in a magnetic field is forced sideways in exactly the same way that a wire carrying current would be acted upon. The cause of the action is the same in both cases, namely the tendency of a current-carrying conductor in a magnetic field to be deflected sideways. The direction in which the arc will move can be determined by the left-hand rule, just as in the case of a motor.

This principle is made use of in magnetic blowouts as applied to trolley-car controllers, contactors, etc. When the contact points separate, there is a magnetic field in the space between them which forces the arc out. This field is usually produced by a coil connected in series with the circuit to be opened. Then the heavier the current to be opened, the stronger will be the field acting to blow the arc. The movement of the arc in the field accomplishes two things. First, the arc is lengthened and moved out to where it will do no harm. A longer arc, we have seen, will break more easily. Second, it is moved over to a cool surface. It has been shown that one condition necessary for the maintenance of an arc at low voltage is an incandescent spot on the surface of the electrode. If the arc is moved off this spot, it will have a strong tendency to go out. This second point is not always fully appreciated.

Prob. 15–15. Using the curve of Fig. 461, draw the characteristic for the combination of this arc in series with 8 ohms resistance. Will the combination be stable at 6 amperes? What is the apparent resistance of the combination at this current value? At 8 amperes?

Prob. 16–15. Make a sketch showing a switch with magnetic blowout, indicating the coil connections and the direction in which the arc will be blown. Will the action be correct if the current to be broken is reversed in direction?

SUMMARY OF CHAPTER XV

THE FLOW OF ELECTRICITY THROUGH GASES is not governed simply by Ohm's Law.

THERMIONIC CONDUCTION is the conduction of electricity by electrons which are evaporated from a hot filament. The hot filament, together with a cold plate, is sealed in a vacuum tube, and voltage is applied between the hot filament and the cold plate.

RICHARDSON'S LAW. The amount of current which passes in a thermionic tube, at a given filament temperature, increases as the voltage between plate and filament increases up to a certain point, where saturation occurs. The saturation point depends upon the temperature of the hot filament. The saturation current is given by the equation

$$i_s = \frac{AT^2}{2.718^{\frac{K}{T}}} \text{ amperes per square centimeter.}$$

AN AMPLIFIER OR REPEATER is made by placing a grid between the hot filament and cold plate. The current passing from plate to filament is caused to vary over a wide range by slight changes in the voltage of the grid. The performance of the tube is specified by means of curves of plate voltage against plate current for various values of grid voltage. A tube containing a filament, grid, and plate is called a triode.

THE AMPLIFICATION FACTOR of a thermionic tube is the change in plate voltage required to counteract a change in grid voltage, divided by that change in grid voltage.

THE PLATE RESISTANCE of a tube is the change in plate voltage which produces a given change in plate current, divided by that change in plate current.

THE VOLTAGE GAIN of a thermionic amplifier is given by the expression,

$$\text{gain} = \mu \frac{R}{r_p + R}$$

in which

μ = amplification factor;
r_p = plate resistance;
R = load resistance.

A RECTIFIER is a device which permits current flow in one direction only. Thermionic tubes act as rectifiers because electrons are produced only from the filament. In radio work the **DETECTOR** is a rectifier.

VACUUM-TUBE OSCILLATORS generate high-frequency alternating current. The grid of the oscillator tube is excited by feeding back a part of the plate output.

A SCREEN-GRID TUBE contains an additional grid between the control grid and the plate. Its purpose is to prevent plate-voltage variations from effecting the control-grid voltage. This type of tube is also called a **TETRODE.**

A PENTODE is a screen-grid tube with a third grid added between the screen grid and the plate. This grid is called a suppressor; it is kept at the same potential as the filament and serves to prevent secondary electrons from leaving the plate.

THE X-RAY TUBE is a special diode operating a very high voltage. The velocity of the electrons hitting the target is many thousands of miles per second. The disturbances produced by the impacts of these high-speed electrons cause the material of the target to emit high-frequency radiations called X-rays. These rays are able to penetrate materials which are opaque to ordinary light.

IN A PHOTOELECTRIC CELL electrons are emitted by the action of light on a special cathode. These electrons are drawn over to a plate by applying voltage between anode and cathode. The current in the tube is proportional to the amount of light reaching the cathode.

THE CATHODE RAY consists of a stream of high-speed electrons which have passed through a hole in the plate of a diode. The ray can be deflected by applying a magnetic field perpendicular to the ray, or by passing the ray between parallel plates across which there is a voltage. Cathode-ray tubes are used extensively in oscillographs for studying wave forms, and also as receivers in television systems.

CONDUCTION THROUGH GAS may take place at low pressure. Free electrons in the gas are set in motion by electric force. These electrons hit molecules and ionize them by dislodging electrons. The electrons and positive ions formed by ionization conduct the electricity through the gas in tubes which do not contain heated filaments. When small amounts of gas are added to thermionic diodes, the positive ions produced by ionization neutralize space charge and permit larger currents to flow at lower voltages.

MERCURY-ARC RECTIFIERS are now widely used in place of synchronous converters for electric railway power supply.

TO PRODUCE A CUMULATIVE DISCHARGE the electric potential must be sufficiently high to give the electron enough acceleration, and the **MEAN FREE PATH** of the electron must be long enough to allow it to acquire sufficient velocity to ionize a molecule.

SPARKS ARE PRODUCED when the gas pressure is so great as to require a comparatively high voltage to give ionizing velocity to the electrons. Cumulative ionization occurs very rapidly and the discharge is sudden.

CORONA IS PRODUCED when the ionization of one section of gas between electrodes does not produce an ionizing stress in all other sections. This is often the condition on high-voltage transmission lines.

ARCS ARE CHARACTERIZED BY THE VAPORIZATION of electrodes and ionization of the vapor. Arcs differ from sparks in being confined to the comparatively narrow path of vapor flow between electrodes.

PROBLEMS ON CHAPTER XV

Prob. 17–15. In a certain experiment an emission current of 1 ampere must be obtained from a heated cathode. The maximum allowable length of the cathode is 2 inches, and it is to be heated to a temperature of 2700 degrees absolute. What diameter tungsten rod must be used for this cathode?

Prob. 18–15. If the cathode of Prob. 17–15 is used as part of a diode, and a voltage of 880 volts is required to draw saturation current, what is the power lost in the plate of the tube?

Prob. 19–15. At what velocity will electrons arrive at the plate of the diode of Prob. 18–15?

Prob. 20–15. If an electron velocity of 2.76×10^8 centimeters per second is required to ionize a molecule of neon, what is the ionization potential of neon?

Prob. 21–15. Assuming that the velocity of electrons necessary to ionize air at standard atmospheric pressure and temperature is 2×10^8 centimeters per second, what is the approximate mean free path of the electron under these conditions?

Prob. 22–15. The space between two flat, metal plates, spaced 1 centimeter apart is filled with argon gas. The pressure of the gas is such that the mean free path of an electron is 0.4 millimeter. If the ionization potential of argon is 15.7 volts per centimeter, what velocity must an electron reach in order to ionize the gas? What voltage must be applied between the plates in order to produce ionization?

Prob. 23–15. A 250-type triode has a plate resistance of 2000 ohms, and an amplification factor of 3.8 when operated with a plate voltage of 350 volts and a grid bias of -63 volts. What is the voltage gain in this tube when supplying a load resistance of 4100 ohms?

Prob. 24–15. What a-c voltage is produced across the load resistance of the tube in Prob. 23–15 when an a-c voltage of 15 volts (r.m.s.) is applied to the grid?

Prob. 25–15. What alternating-current power is supplied to the load resistance of the tube in Prob. 23–15 under the operating conditions of Prob. 24–15?

Prob. 26–15. What will be the voltage gain in a pentode having a plate resistance of 140,000 ohms and an amplification factor of 120, when operating into a plate load resistance of 15,000 ohms?

Prob. 27–15. The pentode of Prob. 26–15 is to be operated with a voltage gain of 50. What value of plate load resistance is required?

Prob. 28–15. The voltage between the plate and filament of the pentode of Prob. 26–15 is 100 volts, and the d-c plate current is 7 milliamperes. If a battery is to be used to supply the plate voltage of this tube, what must the voltage of the battery be? If the plate load resistance is changed so that the gain of the tube is 40, what must the voltage of the plate battery be in order to keep the same voltage between plate and filament?

Prob. 29–15. Two 15-inch spheres are to be used to test the voltage between transformer terminals. Assuming that the curvature of the spheres is sufficient to lower the apparent break-down strength of air by 45 per cent, how far should the spheres be separated in order to break down at 270,000 volts?

Prob. 30–15. The arc, for which the voltage-current curve is given in Fig. 461, is to be operated at 6 amperes, across a 220-volt circuit. What is the smallest value of stabilizing resistance which must be connected in series with the arc?

CHAPTER XVI

THE ELECTROSTATIC CIRCUIT

The modern problem of the economical transmission of large quantities of power demands voltages as high as 290,000 volts. This puts the greatest importance on the problem of insulation. The insulation part of a circuit is called the **electrostatic circuit,** and consists of all that part of the circuit lying between the conductors.

297. The Electrostatic Circuit. The electrostatic circuit can be treated in a manner similar to that used with the magnetic circuit. For instance, let Fig. 462 represent an air gap in a magnetic circuit and Fig. 463 an air gap of the same dimensions between two plates

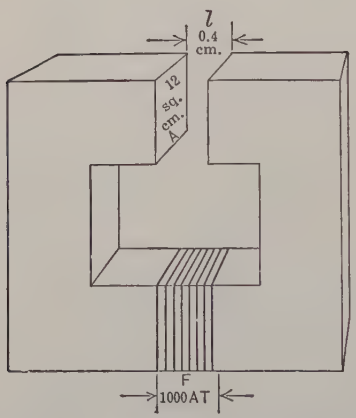


Fig. 462. A magnetic circuit with an air gap.

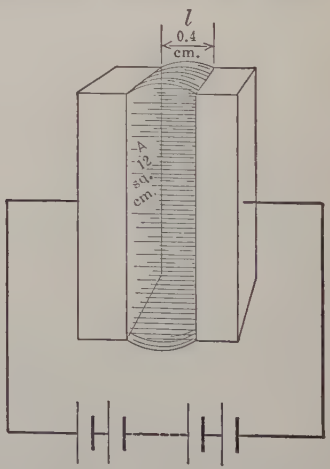


Fig. 463. An electrostatic circuit with an air gap.

or electrodes of an electric circuit. The rest of the electric circuit is composed of some good conductors of electricity so that the air gap only will be considered as the electrostatic circuit. Of course, there is an electrostatic condition all around the circuit because the electrons are pushed out of or into the surface of the conductor wherever an electric pressure exists. Similarly, in the magnetic

circuit a magnetic field exists everywhere in the vicinity of the iron circuit, but we will consider the air gap only. Let us assume that there are 1000 ampere-turns magnetomotive force across the 0.40-cm air gap in the magnetic circuit, and 1000 volts electromotive force across the 0.40-cm air gap in the electrostatic circuit.

We say that there is a magnetic flux across the air gap in the magnetic circuit. Similarly, we say there is an electrostatic flux across the air gap in the electric circuit. For simplicity we will assume no fringing or leakage of flux in either case.

To compute the magnetic flux, in the air gap, we divide the **magnetomotive force** across the gap by the **reluctance** of the gap.

$$\phi = \frac{F}{\mathcal{R}} \quad \text{or} \quad \frac{0.4\pi NI}{\mathcal{R}}$$

where ϕ = flux in magnetic **lines.**

F or $0.4\pi NI$ = magnetomotive force in **gilberts.**

$\mathcal{R}$ = reluctance in **reluctance units.***

Similarly, we find the electrostatic flux in the air gap of the electrostatic circuit, by dividing the **electromotive force** by the **elastance** of the gap

$$\psi = \frac{E}{S}$$

where ψ = flux in electrostatic **lines.**

E = electromotive force in **statvolts.**

S = elastance in **darafs.**

These units will be unfamiliar at first but are chosen because they are easily learned, are most convenient to use, and can easily be reduced to units in any other system.

A **statvolt** is a larger unit of pressure than the volt, being equal to 300 volts. A **statcoulomb** is a smaller unit of charge than the coulomb, since one coulomb equals 3×10^9 statcoulombs. From each statcoulomb of charge on the positive plate there are 12.57 ($= 4\pi$) lines which leave the plate and go over to a corresponding statcoulomb of charge on the negative plate. Expressing this as an equation,

$$\psi = 4\pi Q,$$

* As was pointed out in Chapter VI, there is at present no name for the unit of reluctance. To avoid confusion, they are designated here as "reluctance units." The unit will have the correct value if the reluctance is always computed from the gap dimensions in **centimeters.**

if Q is expressed in **statcoulombs.** If Q is given in **coulombs,** the number of lines is given by

$$\psi = 4\pi \times 3 \times 10^9 Q$$
$$= 12\pi \times 10^9 Q.$$

An electrostatic line starts with the charge on the positive plate and ends at the charge on the negative plate, and is the path which a positive charge of electricity would take if free to move, just as a magnetic line is the path which a positive pole would take if free to move.

The space occupied by the electrostatic lines is called an **electrostatic field** or simply an **electric field,** just as the space occupied by magnetic lines is called a **magnetic field.** If we measure the electromotive force (E) in statvolts and the flux (ψ) in lines, the **elastance** (S) is measured in **darafs,** just as the **reluctance** ($\mathcal{R}$) is measured in reluctance units, when the magnetomotive force (F) is measured in gilberts and the flux in magnetic lines (ϕ).

The **elastance** represents the **hindrance** which the electromotive force meets in setting up the electrostatic lines (ψ), just as the **resistance** represents the hindrance which the electromotive force meets in setting up the electric current (I), and just as the **reluctance** represents the hindrance which the magnetomotive force meets in setting up the magnetic lines (ϕ).

The **elastance** of a centimeter cube of air is taken as the unit of elastance and has been given the name **daraf.*** The only new terms and quantities to be learned are:

1. One daraf = elastance of a centimeter cube of air.
2. One coulomb produces $12\pi \times 10^9$ electrostatic flux lines.

298. The Computation of the Elastance of an Air Gap. The elastance of an electrostatic circuit being analogous to the reluctance of a magnetic circuit, and to the resistance of an electric circuit, is calculated by the same general formula.

The resistance of a round copper wire is

$$R = \frac{10.4\, l}{A \text{ (or } d^2)}$$

* The name **daraf** (farad, spelled backwards) has been suggested for elastance, because **elastance** is the reciprocal of **capacitance.** See Karapetoff, "The Electric Circuit."

where R = resistance of the wire in **ohms.**
10.4 = the resistance of a **mil foot** of copper.
l = the length of the wire in **feet.**
A (or d^2) = the cross-section area of the wire in **circular mils.**

The reluctance of an air gap in a magnetic circuit is

$$\mathcal{R} = \frac{1 \times l}{A}$$

where $\mathcal{R}$ = the reluctance of the gap in **reluctance units.**
l = the length of the gap in **centimeters.**
A = the cross-section area of the gap in **square centimeters.**
1 = the reluctance of a **centimeter cube** of air.

Similarly, the elastance of an air gap between two plates in an electrostatic circuit is

$$S = \frac{1 \times l}{A}$$

where S = elastance in **darafs.**
l = length of gap in **centimeters.**
A = area of gap in **square centimeters.**
1 = elastance of one **centimeter cube** of air.

Example 1. In the magnetic circuit of Fig. 462 the air gap is 0.40 centimeter long and has a cross-section area of 12 square centimeters. The magnetomotive force across the air gap is 1000 ampere-turns. What magnetic flux passes between the pole faces?

Solution.

The reluctance of the air gap is

$$\mathcal{R} = \frac{l}{A}$$

$$= \frac{0.4}{12}$$

$$= 0.0333 \text{ reluctance unit.}$$

The magnetic flux is

$$\phi = \frac{F}{\mathcal{R}}$$

$$= \frac{0.4\pi \times 1000}{0.0333}$$

$$= 37{,}700 \text{ lines.}$$

Example 2. In the dielectric circuit of Fig. 463 the air gap is 0.40 centimeter long and has a cross-section area of 12 square centimeters. The electromotive force across the gap is 1000 volts. What electrostatic flux passes between the plates?

Solution.

The elastance of the air gap is

$$S = \frac{l}{A}$$
$$= \frac{0.40}{12}$$
$$= 0.0333 \text{ daraf.}$$

The electrostatic flux is

$$\psi = \frac{E}{S}$$
$$= \frac{1000}{300 \times 0.0333}$$
$$= 100 \text{ lines.}$$

Of course we might find the number of lines by finding the number of statcoulombs of charge on the plates, and then multiply by 4π. To do this we must first find the capacitance and multiply by the voltage, for $Q = CE$.

From Chapter XIII

$$C = \frac{0.0884\, KA}{l \times 10^6}$$

$$C = \frac{0.0884 \times 1 \times 12}{0.4 \times 10^6}$$
$$= \frac{2.65}{10^6} \mu\text{f.}$$

$$Q = CE$$
$$= \frac{2.65}{10^6} \times 1000$$
$$= \frac{2.65}{10^3} \text{ microcoulombs}$$
$$= \frac{2.65}{10^9} \text{ coulombs}$$
$$= \frac{2.65}{10^9} \times 3 \times 10^9$$
$$= 7.95 \text{ statcoulombs}$$
$$\psi = 7.95 \times 4\pi$$
$$= 100 \text{ lines.}$$

The first method is much shorter.

Prob. 1–16. What is the elastance of an air gap 0.02 centimeter in length between two parallel plates, 24 × 10 centimeters in cross-section area?

Prob. 2–16. How many electrostatic lines would be set up in the air gap of Prob. 1–16 by an e.m.f. of 250 volts across it?

299. The Computation of the Elastance of any Dielectric. We have seen that the **elastance** of a dielectric is analogous to the magnetic **reluctance** of a piece of material, and represents the **hindrance** which the material offers to setting up electrostatic lines in it.

But instead of using the **reluctivity** of a magnetic material in our equations, we generally use the **permeability** (μ) which represents the **ease** with which magnetic lines can be set up in a material. The permeability is the reciprocal of the reluctivity.

Similarly instead of using the **elastivity** of an insulator in our equation we generally use the **dielectric constant** K which represents the **ease** with which electrostatic lines may be set up in the material. The dielectric constant is the reciprocal of the elastivity, that is, it is the reciprocal of the elastance of a centimeter cube of the material, just as permeability is the reciprocal of the reluctance of a centimeter cube. We have already used this dielectric constant, as found in Table V, in the equation for the capacitance. The following examples represent this other use of it in finding the elastance of any material.

Example 3. What is the elastance of a piece of mica 0.01 centimeter thick and having an area of 120 square centimeters?

Solution.

$$S = \frac{l}{A}\frac{1}{K}$$
$$= \frac{0.01}{120} \times \frac{1}{5.8}$$
$$= 0.00001437 \text{ daraf.}$$

Example 4. If the mica sheet of Example 3 is placed between two plates each having 120 square centimeters area, and 100 volts e.m.f. are put across the dielectric, what electrostatic flux will be set up in the mica?

Solution.

$$\psi = \frac{E}{S}$$
$$= \frac{100}{300 \times 0.00001437}$$
$$= 23{,}200 \text{ electrostatic lines.}$$

Example 5. What charge will there be on the condenser in Example 4?

Solution.

$$Q = \frac{\psi}{3 \times 10^9 \times 4\pi}$$
$$= \frac{23{,}200}{3 \times 10^9 \times 12.57}$$
$$= 6.15 \times 10^{-7} \text{ coulombs}$$
$$= 6.15 \times 10^{-7} \times 10^6 \text{ microcoulombs}$$
$$= 0.615 \text{ microcoulomb.}$$

Example 6. What capacitance will the condenser of Example 4 have?

Solution.

$$C = \frac{Q}{E}$$
$$= \frac{0.615}{100}$$
$$= 0.00615 \ \mu\text{f.}$$

or

$$C = \frac{0.0884\ KA}{l \times 10^6}$$
$$= \frac{0.0884 \times 5.8 \times 120}{0.01 \times 10^6}$$
$$= 0.00615 \ \mu\text{f.}$$

Prob. 3–16. What cross-section area must the dielectric of a mica condenser have in order that 80 electrostatic lines be set up by an e.m.f. of 300 volts? The thickness of the dielectric is 0.06 centimeter.

Prob. 4–16. An e.m.f. of 1200 volts in a condenser sets up an electrostatic field of 90 lines. What is the elastance of the dielectric?

Prob. 5–16. What flux will be set up in a condenser consisting of a dielectric of a sheet of flint glass 250 square inches in area and $\frac{1}{16}$ inch thick, between metal plates, when 50,000 volts are applied to the plates?

Prob. 6–16. What charge will 2300 volts put on a condenser made of two metal plates of 144 square centimeters in area with a sheet of gutta percha 1.0 mm thick between them?

Prob. 7–16. What capacitance will the condenser of Prob. 6–16 have?

Prob. 8–16. How thick will a sheet of hard crown glass be, if it has a cross-section area of 75 square centimeters and 350 volts sets up a flux of 5000 lines?

300. The Elastance of Dielectrics in Series. The elastance of dielectrics in series is the sum of the separate elastances, just as the

resistance of a series of resistances is the sum of the separate resistances.

Example 7. The dielectric of a certain condenser consists of two sheets, one of flint glass 0.25 centimeter thick, the other of gutta percha 0.35 centimeter thick. The area of each is 100 square centimeters. What is the elastance of the condenser?

Solution.

The elastance of the glass,

$$S_{gl} = \frac{l}{A} \times \frac{1}{K}$$
$$= \frac{0.25}{100 \times 9.9}$$
$$= 0.000253 \text{ daraf.}$$

The elastance of the gutta percha

$$S_{gu} = \frac{0.35}{100} \times \frac{1}{4.1}$$
$$= 0.000854 \text{ daraf.}$$

The elastance of the series combination

$$S_c = 0.000253 + 0.000854$$
$$= 0.00111 \text{ daraf.}$$

Prob. 9–16. What flux will 1800 volts set up in a condenser the dielectric of which has a cross-section of 900 square centimeters and consists of a sheet of ebonite 0.6 centimeter thick and a sheet of mica 0.25 centimeter thick?

Prob. 10–16. What will be the capacitance of the condenser in Prob. 9–16?

Prob. 11–16. A condenser consists of two metal plates of 800 square inches area, spaced $\frac{1}{2}$ inch. Between the plates is a sheet of hard crown glass $\frac{3}{16}$ inch thick and 800 square inches in area. How great a charge will 550 volts put on this condenser?

Prob. 12–16. How much will the capacitance of the condenser in Prob. 11–16 be increased or decreased if in addition to the sheet of glass a sheet of mica $\frac{1}{8}$ inch thick and 800 square inches in area is put between the plates? The spacing of the plates remains as in Prob. 11–16.

Prob. 13–16. How many sheets of mica 12 × 10 centimeters in area and 0.050 centimeter thick must be used to make up a condenser in which 1000 volts will set up a flux of 500,000 lines? Remember the mica sheets are in parallel in a condenser.

Prob. 14–16. What will be the capacitance of the condenser of Prob. 13–16?

301. Electrostatic Flux Density. We are interested not only in the total electrostatic flux (ψ) in a dielectric, but more particularly in the flux density (D); that is, in the number of electrostatic lines per square centimeter cross-section of the dielectric. In the condensers which we have been studying we have considered the flux to be uniformly distributed across the area of the dielectric. At any point in the dielectric, the flux density is represented by the equation

$$D = \frac{\psi}{A}$$

in which

D = flux density in electrostatic lines per square centimeter.
ψ = total number of electrostatic lines.
A = dielectric area at right angles to lines in square centimeters.

This equation is closely analogous to the equation for magnetic flux density.

$$B = \frac{\phi}{A}$$

in which

B = flux density in magnetic lines per square centimeter.
ϕ = total number of magnetic lines.
A = area at right angles to lines in square centimeters.

Whether the flux is uniformly distributed throughout the area of the dielectric or not, D always represents the lines per square centimeter at any chosen point.

Example 8. There are 15,000 electrostatic lines uniformly distributed through the dielectric of a condenser. The cross-section area of the dielectric is 120 square centimeters. What is the electrostatic flux density?

Solution.

$$D = \frac{\psi}{A}$$
$$= \frac{15{,}000}{120}$$
$$= 125 \text{ lines per square centimeter.}$$

Prob. 15–16. A condenser having 2000 square centimeters of dielectric is charged with 15.2 microcoulombs. What is the electrostatic flux density in the dielectric?

Prob. 16–16. A voltage of 110 volts is applied to a condenser of 8 microfarads capacitance. The dielectric has a cross-section area of 240,000 square centimeters. What is the flux density in the dielectric?

Prob. 17–16. The dielectric of a certain condenser is made of ebonite $\frac{1}{16}$ inch thick and has an area of 840 square inches. What is the flux density (if the flux is uniformly distributed) when 500 volts are applied to the condenser?

Prob. 18–16. What is the total flux in the dielectric of the condenser of Prob. 17–16?

302. Ratio of Electrostatic Flux Density to Electric Stress. We have learned that when a dielectric, that is, an insulator, is broken down by the application of high voltage, it is the electric stress or voltage per centimeter that does the harm. Thus if the electric stress across an air gap exceeds 30,000 volts per centimeter, it will break down the air and cause it to conduct electricity. It takes about 350,000 volts per centimeter to break down glass.

When the electrostatic field is uniformly distributed through the dielectric the electric stress can be found from the formula

$$G = \frac{E}{l}$$

where G = electric stress in the dielectric in **statvolts per centimeter.**

E = **statvolts** across dielectric.

l = thickness of dielectric in **centimeters.**

Example 9. What is the electric stress in the dielectric in a sheet of ebonite 0.08 centimeter thick if a pressure of 500 volts is maintained across it?

Solution.

$$G = \frac{E}{l}$$

$$= \frac{500}{0.08}$$

$$= 6250 \text{ volts per centimeter}$$

$$= 20.83 \text{ statvolts per centimeter.}$$

The electric stress (G) in volts per centimeter corresponds closely to the magnetic stress (H) in gilberts per centimeter which exists in a magnetic circuit, the equation for which is

$$H = \frac{F}{l}$$

where H = magnetomotive force per centimeter, or magnetic stress in oersteds.
F = total magnetomotive force in gilberts.
l = length of magnetic circuit across which the magnetomotive force of F exists.

Example 10. What magnetic stress exists in an air gap 0.40 centimeters long across which a magnetomotive force of 1200 gilberts exists?

Solution.

$$H = \frac{F}{l}$$
$$= \frac{1200}{0.40}$$
$$= 3000 \text{ oersteds.}$$

The importance of being able to determine with fair approximation, the value of the electric stress G is the reason for developing the ideas of dielectric constant K and flux density D, for from the values of these two factors at any point in a dielectric, we can compute the electric stress G.

It will be remembered that in a magnetic circuit, the permeability (μ) was defined as the flux density which one oersted will produce, or the ratio of the flux density (B) to the magnetic pressure H. That is,

$$\mu = \frac{B}{H}$$

where μ = the permeability of the material, or the flux density which one oersted will produce.
B = the flux density in lines per square centimeter.
H = magnetic pressure in oersteds.

In an exactly similar manner, we may define the dielectric constant K as the dielectric flux density which a stress of one statvolt per centimeter (G) will set up; or it is the ratio of the flux density D to the electric pressure G. That is,

$$K = \frac{D}{G}$$

where
K = dielectric constant of the material, or the flux density which one statvolt per centimeter will produce.
D = dielectric flux density in lines per square centimeter.
G = electric stress in statvolts per centimeter.

Thus we may use Table V of the Appendix as the flux densities which a pressure of one statvolt per centimeter will produce in the various materials.

Example 11. What electrostatic flux density will 10,000 volts per centimeter produce in an air gap?

Solution.

$$D = KG$$
$$= \frac{1 \times 10{,}000}{300}$$
$$= 33.3 \text{ lines per square centimeter.}$$

Or one volt per centimeter pressure will produce $\frac{1}{300}$ line per square centimeter.

10,000 volts per centimeter will produce $10{,}000 \times \frac{1}{300}$ or 33.3 lines per square centimeter.

The more common form of the equation, however, is

$$G = \frac{D}{K}.$$

Example 12. What is the electric stress in a piece of mica when the dielectric flux density is 100 lines per square centimeter?

Solution.

$$G = \frac{D}{K}$$
$$= \frac{100}{5.8}$$
$$= 17.24 \text{ statvolts per centimeter}$$
$$= 5170 \text{ volts per centimeter.}$$

Example 13. The dielectric of a certain condenser consists of two sheets, one of flint glass 0.60 centimeter in thickness, the other of ebonite 0.15 centimeter in thickness. The cross-section area of each is 80 square centimeters. When 10,000 volts are placed across the condenser, what will be the electric pressure on each dielectric?

Solution.

$$\text{Elastance of flint glass} = \frac{l}{A \times K}$$
$$= \frac{0.60}{80 \times 9.9}$$
$$= 0.000758 \text{ daraf.}$$

$$\text{Elastance of ebonite} = \frac{0.15}{80 \times 2.7}$$
$$= 0.000694 \text{ daraf.}$$
$$\text{Total elastance} = 0.000758 + 0.000694$$
$$= 0.00145 \text{ daraf.}$$
$$\text{Total flux} = \frac{10{,}000}{300 \times 0.00145}$$
$$= 23{,}000 \text{ lines.}$$
$$\text{Flux density} = \frac{23{,}000}{80}$$
$$= 287.5 \text{ lines per square centimeter.}$$
$$G \text{ in glass} = \frac{D}{K}$$
$$= \frac{287.5}{9.9}$$
$$= 29 \text{ statvolts per centimeter}$$
$$= 8700 \text{ volts per centimeter.}$$
$$G \text{ in ebonite} = \frac{287.5}{2.7}$$
$$= 106.5 \text{ statvolts per centimeter}$$
$$= 32{,}000 \text{ volts per centimeter.}$$

Prob. 19–16. A condenser has three sheets of dielectric between its metal plates, of the following constants. The area of each is 100 square centimeters.

1 sheet of paraffin 0.26 centimeter thick.
1 sheet of gutta percha 0.16 centimeter thick.
1 sheet of lead glass 0.09 centimeter thick.

When 18,000 volts are put across the condenser, what is the electric stress in each dielectric?

Prob. 20–16. There are 80,000 volts between two parallel metal plates spaced 2 centimeters apart. The space is filled with 2 sheets of glass each 1 centimeter thick, one of lead glass and the other of flint glass. What is the electric stress in each sheet of glass? The plate area is 250 square centimeters.

Prob. 21–16. A condenser of 20 microfarads capacitance is operating on 500 volts. The cross-section of the dielectric is 80,000 square centimeters. What is the electric stress in the dielectric which is composed of mica sheets?

303. Distribution of Stresses in Insulation. The preceding article has considerable bearing on the design of insulation. We

see from Prob. 20 that in a condenser having two dielectrics of the same thickness in series, the pressure across the different dielectrics is inversely proportional to their dielectric constants. The one having the greater dielectric constant has the lower stress in it.

This is strongly illustrated as follows:

Suppose that we have two plates as in Fig. 464 separated 2.50 cm and subjected to a voltage of 60,000 volts. This is 24,000 volts per cm and is about all such a gap will stand when the plates are separated by air, even with the corners well rounded to avoid local high flux densities. Suppose we insert a plate of glass 1.25 cm thick as shown, with a dielectric constant 5, in order to protect against possible breakdown. Examine the new conditions.

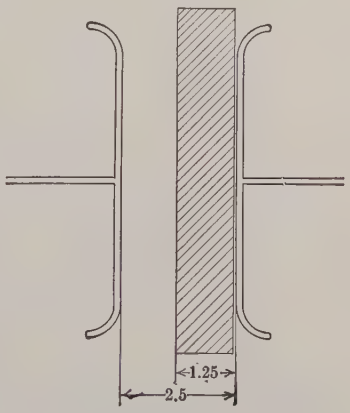


FIG. 464. A glass plate inserted in an air gap.

We will consider one square centimeter, for convenience. The elastance of the glass, 1.25 cm thick and 1 square centimeter in cross-section, is

$$S_g = \frac{1.25}{1 \times 5}$$
$$= 0.25 \text{ daraf.}$$

The elastance of the air, 1.25 cm thick and 1 square centimeter in cross-section,

$$S_a = \frac{1.25}{1 \times 1}$$
$$= 1.25 \text{ darafs.}$$

Total elastance of air and glass

$$S = 0.25 + 1.25$$
$$= 1.50 \text{ darafs.}$$

Flux through both air and glass

$$\psi = \frac{E}{S}$$
$$= \frac{60{,}000}{300 \times 1.5}$$
$$= 133 \text{ lines.}$$

Since we are considering 1 square centimeter only, this is also the value of flux density in both. Thus

$$D = 133 \text{ lines per square centimeter}$$

$$G_g = \frac{133}{5}$$

$$= 26.7 \text{ statvolts per cm},$$

$$= 8000 \text{ volts per cm.}$$

The stress in the air at first is

$$G_a = \frac{133}{1}$$

$$= 133 \text{ statvolts per cm},$$

$$= 40{,}000 \text{ volts per cm.}$$

Thus the introduction of a sheet of insulating material with a high dielectric constant causes most of the stress to be brought to bear on the remaining material of low dielectric constant. In this case, the material with low dielectric constant is air and the stress is raised from 24,000 volts per centimeter to 40,000 volts per centimeter which will surely break it down, and ionize it,* making it a good conductor. This in turn puts practically the whole 60,000 volts across the 1.25 cm of glass, which it surely can stand. However, if the voltage is alternating, a heating of the air, and consequently of the glass, will occur which may crack the glass.

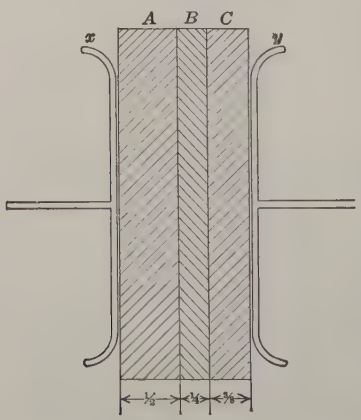


Fig. 465. Three dielectrics of different constants inserted in an air gap.

The above example illustrates the fact that to replace part of a dielectric with a low dielectric constant by one with a high dielectric constant does not always result in better insulation, and may even produce poorer insulation.

Prob. 22–16. Two plate electrodes similar to those in Fig. 464 are placed 2 centimeters apart and a potential of 20,000 volts applied. If a sheet of dielectric 0.5 centimeter thick and having a dielectric constant of 8 is inserted between the electrodes, what will be the new distribution of electric stress?

* See discussion on ionization in Chapter XV.

Prob. 23–16. The three dielectric sheets shown in Fig. 465 have the following relative dielectric constants: A, 2.5; B, 3; C, 4.5. What is the potential distribution across the different sheets when 25,000 volts are applied between the plates x and y? All dimensions are in centimeters.

Prob. 24–16. Assuming the same data as in Prob. 23–16, compute the voltage distribution in Fig. 465:

(a) With A removed and B and C as in the figure,
(b) With B removed and A and C as in the figure,
(c) With C removed and A and B as in the figure.

Prob. 25–16. What dielectric sheet or sheets of arrangement in Prob. 24–16 must be removed to produce a condition most likely to cause a breakdown of the air? Assume the dielectric sheets to have much greater dielectric strength than air.

304. Electric Stresses in Cables. A cable, we have seen, forms a cylindrical condenser, the conductor forming one plate and the sheath the other, while the insulating material between them forms the dielectric. Since the areas of these plates are not the same, the flux densities and consequently the electric stresses are not the same throughout the dielectric, being greatest at the surface of the conductor.

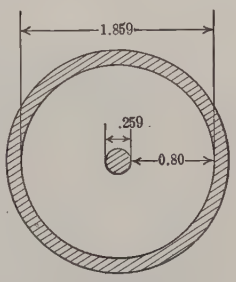


FIG. 466. A cable with a lead sheath.

Consider the stress distribution in the submarine cable shown in Fig. 466, if we put a voltage of 20,000 volts between conductor and sheath. The conductor is a No. 10 copper wire 0.259 cm in diameter, and there is 0.80 cm gutta percha insulation between it and the sheath. For convenience we will consider a 1-centimeter length of the cable.

The capacitance of a cable is given in Chapter XIII by the equation

$$C = \frac{3.88\,Kl}{10^2 \log_{10} \frac{D}{d}}$$

where C = capacitance of cable in microfarads.
K = relative dielectric constant.
l = length of cable in miles.
D = outside diameter of dielectric.
d = inside diameter of dielectric.

The capacitance of 1 centimeter of this cable in farads with the dielectric of gutta percha is

$$C = \frac{3.88 \times 4.1 \times 1}{10^2 \times 10^6 \times \log \frac{1.859}{0.259} \times 5280 \times 12 \times 2.54}$$
$$= 1.155 \times 10^{-12} \text{ farad.}$$

The charge which 20,000 volts would put on one centimeter of this cable is

$$Q = EC$$
$$= 20{,}000 \times 1.155 \times 10^{-12}$$
$$= 2.31 \times 10^{-8} \text{ coulombs.}$$

In statcoulombs this is

$$2.31 \times 10^{-8} \times 3 \times 10^9$$
$$= 69.3 \text{ statcoulombs.}$$

This means that in every centimeter of this cable there are $69.3 \times 4\pi$ or 870 electrostatic lines radiating from the conductor to the sheath, the field being of the shape shown in Fig. 467. Note that the field varies in density from the conductor to the sheath, being densest at the surface of the conductor and thinnest at the sheath.

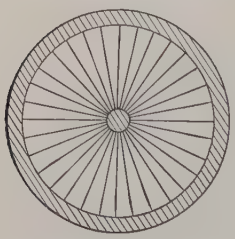


FIG. 467. The electrostatic flux distribution in the space between conductor and sheath of a cable.

The outside area of the dielectric for one centimeter of cable is

$$A_0 = 1 \times 3.14 \times 1.859$$
$$= 5.84 \text{ square centimeters.}$$

The flux density at the outside of the dielectric

$$D_0 = \frac{870}{5.84}$$
$$= 149 \text{ lines per square centimeter.}$$

The electric stress at the outer edge of the dielectric is

$$G_0 = \frac{D_0}{K}$$
$$= \frac{149}{4.1}$$
$$= 36.3 \text{ statvolts per centimeter}$$
$$= 10{,}900 \text{ volts per centimeter.}$$

This stress is very low for gutta percha and there would be no danger of the insulation being broken down if the stress through it were uniform.

But the area at the inner face of the dielectric is

$$A_i = 1 \times 3.14 \times 0.259$$
$$= 0.813 \text{ square centimeter.}$$

The flux density at the inner face is

$$D_i = \frac{870}{0.813}$$
$$= 1070 \text{ lines per square centimeter.}$$

The electric stress at this face is

$$G_i = \frac{1070}{4.1}$$
$$= 261 \text{ statvolts per centimeter}$$
$$= 78{,}300 \text{ volts per centimeter.}$$

This stress is about 56 per cent of the break-down value for gutta percha. It would be poor practice to use the material at such a high value of stress, not only because of the risk of breakdown, but also because of excessive losses in the insulation. Commercially, this cable would probably be rated at from 7000 to 8000 volts, instead of 20,000 volts.

The principle brought out by this example is that the electrostatic flux density and hence the electric stress may vary throughout an insulator, and should be computed for the point where it is greatest. For, if the insulation breaks down at one point it will throw the whole voltage on the rest of the insulation, which action may break down the whole structure. A cable with two dielectric substances is treated as two condensers in series.

Prob. 26–16. What is the maximum voltage stress in a cable, the conductor of which is No. 4 (B. & S.) copper wire, and the dielectric is a 0.6 inch layer of oiled paper with a relative dielectric constant of 3.6? The sheath is of lead, and the voltage between the conductor and sheath is 65,000 volts.

Prob. 27–16. What would be the maximum voltage stress on the conductor in Prob. 26–16 if the conductor were No. 0000 (B. & S.), the other data remaining as in Prob. 26–16?

Prob. 28–16. If the thickness of the insulation in Prob. 26 were doubled, what would be the maximum voltage stress?

Prob. 29–16. If the greatest voltage stress allowed in the insulation of Prob. 26–16 is 60,000 volts per centimeter, what voltage should be allowed between the conductor and the sheath?

305. The Construction of Insulators. In the construction of insulators for high voltage, great care must be exercised that no air pockets are left in the material. The porcelain part of the insulator, for instance, may stand an electric stress of 150,000 volts per centimeter, while air breaks down at 30,000 volts per centimeter. Thus, due to ionization of the molecules in an air pocket, there may be heat generated which will cause mechanical strain in the porcelain, and may even cause a crack. The presence of an air bubble also causes a change in the distribution of electrostatic flux, as shown in Fig. 468. Since the dielectric constant of the porcelain is higher than that of air, the flux lines tend to go through the porcelain rather than through the air. Hence at the edges of the bubble, the lines tend to concentrate, as shown at points A and B, Fig. 468. But since the electric stress is proportional to the density of the electrostatic field, the stress at A and B must be greater than the average throughout the body of the material. With large air bubbles, or bubbles having sharply rounded edges, the local stress may be sufficient to cause insulator failure.

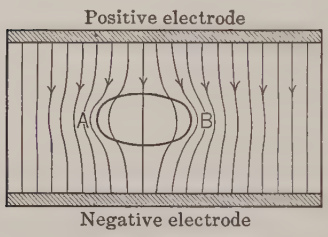


Fig. 468. An air bubble enclosed in a solid dielectric causes increased dielectric stress at the points A and B.

The proper design of an insulator brings in many factors. For instance, in designing an insulator for a transmission line, we must first construct it so that there is sufficient dielectric strength of the material to avoid puncture under the maximum stress which will occur. Second, it must have sufficient surface so that it will not break down between wire and ground over the surface of the material. This is one reason why insulators are constructed with a deeply corrugated surface or with petticoats, as shown in Fig. 469. Also, these petticoats must be of such shape that when it rains, there will be enough surface of the insulator left dry to stand the applied potential. In addition, of course, the insulator must be strong enough to stand the mechanical stresses which are brought to bear upon it, and these are often high when a heavy transmis-

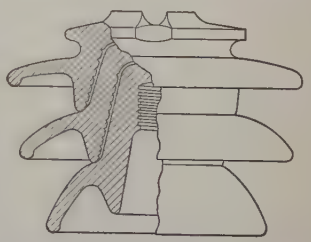


Fig. 469. An insulator of the pin type.

sion wire is being supported. Finally, there must be no point close to the insulator at which the electric stress is so great that it tears the electrons from the atoms and thus ionizes the air, and makes it a conductor of electricity. And, as we have said, there must be no bubbles of air in the material.

SUMMARY OF CHAPTER XVI

OHM'S LAW FOR THE ELECTROSTATIC CIRCUIT is

$$\psi = \frac{E}{S}$$

in which

ψ = lines of electrostatic force
E = voltage across dielectric in statvolts
S = elastance of dielectric in DARAFS.

The ELECTROSTATIC LINES (ψ) correspond to the magnetic lines (ϕ), and the elastance (S) to the reluctance ($\mathcal{R}$).

A DARAF is the elastance of a dielectric circuit in which one statvolt sets up one electrostatic line.

4π ELECTROSTATIC LINES are drawn to each statcoulomb.

THE ELASTANCE OF A DIELECTRIC equals

$$S = \frac{1}{A} \times \frac{1}{K} \text{ darafs}$$

in which $\frac{1}{K}$ is the elastance of a centimeter cube of the material. It is the reciprocal of the value of K in Table V.

The ELASTANCE of dielectrics in series is the sum of the elastances of the separate dielectrics.

The electrostatic flux density is

$$D = \frac{\psi}{A} \text{ lines per square centimeter.}$$

The dielectric stress is

$$G = \frac{D}{K} \text{ statvolts per centimeter.}$$

The DISTRIBUTION OF STRESSES IN INSULATION can often be computed from the above equation. In a cable it is greatest at the surface of the conductor.

1 statvolt = 300 volts.
1 coulomb = 3×10^9 statcoulombs.

PROBLEMS ON CHAPTER XVI

Prob. 30–16. Three condensers having capacitances of 4, 8, and 12 microfarads, respectively, are connected in series across a 600-volt d-c generator. How many electrostatic lines of flux are there in each condenser?

Prob. 31–16. What is the elastance of the series combination of condensers in Prob. 30–16?

Prob. 32–16. Two parallel discs, each 5 centimeters in diameter form an air gap across which a potential of 500 volts is applied. If 120 electrostatic flux lines are thus produced, what is the spacing of the plates? Assume a uniform distribution of flux with no fringing.

Prob. 33–16. A cable is made up of a No. 6 B. & S. copper conductor which is covered with a $\frac{1}{4}$ inch layer of gutta percha. The gutta percha is covered with a layer of oiled paper $\frac{3}{8}$ inch thick, with a dielectric constant of 3.2. The outer covering is a lead sheath. If the maximum stress allowed in the gutta percha is 80,000 volts per inch, and in oiled paper, 150,000 volts per inch, for what voltage can the cable be used?

Prob. 34–16. If the positions of the dielectrics in Prob. 33–16 are reversed, for what voltage can the cable be used? The thickness of each dielectric is to remain unchanged.

Prob. 35–16. What is the capacitance to sheath per mile of a rubber-covered single-conductor cable having a wire of No. 0000 B. & S. and insulation $\frac{1}{2}$ inch thick? $K = 3.5$.

Prob. 36–16. If impregnated paper is substituted for the rubber in the cable of Prob. 35–16, what will be the capacitance per mile? The dielectric constant of impregnated paper may be taken as 2.8.

Prob. 37–16. If a 1000-volt potential is applied to the cable of Prob. 35–16, plot the voltage stress throughout the insulation.

Prob. 38–16. How thick should the insulation be made in Prob. 35–16 if the maximum voltage stress in the insulation is to be 75,000 volts per inch and voltage between conductor and sheath is 20,000 volts?

Prob. 39–16. How thick should be the paper insulation of the cable in Prob. 36–16 if the maximum voltage stress in the paper is to be 60,000 volts per centimeter and the voltage between conductor and sheath is 30,000 volts?

APPENDIX

SIMPLE TRIGONOMETRIC FUNCTIONS

From Jameson's " Mechanics," John Wiley & Sons, Inc.

Let ABC, Fig. 1*a*, be any given angle, and let P be **any** point on the line BC. From P draw PN perpendicular to the side BA, thus forming the right-angled triangle PNB. Suppose we measure BP and find it to be 10 inches long and PN and find it to be 5.05 inches long. Then

$$\frac{PN}{BP} = \frac{5.05}{10} = 0.505.$$

Now let us choose another point on BC, as P', draw the perpendicular $P'N'$ and measure lines BP' and $P'N'$. If BP' is $\frac{4}{5}$ as long as BP, or 8 inches, then $P'N'$ will be $\frac{4}{5}$ as long as PN, or $5.05 \times \frac{4}{5} = 4.04$ inches. The ratio $\frac{P'N'}{BP'} = \frac{4.04}{8} = 0.505$, or the same as that of PN to BP. So we might choose any point on BC and always obtain the same ratio of the perpendicular to the hypothenuse as long as we use the same angle ABC.

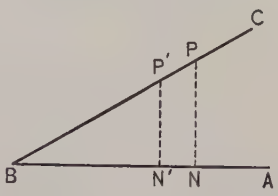


FIG. 1*a*. Triangles BPN and $BP'N'$ are similar.

Similarly, $\frac{BN}{BP}$ gives a second constant ratio for the given angle, and $\frac{PN}{BN}$ a third, all of which are independent of the position of P and dependent only on the angle ABC.

If the angle ABC be changed, then the ratios $\frac{PN}{BP}$, etc., will have new values, but these again will be the same, no matter where P is taken on the line BC. Each angle thus has a certain number of constant ratios among which are the three here given, and these ratios are given distinguishing names.

The ratio $\frac{PN}{BP}$ is called the **sine** of the angle ABC.

The ratio $\frac{BN}{BP}$ is called the **cosine** of the angle ABC.

The ratio $\frac{PN}{BN}$ is called the **tangent** of the angle ABC.

Definitions. Let B be an angle (not the right angle) of a right-angled triangle. The **sine** of the angle B is the ratio of the side **opposite** the angle to the hypothenuse of the triangle.

The **cosine** of the angle B is the ratio of the side **adjacent** the angle to the hypothenuse of the triangle.

The **tangent** of the angle B is the ratio of the side **opposite** to the side **adjacent.**

In calculations, sine, cosine, and tangent are always written for brevity, sin, cos, tan. Thus, sin $30° = 0.500$; cos $45° = 0.707$; tan $50° = 1.19$.

The values of these ratios have been calculated for all angles, and are given in what are called tables of trigonometric functions. Such tables, with the values carried out to three decimals, will be found on the following pages.

Much time can be saved by the use of a conversion chart* which will give the direct products or quotients of numbers and trigonometric functions.

* Woodruff's Polar-Rectangular Conversion Chart. John Wiley & Sons, Inc.

TABLE I

RESISTIVITY, TEMPERATURE COEFFICIENT, AND RECIPROCAL OF TEMPERATURE COEFFICIENT

Material	Resistivity in ohms per mil-foot at 20° C.	Temperature coefficient per degree C. per ohm at 0° C.	Reciprocal of temperature coefficient at 0° C.
Aluminum	17.0	0.00420	238.1
Antimony	251	0.00388	257.7
Bismuth	718	0.00435	230.0
Brass (annealed)	42	0.00208	480.8
Constantan	295	*	.....
Copper			
Annealed Intnl. Standard	10.4	0.00426	234.5
Hard drawn	10.7	0.00413	242.1
Pure	10.2	0.00410	244.0
German Silver	199	0.00036	2780
Gold	14.7	0.00365	274.0
Iron, Commercial	66.4 to 81.4	0.00618	161.8
Hard cast	590	.....	.....
Lead	132.5	0.00466	214.6
Manganin	265	*	.....
Monel Metal	253	0.00208	480.8
Mercury	577	0.00088	1136
Nickel	47	0.006	167
Platinum	60.2	0.0037	270
Platinum-iridium	148	0.0012	833
Silver	9.89 to 11.2	0.00411	243.3
Steel, Hard	275	0.0016	625
Soft	95.7	0.00458	218.3
Rail	83.5 to 130	.....	.....
Tantalum	93.3	0.0033	303
Tin	69.2	0.00458	218.3
Tungsten	33.2	0.0049	204
Zinc	37.4	0.0040	250
Advance Metal	294	*	.....
Ialia	301	−0.00003	−33,300
Superior	520	0.00082	1220
Nichrome	600	0.00044	227
Nichrome II	660	0.00016	6250
Calorite	720	.....	.....

* Nearly zero.

TABLE II

Resistance of International Standard Annealed Copper

American Wire Gauge (B. & S.)

B. & S. gauge, No.	Diameter in mils, d	Area in circular mils, d^2	Ohms per 1000 ft. at 20° C. or 68° F.	Pounds per 1000 ft.	B. & S. gauge, No.	Diameter in mils, d	Area in circular mils, d^2	Ohms per 1000 ft. at 20° C. or 68° F.	Pounds per 1000 ft.
0000	460.00	211,600	0.04901	640.5	21	28.462	810.10	12.80	2.452
000	409.64	167,810	0.06180	508.0	22	25.347	642.40	16.14	1.945
00	364.80	133,080	0.07793	402.8	23	22.571	509.45	20.36	1.542
0	324.95	105,530	0.09827	319.5	24	20.100	404.01	25.67	1.223
					25	17.900	320.40	32.37	0.9699
1	289.30	83,694	0.1239	253.3	26	15.940	254.10	40.81	0.7692
2	257.63	66,373	0.1563	200.9	27	14.195	201.50	51.47	0.6100
3	229.42	52,634	0.1970	159.3	28	12.641	159.79	64.90	0.4837
4	204.31	41,742	0.2485	126.4	29	11.257	126.72	81.83	0.3836
5	181.94	33,102	0.3133	100.2	30	10.025	100.50	103.2	0.3042
6	162.02	26,250	0.3951	79.46	31	8.928	79.70	130.1	0.2413
7	144.28	20,816	0.4982	63.02	32	7.950	63.21	164.1	0.1913
8	128.49	16,509	0.6282	49.98	33	7.080	50.13	206.9	0.1517
9	114.43	13,094	0.7921	39.63	34	6.305	39.75	260.9	0.1203
10	101.89	10,381	0.9989	31.43	35	5.615	31.52	329.0	0.0954
11	90.742	8,234.0	1.260	24.93	36	5.000	25.00	414.8	0.0757
12	80.808	6,529.9	1.588	19.77	37	4.453	19.83	523.1	0.0600
13	71.961	5,178.4	2.003	15.68	38	3.965	15.72	659.6	0.0476
14	64.084	4,106.8	2.525	12.43	39	3.531	12.47	831.8	0.0377
15	57.068	3,256.7	3.184	9.858	40	3.145	9.89	1049	0.0299
16	50.820	2,582.9	4.016	7.818					
17	45.257	2,048.2	5.064	6.200					
18	40.303	1,624.3	6.385	4.917					
19	35.890	1,288.1	8.051	3.899					
20	31.961	1,021.5	10.15	3.092					

TABLE III

Allowable Carrying Capacities of Wires

National Electrical Code, 1936

The following table is for copper wires of ninety-eight per cent conductivity, and must be followed in placing interior conductors.

For insulated aluminum wire, the safe carrying capacity is *eighty-four per cent* of that given for copper wire with the same kind of insulation.

B. & S. gauge	Diameter of solid wires in mils	Area in circular mils	Rubber insulation. Amperes	Varnished cloth insulation. Amperes	Other insulation.* Amperes
18	40.3	1,624	3		6
16	50.8	2,583	6		10
14	64.1	4,107	15	18	20
12	80.8	6,530	20	25	30
10	101.9	10,380	25	30	35
8	128.5	16,510	35	40	50
6	162.0	26,250	50	60	70
5	181.9	33,100	55	65	80
4	204.3	41,740	70	85	90
3	229.4	52,630	80	95	100
2	257.6	66,370	90	110	125
1	289.3	83,690	100	120	150
0	325	105,500	125	150	200
00	364.8	133,100	150	180	225
000	409.6	167,800	175	210	275
		200,000	200	240	300
0000	460	211,600	225	270	325
		250,000	250	300	350
		300,000	275	330	400
		350,000	300	360	450
		400,000	325	390	500
		500,000	400	480	600
		600,000	450	540	680
		700,000	500	600	760
		750,000	525	630	800
		800,000	550	660	840
		900,000	600	720	920
		1,000,000	650	780	1000
		1,100,000	690	830	1080
		1,200,000	730	880	1150
		1,300,000	770	920	1220
		1,400,000	810	970	1290
		1,500,000	850	1020	1360
		1,600,000	890	1070	1430
		1,700,000	930	1120	1490
		1,800,000	970	1160	1550
		1,900,000	1010	1210	1610
		2,000,000	1050	1260	1670

* Includes bare conductors.

TABLE IV

Trigonometric Functions

A	Sin A	Cos A	Tan A	A	Sin A	Cos A	Tan A
0	.000	1.000	.000				
1	.018	1.000	.018	46	.719	.695	1.04
2	.035	.999	.035	47	.731	.682	1.07
3	.052	.999	.052	48	.743	.669	1.11
4	.070	.998	.070	49	.755	.656	1.15
5	.087	.996	.088	50	.766	.643	1.19
6	.104	.994	.105	51	.777	.629	1.23
7	.122	.992	.123	52	.788	.616	1.28
8	.139	.990	.140	53	.799	.602	1.33
9	.156	.988	.158	54	.809	.588	1.38
10	.174	.985	.176	55	.819	.574	1.43
11	.191	.982	.194	56	.829	.559	1.48
12	.208	.978	.213	57	.839	.545	1.54
13	.225	.974	.231	58	.848	.530	1.60
14	.242	.970	.249	59	.857	.515	1.66
15	.259	.966	.268	60	.866	.500	1.73
16	.276	.961	.287	61	.875	.485	1.80
17	.292	.956	.306	62	.883	.470	1.88
18	.309	.951	.325	63	.891	.454	1.96
19	.326	.946	.344	64	.899	.438	2.05
20	.342	.940	.364	65	.906	.423	2.14
21	.358	.934	.384	66	.914	.407	2.25
22	.375	.927	.404	67	.920	.391	2.36
23	.391	.920	.424	68	.927	.375	2.48
24	.407	.914	.445	69	.934	.358	2.60
25	.423	.906	.466	70	.940	.342	2.75
26	.438	.899	.488	71	.946	.326	2.90
27	.454	.891	.510	72	.951	.309	3.08
28	.470	.883	.532	73	.956	.292	3.27
29	.485	.875	.554	74	.961	.276	3.49
30	.500	.866	.577	75	.966	.259	3.73
31	.515	.857	.601	76	.970	.242	4.01
32	.530	.848	.625	77	.974	.225	4.33
33	.545	.839	.649	78	.978	.208	4.70
34	.559	.829	.675	79	.982	.191	5.14
35	.574	.819	.700	80	.985	.174	5.67
36	.588	.809	.726	81	.988	.156	6.31
37	.602	.799	.754	82	.990	.139	7.12
38	.616	.788	.781	83	.992	.122	8.14
39	.629	.777	.810	84	.994	.104	9.51
40	.643	.766	.839	85	.996	.087	11.43
41	.656	.755	.869	86	.998	.070	14.30
42	.669	.743	.900	87	.999	.052	19.08
43	.682	.731	.932	88	.999	.035	28.64
44	.695	.719	.966	89	1.000	.018	57.29
45	.707	.707	1.000	90	1.000	.000	Infinity

TABLE V

DIELECTRIC CONSTANTS (AVERAGE VALUES)

Material	Dielectric Constant	Material	Dielectric Constant
Air	1	Acetone	26.6
Ebonite	2.7	Alcohol (0° C.)	
Glass		amyl	17.4
flint	9.9	ethyl	28.4
hard crown	7.0	methyl	35
lead	6.6	Ammonia	22
Gutta percha	4.1	Benzene	2.3
Mica	5.8	Glycerine	56.2
Paraffin	2.1	Petroleum	2.1
Shellac	3.1	Water (pure)	81

TABLE VI

DIELECTRIC STRENGTH (AVERAGE VALUES)

Material	Dielectric Strength in Volts per Centimeter	Material	Dielectric Strength in Volts per Centimeter
Air	30,000	Gutta percha	140,000
Bakelite	210,000	Mica	500,000
Ebonite	700,000	Paraffin	290,000
Glass	350,000	Transil oil	100,000

TABLE VII

Useful Numbers

$\pi = 3.1416 = \dfrac{\text{circumference}}{\text{diameter}}$.

$\pi^2 = 9.8696$; $\dfrac{1}{\pi} = .3183$.

Area of circle $= \pi r^2 = \dfrac{\pi d^2}{4} = .7854 d^2$.

Surface of cyl. $= 2\pi rl + 2\pi r^2$.

Volume of cyl. $= \pi r^2 l$.

Surface of sphere $= 4\pi r^2$.

Volume of sphere $= \dfrac{\pi d^3}{6} = \dfrac{4\pi r^3}{3}$.

Metric-English Equivalents

1 cm	=	.39 in	1 in	=	2.54 cm
1 m	=	39.37 in	1 ft	=	30.48 cm
1 m	=	3.28 ft	1 ft	=	.305 m
1 km	=	.62 mile	1 mile	=	1.61 km
1 g	=	.035 oz (avoir.)	1 oz	=	28.35 g
1 kg	=	2.205 lb (avoir.)	1 lb	=	453.6 g
1 sq cm	=	.155 sq in	1 sq in	=	6.45 sq cm
1 cc	=	.061 cu in	1 cu in	=	16.39 cc

Units of Force, Work, Power, etc.

1 dyne = .00102 gram
1 ft-lb = 1.356×10^7 ergs.
1 joule = 10^7 ergs.
1 horsepower = 33,000 ft-lb/min.
1 horsepower = 550 ft-lb/sec
1 horsepower = 7.46×10^9 ergs/sec.
1 horsepower = 746 watts.
1 watt = .00134 horsepower.
1 watt = 10^7 ergs/sec = 1 joule/sec.

Mechanical Equivalents of Heat

1 g of water heated 1° C. = 4.2×10^7 ergs.
1 lb of water heated 1° C. = 1400 ft-lb.
1 lb of water heated 1° F. = 780 ft-lb.

The combustion of 1 lb of coal produces about 14,000 Btu.

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